

2.2.5 Uniform Color Spaces

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Abstract: In 1976, the CIE specified CIELAB and CIELUV, two perceptually uniform color spaces to estimate the magnitude of the difference between two color stimuli. These color spaces were also designed to provide color-difference equations and to interpret color difference in terms of dimensions in lightness, hue, and chroma. The CIELAB and CIELUV color-difference equations have been widely used in industry. However, they do not accurately quantify small- to medium-size color difference. More advanced equations based upon the modification of the CIELAB color-difference equation were developed. The CIEDE2000 color-difference equation is the current CIE recommendation for computing small color differences. Future research in the area of the color difference may be based upon a more uniform color space, based on color-vision theory and be capable of accounting for different viewing parameters.

1 Introduction

The CIE (Commission internationale de l'éclairage or International Commission on Illumination) 1931 XYZ system has been effective to measure the luminance and chrominance of a color (see [Chap. 2.2.2](#)). However, the distribution of colors in its x, y chromaticity diagram is very non-uniform. This leads to the problem of change in chromaticity or luminosity and visual perception are not linearly related. Equal changes in x, y or Y do not correspond to perceived differences of equal magnitude. In 1976, the CIE suggested two color spaces, CIELAB and CIELUV, as a way to overcome the limitations of the CIE XYZ system [1]. The visual magnitudes of color differences are intended to be approximately proportional to the distance in these spaces.

2 The CIELAB and CIELUV Spaces

CIELAB and CIELUV were primarily suggested to provide color-difference equations but they were also designed to express color differences correlated with perception attributes: hue, lightness, and chroma. These spaces are represented by plotting the three attributes along axes at right angles to one another.

Both CIELAB (concerned with subtractive mixture, e.g., surface colorant) and CIELUV (for additive mixture of colored light, e.g., television) color spaces have the same lightness scales L^* , which is defined in terms of the ratio of the Y tristimulus value of the color considered to that of the reference white Y_n as follows:

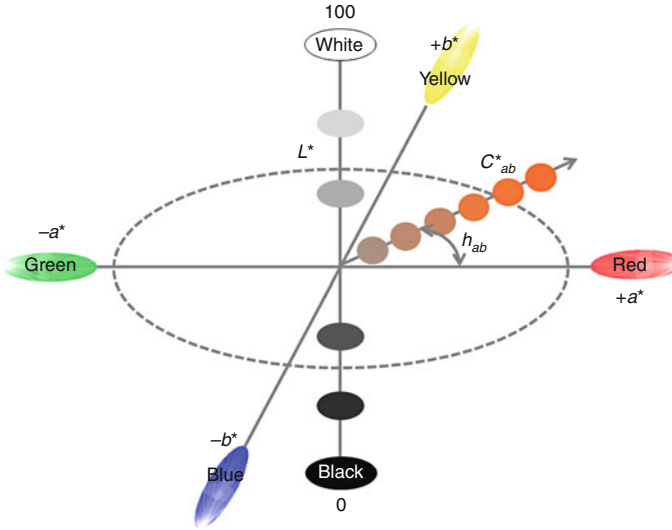
$$\begin{aligned} L^* &= 116(Y/Y_n)^{1/3} - 16 && \text{for } Y/Y_n > 0.008856 \\ L^* &= 903.3(Y/Y_n) && \text{for } Y/Y_n \leq 0.008856 \end{aligned} \quad (1)$$

The opponent color axes, approximately red-green (a^*) versus yellow-blue (b^*) for the CIELAB color space (as shown in [Fig. 1](#)) are defined in [Eq. 2](#). Chroma (C_{ab}^*) and hue (h_{ab}) are calculated using a^* and b^* as [Eqs. 3](#) and [4](#) respectively.

$$\begin{aligned} a^* &= 500[f(X/X_n) - f(Y/Y_n)] \\ b^* &= 200[f(Y/Y_n) - f(Z/Z_n)] \end{aligned} \quad (2)$$

where

$$\begin{aligned} f(I) &= I^{1/3} && \text{for } I > 0.008856 \\ f(I) &= 7.787I + 16/116 && \text{for } I \leq 0.008856 \end{aligned}$$



■ Fig. 1
A schematic representation of the CIELAB color space

$$C_{ab}^* = (a^{*2} + b^{*2})^{1/2} \quad (3)$$

$$h_{ab} = \arctan(b^*/a^*) \quad (4)$$

Similarly, the CIELUV color space contains the opponent color axes, approximately red-green (u^*) versus yellow-blue (v^*), which are defined in ► Eq. 5.

$$\begin{aligned} u^* &= 13L^*(u' - u'_n) \\ v^* &= 13L^*(v' - v'_n) \end{aligned} \quad (5)$$

Saturation (s_{uv}), Chroma (C_{uv}^*), which can also be correlated for saturation in the CIELUV space and hue (h_{uv}) are calculated respectively in ► Eqs. 6–8.

$$s_{uv} = 13[(u' - u'_n)^2 + (v' - v'_n)^2]^{1/2} \quad (6)$$

$$C_{uv}^* = (u^{*2} + v^{*2})^{1/2} = L^* s_{uv} \quad (7)$$

$$h_{uv} = \arctan(v^*/u^*) \quad (8)$$

3 Applications to Colorimetry

The CIE system offers a precise means of specifying a color stimulus under a set of viewing conditions and suggested CIELAB and CIELUV uniform color spaces as useful representations of colors that correlate with perceptual attributes [1]. Another important use of the CIE system is the evaluation of perceived color difference. Color-difference equations are designed to provide quantitative representations of the perceived color differences between pairs of colored

samples. Color differences specified by CIELAB (Eq. 9) and CIELUV (Eq. 10) spaces are measured by the Euclidean distance between the co-ordinates for two stimuli. One unit represents approximately one just-noticeable difference (JND) for a pair of samples viewed side by side [2].

$$\Delta E_{ab}^* = (\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2})^{1/2} \text{ or } (\Delta L^{*2} + \Delta H_{ab}^{*2} + \Delta C_{ab}^{*2})^{1/2} \quad (9)$$

where

$$\Delta H_{ab}^* = [(\Delta E_{ab}^*)^2 - (\Delta L^*)^2 - (\Delta C_{ab}^*)^2]^{1/2}$$

$$\Delta E_{uv}^* = (\Delta L^{*2} + \Delta u^{*2} + \Delta v^{*2})^{1/2} \text{ or } (\Delta L^{*2} + \Delta H_{uv}^{*2} + \Delta C_{uv}^{*2})^{1/2} \quad (10)$$

where

$$\Delta H_{uv}^* = [(\Delta E_{uv}^*)^2 - (\Delta L^*)^2 - (\Delta C_{uv}^*)^2]^{1/2}$$

4 Optimized Color-Difference Equations

CIELAB and CIELUV were derived from non-linear transformations of the CIE XYZ system and have been widely used in color industries. The two CIE recommended equations, however, do not accurately quantify small- to medium-size color difference [3]. Many attempts have been made to modify the CIELAB color-difference equation to develop more advanced equations including JPC79 [4], CMC(*l:c*) [5], BFD(*l:c*) [6, 7], CIE94 [8], and CIEDE2000 [9].

4.1 JPC79 and CMC(*l:c*) color-difference equations

McDonald accumulated a large number of data involving polyester thread pairs and carried out visual pass/fail color-matching assessments [4]. The visual results were used to derive the JPC79 equation. At a later stage the JPC79 equation was modified, due to the problem that some anomalies were found for colors close to neutral and black [10], and renamed as the CMC(*l:c*) equation.

$$\Delta E_{\text{CMC}(l:c)} = \left[(\Delta L_{ab}^*/(S_L))^2 + (\Delta C_{ab}^*/(cS_C))^2 + (\Delta H_{ab}^*/S_H)^2 \right]^{1/2} \quad (11)$$

where

$$S_L = 0.040975 L_{ab, std}^* / (1 + 0.01765 L_{ab, std}^*) \quad \text{if } L_S^* \geq 16$$

otherwise

$$S_L = 0.511$$

and

$$S_C = 0.0638 C_{ab, std}^* / (1 + 0.0131 C_{ab, std}^*) + 0.638,$$

$$S_H = S_C(Tf + 1 - f).$$

The terms T and f are given by

$$f = [(C_{ab, std}^*)^4 / ((C_{ab, std}^*)^4 + 1900)]^{1/2}$$

and

$$T = 0.36 + |0.4 \cos(h_{ab, std} + 35)| \text{ if } h_{ab, std} \leq 164 \text{ or } h_{ab, std} \geq 345$$

Otherwise

$$T = 0.56 + |0.2 \cos(h_{ab, std} + 168)|.$$

The CMC($l:c$) equation is based upon the CIELAB color space and the terms L^* , C_{ab}^* , and H_{ab}^* are corresponded to the CIELAB lightness, chroma, and hue respectively. The terms S_L , S_C , and S_H define the lengths of the semi-axes of the tolerance ellipsoid at the position of the standard in CIELAB space in each of the lightness (S_L), chroma (S_C), and hue (S_H) directions. The ellipsoids were fitted to visual tolerances determined from psychophysical experiments and the dimension of the ellipsoid is a function of the position of the standard in the color space.

The parametric terms l and c allow the ratio between lightness and chroma components to be adjusted. It is considered that there is greater acceptance for shifts in lightness dimension than in chromatic (chroma and hue) dimension. For predicting the perceptibility of color differences, it was recommended that both l and c equal to 1 whereas for predicting acceptability of color differences it was recommended that l and c equals to 2 and 1 respectively. The subscript *std* refers to the standard of a pair of samples.

The CMC($l:c$) equation has been widely used in a number of industries and became an ISO standard for the textile industry in 1995 [9]. It was also adopted as a British standard (BS 6923) and an AATCC test method (AATCC 173).

4.2 BFD($l:c$) color-difference equation

Luo and Rigg [6, 7, 11] accumulated a large set of experimental data relating to small and medium color differences between pairs of surface colors and developed the BFD($l:c$) color-difference equation. The structure of the BFD($l:c$) equation is similar to that of the CMC($l:c$) equation. However, it was found that an additional term in the BFD($l:c$) equation is considered which take into account the fact that the chromaticity ellipses do not all point toward the neutral point as assumed in the CMC($l:c$) equation. The effect is most significant in the blue region.

$$\Delta E_{BFD} = [(\Delta L_{BFD}/l)^2 + (\Delta C_{ab}^*/(cD_C))^2 + (\Delta H_{ab}^*/D_H)^2 + R_T(\Delta C_{ab}^* \Delta H_{ab}^*/D_C D_H)]^{1/2} \quad (12)$$

where

$$L_{BFD} = 54.6 \log(Y + 1.5) - 9.6$$

$$D_C = 0.035 \overline{C_{ab}^*} / (1 + 0.00365 \overline{C_{ab}^*}) + 0.521$$

$$D_H = D_C(GT' + 1 - G)$$

$$G = [\overline{C_{ab}^*}^4 / (\overline{C_{ab}^*}^4 + 14000)]^{1/2}$$

$$T' = 0.627 + 0.055 \cos(\overline{h_{ab}} - 254^\circ) - 0.040 \cos(2\overline{h_{ab}} - 136^\circ) + 0.070 \cos(3\overline{h_{ab}} - 32^\circ) \\ + 0.049 \cos(4\overline{h_{ab}} - 114^\circ) - 0.015 \cos(5\overline{h_{ab}} - 103^\circ) R_T = R_C R_H$$

$$R_H = -0.260 + 0.055 \cos(\overline{h_{ab}} - 308^\circ) - 0.379 \cos(2\overline{h_{ab}} - 160^\circ) - 0.636 \cos(3\overline{h_{ab}} - 254^\circ) \\ + 0.226 \cos(4\overline{h_{ab}} - 140^\circ) - 0.194 \cos(5\overline{h_{ab}} - 280^\circ)$$

$$R_C = [\overline{C_{ab}^*}^6 / (\overline{C_{ab}^*}^6 + 7 \times 10^7)]^{1/2}$$

The terms $\overline{C_{ab}^*}$ and $\overline{h_{ab}}$ refer to the arithmetic mean values of chroma and hue angle respectively. Both l and c equal to 1 for predicting perceptibility of color differences and l and c , respectively, equals to 1.5 and 1 for predicting acceptability of color differences.

4.3 CIE94 color-difference equation

Berns suggested a color-difference equation derived also by modifying the CIELAB equation [12]. The equation was later recommended by the CIE in 1994 [8] and is named as CIE94 color-difference equation. It has a similar structure to that of the CMC($l:c$) equation but with simpler weighting functions. The CIE94 formula is given by

$$\Delta E_{94}^* = [(\Delta L^* / (K_L S_L))^2 + (\Delta C_{ab}^* / (K_C S_C))^2 + (\Delta H_{ab}^* / (K_H S_H))^2]^{1/2} \quad (13)$$

where

$$S_L = 1,$$

$$S_C = 1 + 0.045 C_{ab, std}^*,$$

$$S_H = 1 + 0.015 C_{ab, std}^*.$$

The parametric factors K_L , K_C , and K_H are included to correct for the variation in experimental conditions. For all applications except for textile industry, a value of 1 is recommended for all the parametric factors. For the textile industry CIE94(2:1:1) is recommended where K_L equals to 2, and K_C and K_H equal to 1.

4.4 CIEDE2000 color-difference equation

It has been realized that both the CMC($l:c$) and CIE94 color-difference equations are standardized but by different organizations: CMC($l:c$) by ISO [13] and CIE94 by CIE [8]. However, there are large discrepancies between the two equations in predicting lightness differences, and have problems predicting grayish and bluish colors [3]. The value of the function in the CMC($l:c$) equation increases markedly as L^* increases, implying that for equal differences in L^* the visual difference should be largest for the smaller L^* values. However, the lightness correction in the CIE94 equation implied that equal differences in L^* would yield equal visual differences for all values of L^* .

The CIE subsequently formed a Technical Committee (TC) 1-47 to develop a generalized color-difference equation. The CIEDE2000 equation was agreed by the CIE [14] and includes not only lightness, chroma, and hue weighting functions, but also an interactive term between chroma and hue differences for improving the performance for blue colors and a scaling factor for the CIELAB a^* scale for improving the performance for colors close to the achromatic axis. The equation is given by:

Step 1: calculate the CIELAB L^* , a^* and b^* values

Step 2: calculate a' , C'_{ab} and h'_{ab}

$$\begin{aligned} L' &= L^*, \\ a' &= (1 + G)a^*, \\ b' &= b^*, \\ C'_{ab} &= (a'^2 + b'^2)^{1/2}, \end{aligned}$$

and

$$h'_{ab} = \arctan(b'/a').$$

where

$$G = 0.5 - 0.5[\overline{C_{ab}^*} / (\overline{C_{ab}^*} + 25^7)]^{1/2}$$

Step 3: calculate $\Delta L'$, $\Delta C'$, and $\Delta H'$

$$\begin{aligned} \Delta L' &= L'_{batch} - L'_{std} \\ \Delta C'_{ab} &= C'_{ab,batch} - C'_{ab,std} \\ \Delta H'_{ab} &= 2(C'_{ab,batch} C'_{ab,std})^{0.5} \sin(\Delta h'_{ab}/2) \end{aligned}$$

where

$$\Delta h'_{ab} = h'_{ab,batch} - h'_{ab,std}$$

Step 4: calculate CIEDE2000 ΔE_{00}

$$\begin{aligned} \Delta E_{00} &= [(\Delta L' / (k_L S_L))^2 + (\Delta C'_{ab} / (k_C S_C))^2 + (\Delta H'_{ab} / (k_H S_H))^2 \\ &\quad + R_T(\Delta C'_{ab} / (k_C S_C))(\Delta H'_{ab} / (k_H S_H))]^{1/2} \end{aligned} \quad (14)$$

where

$$S_L = 1 + [0.015(\overline{L'} - 50)^2] / [20 + (\overline{L'} - 50)^2]^{1/2},$$

$$S_C = 1 + 0.045 \overline{C_{ab}^*},$$

and

$$S_H = 1 + 0.015 \overline{C_{ab}^*} T.$$

where

$$T = 1 - 0.17 \cos(\overline{h'_{ab}} - 30^\circ) + 0.24 \cos(2\overline{h'_{ab}}) + 0.32 \cos(3\overline{h'_{ab}} + 6^\circ) - 0.20 \cos(4\overline{h'_{ab}} - 63^\circ).$$

and

$$R_T = -\sin(2\Delta\theta)R_C$$

where

$$\Delta\theta = 30 \exp \left\{ -\left[\frac{\overline{H'_{ab}} - 275^\circ}{25} \right]^2 \right\}$$

and

$$R_C = 2 \left(\frac{\overline{C_{ab}^*}}{\overline{C_{ab}^*} + 25^7} \right)^{1/2}$$

Note that $\overline{L'}$, $\overline{C_{ab}^*}$, and $\overline{H'_{ab}}$ are the arithmetic mean of the L' , C'_{ab} , and H'_{ab} values of a pair of samples. Caution needs to be taken for neutral colors having hue angles in different quadrants. If the difference is less than 180° , arithmetic mean of the samples should be used; 360° should be subtracted from the larger angle, followed by calculating the arithmetic mean, if otherwise.

The CIEDE2000 equation has been shown to perform better than the CMC($l:c$) and CIE94 equations [9, 15, 16] and it is the current CIE recommendation for computing small color differences.

5 Summary

The need for a uniform color space resulted in the specification of the CIELAB and CIELUV color spaces, as a way to represent colors that correlate with perceptual attributes. Another important use of the CIE system is the evaluation of perceived color difference between pairs of color stimuli. Color differences specified by CIELAB and CIELUV spaces are widely adopted by industry. More advanced color-difference equations were developed based upon the modification of the CIELAB color-difference equation to better quantify small- to medium-size color difference. Note, however, that all these color-difference equations were derived based on the perception of spatially uniform color patches. CIE colorimetry only considers color matching between two stimuli under identical conditions including surround, background, size, shape, texture, and illuminating/viewing geometry. Color matches defined by CIE may no longer be valid if any of these constraints is violated. Future research in the area of color difference may be based upon a more uniform color space rather than modifications of CIELAB. Instead of the empirical approach, it is expected that formulas may be based on color-vision theory and be capable of accounting for different viewing parameters such as sample size, size of color difference, spatial separation, background, and luminance level [17].

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