Building Fuzzy Inference Systems with a New Interval Type-2 Fuzzy Logic Toolbox

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Abstract. This paper presents the development and design of a graphical user interface and a command line programming Toolbox for construction, edition and simulation of Interval Type-2 Fuzzy Inference Systems. The Interval Type-2 Fuzzy Logic System (IT2FLS) Toolbox, is an environment for interval type-2 fuzzy logic inference system development. Tools that cover the different phases of the fuzzy system design process, from the initial description phase, to the final implementation phase, constitute the Toolbox. The Toolbox's best qualities are the capacity to develop complex systems and the flexibility that allows the user to extend the availability of functions for working with the use of type-2 fuzzy operators, linguistic variables, interval type-2 membership functions, defuzzification methods and the evaluation of Interval Type-2 Fuzzy Inference Systems.

Keywords: Interval Type-2 Fuzzy Inference Systems, Interval Type-2 Fuzzy Logic Toolbox, Interval Type-2 Membership Functions, Footprint of Uncertainty.

1 Introduction

Over the past decade, fuzzy systems have displaced conventional technologies in different scientific and system engineering applications, especially in pattern recognition and control systems. The same fuzzy technology, in approximation reasoning form, is resurging also in the information technology, where it is now giving support to decision-making and expert systems with powerful reasoning capacity and a limited quantity of rules. The fuzzy sets were presented by L.A. Zadeh in 1965 [1-3] to process or manipulate data and information affected by unprobabilistic uncertainty/imprecision. These were designed to mathematically represent the vagueness and uncertainty of linguistic problems; thereby obtaining formal tools to work with intrinsic imprecision in different type of problems; it is considered a generalization of the classic set theory. Intelligent Systems based on fuzzy logic are fundamental tools for nonlinear complex system modeling. Fuzzy sets and fuzzy logic are the base for fuzzy systems, where the objective has been to model how the brain manipulates inexact information. Type-2 fuzzy sets are used for

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modeling uncertainty and imprecision in a better way. These type-2 fuzzy sets were originally presented by Zadeh in 1975 and are essentially "fuzzy fuzzy" sets where the fuzzy degree of membership is a type-1 fuzzy set [4,6]. The new concepts were introduced by Mendel and Liang [8,9] allowing the characterization of a type-2 fuzzy set with a inferior membership function and an superior membership function; these two functions can be represented each one by a type-1 fuzzy set membership function. The interval between these two functions represents the footprint of uncertainty (FOU), which is used to characterize a type-2 fuzzy set. The uncertainty is the imperfection of knowledge about the natural process or natural state. The statistical uncertainty is the randomness or error that comes from different sources as we use it in a statistical methodology. Type-2 fuzzy sets have been applied to a wide variety of problems by Castillo and Melin [13].

2 Interval Type-2 Fuzzy Set Theory

A type-2 fuzzy set [6,7] expresses the non-deterministic truth degree with imprecision and uncertainty for an element that belongs to a set. A type-2 fuzzy set denoted by $\tilde{\tilde{A}}$, is characterized by a type-2 membership function $\mu_{\tilde{\tilde{A}}}(x,u)$, where $x \in X$, $u \in J_x^u \subseteq [0,1]$ and $0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1$ is defined in equation (1).

$$\tilde{\tilde{A}} = \left\{(x, \mu_{\tilde{\tilde{A}}}(x)) \mid x \in X\right\} = \left\{ \int_{x \in X} \left[\int_{u \in J_x^u \subseteq [0,1]} f_x(u)/u \right] / x \right\} \tag{1}$$

An example of a type-2 membership function constructed in the IT2FLS Toolbox [15] was composed by a Pi primary and a Gbell secondary type-1 membership functions, these are depicted in Figure 1.

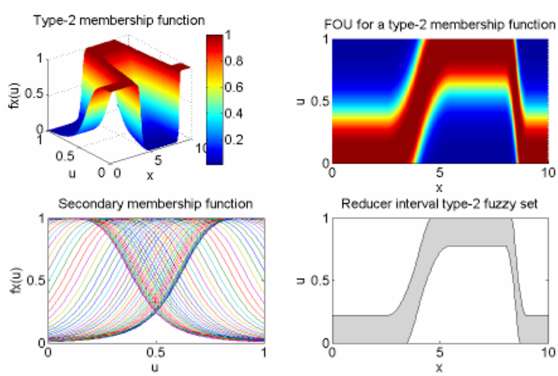

Fig. 1. FOU for Type-2 Membership Functions

If $f_x(u)=1, \forall u \in [\underline{J}^u_x, \overline{J}^u_x] \subseteq [0,1]$, the type-2 membership function $\mu_{\tilde{A}}(x,u)$ is expressed by one inferior type-1 membership function, $\underline{J}^u_x \equiv \underline{\mu}_A(x)$ and one superior type-1 membership function, $\overline{J}^u_x \equiv \overline{\mu}_A(x)$ (Fig. 2), then it is called an interval type-2 fuzzy set [8] denoted by equation (2).

$$\tilde{\tilde{A}} = \left\{ \int_{x \in X} \left[\int_{u \in [\underline{\mu}_A(x), \overline{\mu}_A(x)] \subseteq [0,1]} 1/u \right] / x \right\} \tag{2}$$

If $\tilde{\tilde{A}}$ is a type-2 fuzzy singleton, the membership function is defined by equation (3).

$$\mu_{\tilde{A}}(x) = \begin{cases} 1/1 & si\ x = x' \\ 1/0 & si\ x \neq x' \end{cases} \tag{3}$$

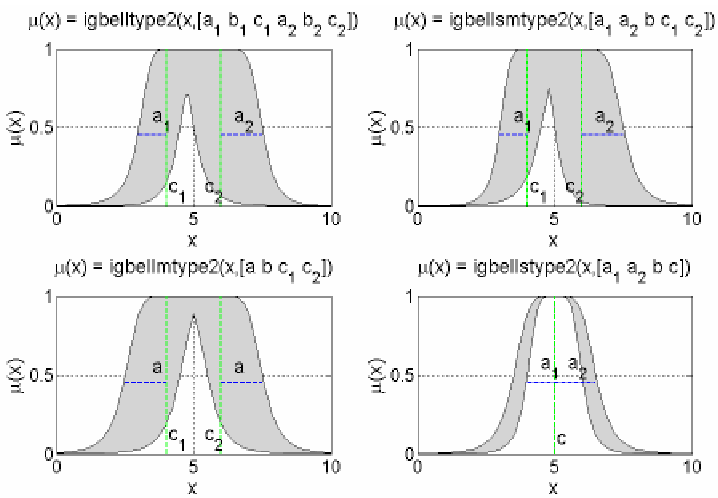

Fig. 2. FOU for Gbell Primary Interval Type-2 Membership Functions

Interval Type-2 Fuzzy Inference System

The human knowledge is expressed as a set of fuzzy rules. The fuzzy rules are basically of the form IF <Antecedent> THEN <Consequent> and express a fuzzy relationship or proposition. In fuzzy logic the reasoning is imprecise, it is approximated, which means that we can infer from one rule a conclusion even if the antecedent doesn't comply completely. We can count on two basic inference methods between rules and inference laws, Generalized Modus Ponens (GMP) [5,6,8,11] and Generalized Modus Tollens (GMT) that represent the extensions or generalizations of classic reasoning. The GMP inference method is known as direct reasoning and is resumed as:

Rule 1:	*IF* x_1 *is* A_{11} *and* x_2 *is* A_{21} *THEN* y_1 *is* C_1
Rule 2:	*IF* x_1 *is* A_{12} *and* x_2 *is* A_{22} *THEN* y_1 *is* C_2
Fact:	x_1 *is B1 and x2 is B2*
Conclusion:	y_1 *is C*

Where A_{11}, A_{12}, A_{21}, A_{22}, C_1, C_2, B_1, *and* B_2 are interval type-2 fuzzy sets. This relationship is expressed as:

$$C_1' = [B_1 \circ (A_{11} \to C_1)] \sqcap [B_2 \circ (A_{21} \to C_1)]$$
$$C_2' = [B_1 \circ (A_{21} \to C_2)] \sqcap [B_2 \circ (A_{22} \to C_2)]$$

$C = C_1' \sqcup C_2'$, where $\sqcap$ =meet and $\sqcup$=join [10,11,13]. Figure 3 shows an example of non-singleton interval type-2 fuzzy logic system with Mamdani reasoning [9], two inputs x_1 and x_2 and output y_1. An Interval type-2 Fuzzy Inference System is a rule base system that uses Interval type-2 fuzzy logic, instead of Boolean logic utilized in data analysis [4,9,11,12]. A rule based Fuzzy Logic System (FLS) contains four components: Rules, fuzzifier, inference engine, and output processor that are inter-connected, as shown in Figure 4.

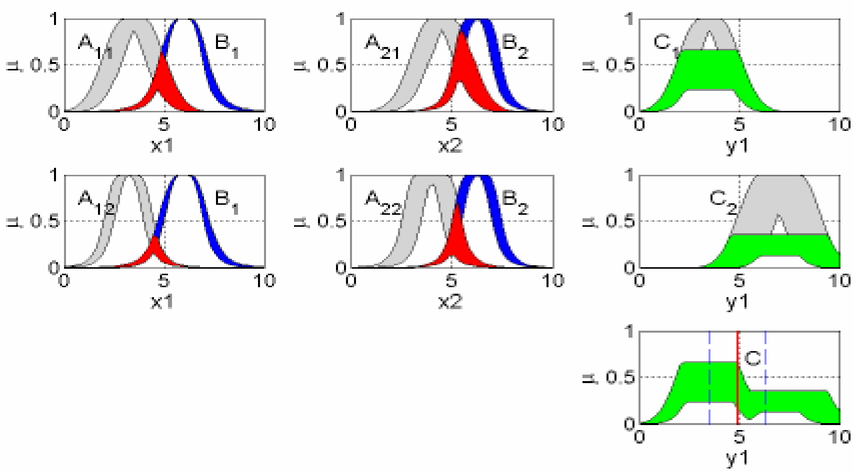

Fig. 3. Interval Type-2 Fuzzy Reasoning

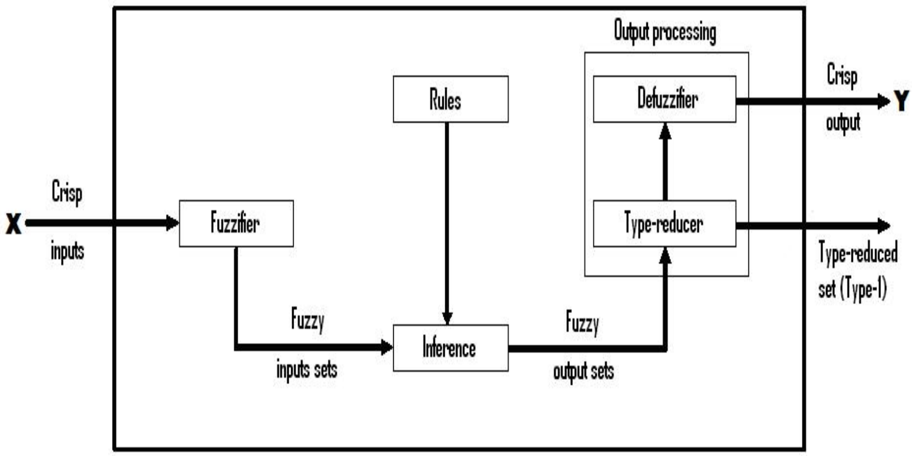

Fig. 4. Type-2 fuzzy logic System

3 Interval Type-2 Fuzzy Logic System Design

The Mamdani and Takagi-Sugeno-Kang Interval Type-2 Fuzzy Inference Models [9] and the design of Interval Type-2 membership functions and operators are implemented in the IT2FLS (Interval Type-2 Fuzzy Logic Systems) Toolbox which was build on top of the Matlab® commercial Fuzzy Logic Toolbox. The IT2FLS Toolbox [15] contains the functions to create Mamdani and TSK Interval Type-2 Fuzzy Inference Systems (newfistype2.m), functions to add input-output variables and their ranges (addvartype2.m), it has functions to add 22 types of Interval Type-2 Membership functions for input-outputs (addmftype2.m), functions to add the rule matrix (addruletype2.m), it can evaluate the Interval Type-2 Fuzzy Inference Systems (evalifistype2.m), evaluate Interval Type-2 Membership functions (evalimftype2.m), it can generate the initial parameters of the Interval Type-2 Membership functions (igenparamtype2.m), it can plot the Interval Type-2 Membership functions with the input-output variables (plotimftype2.m), it can generate the solution surface of the Fuzzy Inference System (gensurftype2.m), it plots the Interval type-2 membership functions (plot2dtype2.m, plot2dctype2.m), a folder to evaluate the derivatives of the Interval type-2 Membership Functions (dit2mf) and a folder with different and generalized Type-2 Fuzzy operators (it2op, t2op).

The implementation of the IT2FLS GUI is analogous to the GUI used for Type-1 FLS in the Matlab® Fuzzy Logic Toolbox, thus allowing the experienced user to adapt easily to the use of IT2FLS GUI [15]. Figures 5 and 6 show the main viewport of the Interval Type-2 Fuzzy Inference Systems Structure Editor called IT2FIS (Interval Type-2 Fuzzy Inference Systems).

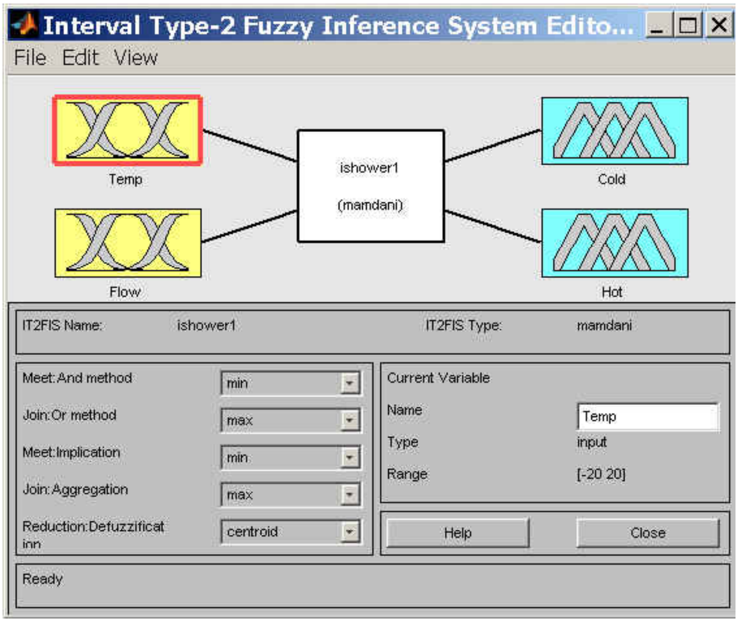

Fig. 5. IT2FIS Editor

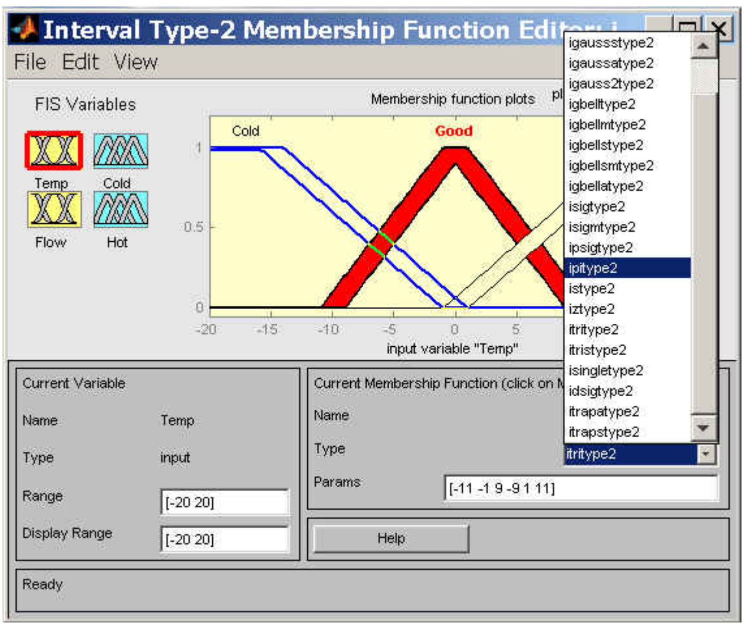

Fig. 6. Interval Type-2 MF's Editor

4 Simulation Results

We present results of a comparative analysis of the Mackey-Glass chaotic time-series forecasting study using an intelligent cooperative architecture of hybrid methods (neuro-genetic, fuzzy-genetic and neuro-fuzzy), with neural networks, type-1 fuzzy inference systems (Mamdani, Takagi-Sugeno-Kang), genetic algorithms (GA) and an interval type-2 fuzzy logic model, for the implicit knowledge acquisition in a time series behavioral data history [14]. Also we present a shower simulation and a truck backer-upper simulation with interval type-2 fuzzy logic systems using the IT2FLS Toolbox.

Mackey-Glass Chaotic Time-Series

To identify the model we make an exploratory series analysis with 5 delays, $L^5x(t)$, 6 periods and 500 training data values to forecast 500 output values. The IT2FLS (Takagi-Sugeno-Kang) system works with 4 inputs, 4 interval type-2 membership functions (igbellmtype2) for each input, 4 rules (Fig. 7) and one output with 4 interval linear functions, it is evaluated with no normalized values. The forecasted root mean square error (RMSE) is 0.0235. Table 1 shows the RMSE differences of six forecasting methods, where CANFIS (CoActive Neuro-Fuzzy Inference Systems) and IT2FLS-TSK (Takagi-Sugeno-Kang) evaluate the best Mackey-Glass series forecasts respectively. The advantage of using the interval type-2 fuzzy logic forecasting method is that it obtains better results, even when data contains high levels of noise.

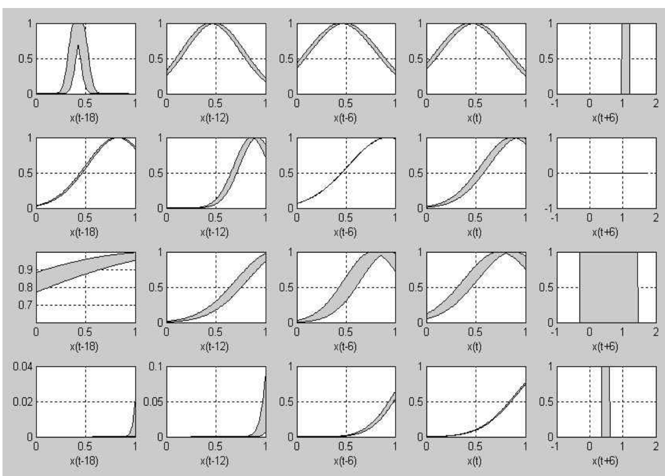

Fig. 7. IT2FLS (TSK) Rules

Table 1. Forecasting of Time Series

METHODS	MACKEY-GLASS			
	RMSE	Trn/Chk	Epoch	Cpu(s)*
FFNN(Feed-Forward Neural Networks)**†	0.0595	500/500	200	13.36
CANFIS	0.0016	500/500	50	7.34
FFNN-GA†	0.0236	500/500	150	98.23
FLS(TKS)-GA†	0.0647	500/500	200	112.01
FLS(Mamdani)-GA†	0.0693	500/500	200	123.21
IT2FLS	0.0235	500/500	6	20.47

* POWER BOOK G4 1.5 Ghz / 512 MB RAM.
** Architecture: 4-13-1. † 30 samples average.

Shower Control Simulation

In this experiment we evaluate the system response to compare the type-1 and type-2 controls with the ISE (Integral of Square Error), IAE (Integral of Absolute value of the Error) and ITAE (Integral of the Time multiplied by the Absolute value of the Error) functionality criteria. The best results were in type-2 controls, as shown in Table 2. In figure 8 we show an interval type-2 fuzzy control scheme and in figures 9 the control results.

Table 2. Control functionality criteria comparison

TYPE FLS	ISE	IAE	ITAE	VARIABLE
Type-1	277.3	76.21	3934	Temperature
Type-2	243.8	64.88	3344	Temperature
Type-1	0.6735	3.5153	172.35	Flow
Type-2	0.6427	3.3039	162.63	Flow

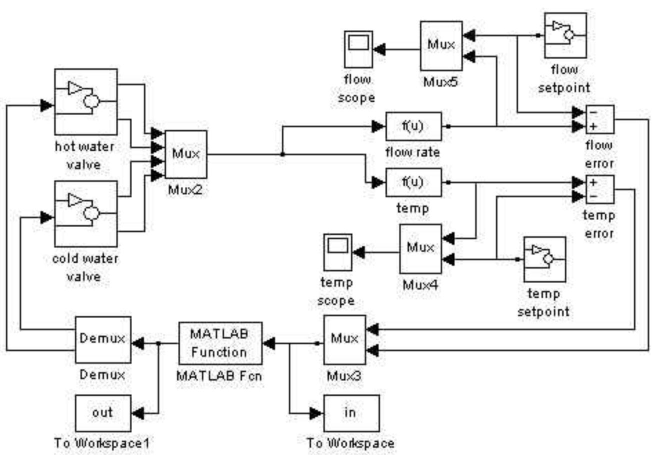

Fig. 8. Simulink interval type-2 fuzzy control scheme

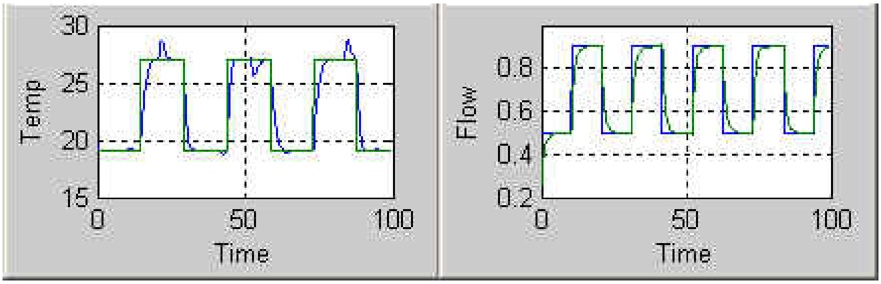

Fig. 9. Temperature and flow interval type-2 fuzzy control

Truck Backer-Upper Control Simulation

In this case study we use a SNR=28 dB signal-to-noise-ratio to generate uncertainty in the plant output variables. We compare the type-1 and type-2 controls using the mean functionality criteria for each trajectory, obtaining the following results: ISE=2.2053, IAE=2.9759 y ITAE=6.2091 for type-1 and ISE=2.0386, IAE=2.8301 y ITAE=5.7254 for type-2. The type-2 controller was better. In figure 10 have the interval type-2 fuzzy control' scheme and in figure 11 the control results of the car trajectories.

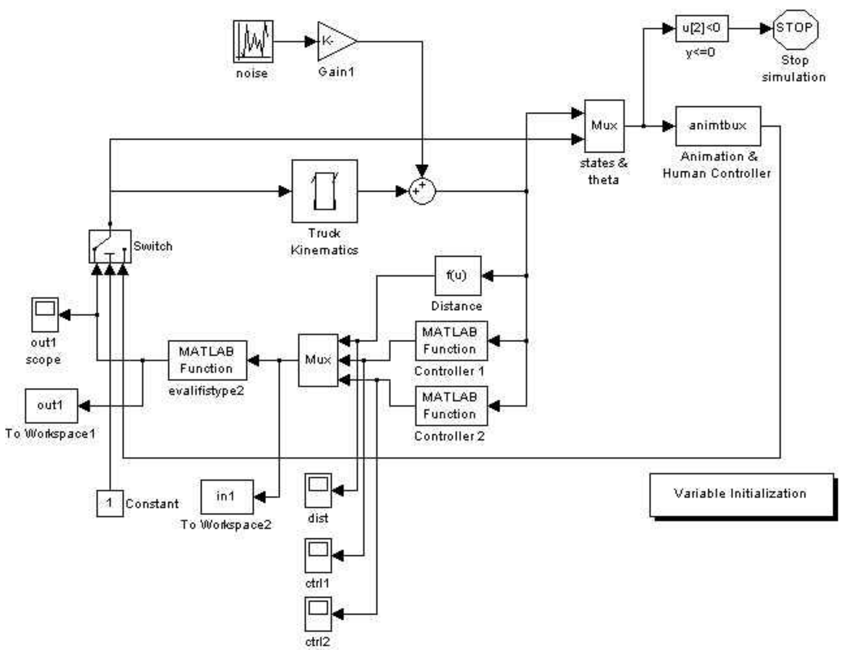

Fig. 10. Simulink interval type-2 fuzzy control scheme

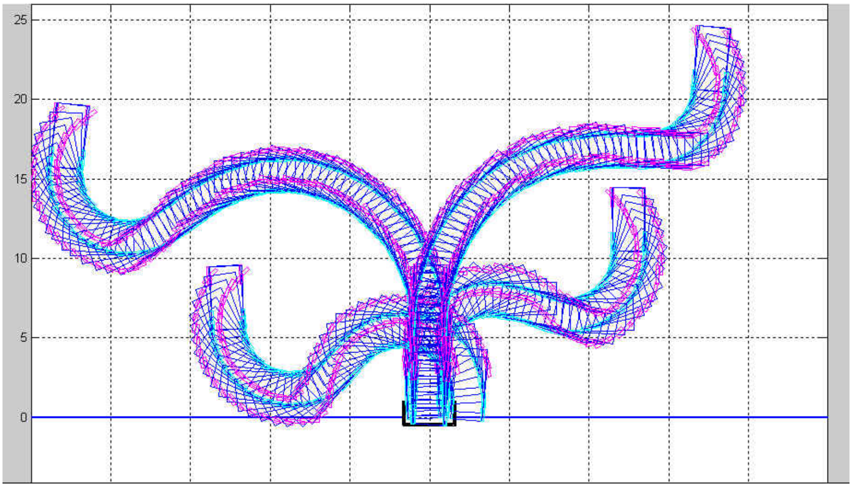

Fig. 11. Trajectory Interval type-2 fuzzy control

5 Conclusions

The time series results show that intelligent hybrid methods and interval type-2 fuzzy models can be derived as a generalization of the autoregressive non-linear models in the context of time series. This derivation allows a practical specification for a general class of prognosis and identification time series models, where a set of input-output variables are part of the dynamics of the time series knowledge base. This helps the application of the methodology to a series of diverse dynamics, with a very low number of causal variables to explain the behavior. The results in the interval type-2 fuzzy control cases of the shower and truck backer upper have similar results to the type-1 fuzzy control with moderate footprints of uncertainty. To better characterize the interval type-2 fuzzy models we need to generate more case studies with better knowledge bases for the proposed problems, therefore classify the interval type-2 fuzzy model application strengths. The design and implementation of the IT2FLS Toolbox is potentially important for research in the interval type-2 fuzzy logic area, thus solving complex problems on the different applied areas. Our future work includes improving the IT2FLS Toolbox with a better graphics user interface (GUI), integrating a learning technique Toolbox to optimize the knowledge base parameters of the interval type-2 fuzzy inference system, and the design of interval type-2 fuzzy neural network hybrid models.

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