Wave-based Control of Flexible Mechanical Systems

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Abstract. There are many contexts, from space structures to disk drive heads, from medical mechanisms to long-arm manipulators, from cranes to light robots, in which it is desired to achieve rapid and accurate position control of a system end-point by an actuator working through a flexible system. The system's actuator must then attempt to reconcile two, potentially conflicting, demands: position control and active vibration damping. Somehow each must be achieved while respecting the other's requirements. Wave-based control is a powerful, relatively new strategy for this important problem that has many advantages over most existing techniques. The central idea is to consider the actuator motion as launching mechanical waves into the flexible system while simultaneously absorbing returning waves. This simple, intuitive idea leads to robust, generic, highly efficient, adaptable controllers, allowing rapid and almost vibrationless re-positioning of the remote load (tip mass). For the first time there is a generic, high-performance solution to this important problem that does not depend on an accurate system model or near-ideal actuator behaviour.

Keywords. Flexible mechanical systems, robot analysis and control, slewing of space structures, active vibration control.

1 Introduction

There are many contexts, from space structures to disk drive heads, from medical mechanisms to long-arm manipulators, from cranes to light robots, in which it is desired to achieve rapid and accurate position control of a load (or system end-point) by an actuator that is separated from the load by an intermediate system which is flexible. While all systems are to some extent flexible, issues related to flexibility become decisive as one tries to design lighter mechanisms, or systems that are more dynamically responsive, or softer, or more energy efficient, or simply long in one dimension.

The system's actuator must then attempt to reconcile two, potentially conflicting, demands: position control and active vibration damping. Somehow each must be achieved while respecting the other's requirements.

Previous approaches have included various classical and state feedback control techniques (often using simplified dynamic models); modal control (often considering a rigid-body, or zero frequency mode separately from vibration modes); sliding mode control; input command shaping; time-optimal control leading to bang-bang control; wave-based control; and control based on real-virtual system models [1] [2] [3] [4] [5] [6]. Each method has special characteristics and drawbacks, discussed in the literature. None is completely satisfactory under all headings.

The wave-based control strategy [7] [8] [9] [10] [11] [12] is a powerful, relatively new method of dealing with flexibility that has been shown to be better than existing methods in most respects. The central idea is to consider the actuator motion as launching mechanical waves into the flexible system while simultaneously absorbing returning waves. This simple, intuitive idea leads to robust, generic, highly efficient, adaptable controllers, allowing rapid and almost vibrationless re-positioning of the system and the remote load (tip mass). For the first time there is a generic solution to this important problem that does not depend on an accurate system model and does not demand close to ideal performance by the actuator. Rather than treating the flexibility as a problem, it works with the flexibility to achieve system control in a natural way.

This chapter will investigate the mathematical foundation for a wave-based interpretation of flexible system dynamics, both lumped and continuous, exemplified in Fig. 1. It will then show how this view can be used to interpret the actuator-system interface as a two-way energy flow, leading to the design of controllers that give optimal performance by controlling this energy flow, in ways that are simple, robust, generic, and energy efficient.

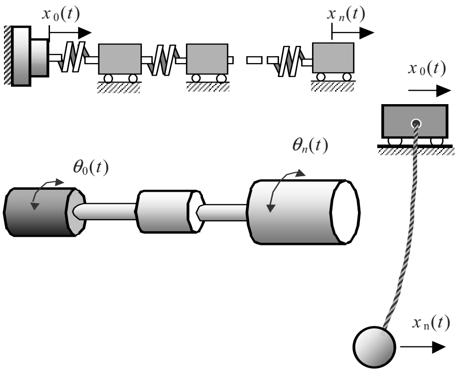

Fig. 1 Typical flexible systems, with actuator position, $x_0(t)$ or $\theta_0(t)$, controlling tip position, $x_n(t)$ or $\theta_n(t)$.

For simplicity, it will be assumed that there is a single actuator, with its own position controller, which is attempting to control the position of the system tip, moving it from rest in one position to rest at a target position. If gravity is active, it is assumed that the initial and final gravitational strains are equal, so that, when the system comes to rest again, the net displacement of the actuator will equal that of the tip. It is further assumed that the actuator position controller has zero steady-state error, so final position accuracy is limited only by the actuator sensor accuracy.

2 Wave Analysis of Flexible Systems

In flexible systems of the above type, the actuator and load are dynamically uncoupled. The interaction between them is mediated by the flexible system dynamics and it involves delays. When the actuator moves it directly affects only the part of the flexible system to which it is attached. A disturbance (or "wave") then travels through the system to the load or tip, and then back towards the actuator, typically dispersing as it goes, in a complex motion. At each end of the system, some of this wave may be reflected and/or absorbed, depending on the instantaneous relationship between the motions of the actuator, or tip, and the motion of the adjacent parts of the flexible system.

The wave-based control strategy depends on (a) understanding, (b) measuring and (c) controlling the (notional) two-way flow of energy and momentum happening at the interface between the actuator and the flexible system. To move the tip from rest to rest the actuator must launch a "wave" into the flexible system and then absorb it, in such a way that when all the energy and momentum of the motion have been extracted, the system is at the target position.

The term "wave" here is very general, and includes not just oscillating motion but also a "step wave" which, after it passes a point, changes the net or DC displacement, implying "rigid body" or "zero frequency" motion. Because the primary focus here is position control, the wave variable is taken as displacement, linear or angular (in meters or radians). In other applications, variables such as force, torque, velocity, or acceleration would be appropriate as wave variables, and the wave control ideas can easily be adapted to suit.

3 Resolving Actuator Motion into 2-Way Waves

The actuator motion, $x_0(t)$ is notionally resolved into two component motions, $a_0(t)$ and $b_0(t)$,

$$x_0(t) = a_0(t) + b_0(t), \tag{1}$$

with a_0 corresponding to an outwards-going, or launch, wave, b_0 corresponding to a return wave, which the actuator attempts to absorb.

For the control application, the resolving in (1) need not be precise: it is necessary to fulfil only certain generic criteria [11]. The simplest definition sets

$$a_0(t) = \tfrac{1}{2}\left(x_0(t) + \int \frac{f(t)}{Z} dt \right) \tag{2}$$

$$b_0(t) = \tfrac{1}{2}\left(x_0(t) - \int \frac{f(t)}{Z} dt \right) \tag{3}$$

where $f(t)$ is the force that the actuator applies to the beginning of the flexible system in the direction of motion, and Z is an impedance term, whose value is not critical. For lumped systems Z can be taken as $\surd(k_1 m_1)$, corresponding to the first spring stiffness, k_1, and first mass, m_1. For the gantry crane, one can set $Z = \surd(\rho T)$ where ρ is the linear density of the cable and T the tension at the top, and for a simple pendulum system, $Z = m\surd(g/L)$, with m the mass and L the length.

For a system of lumped masses and springs in series, a slightly better expression [11] [12] for the s-domain version of the return wave, $B_0(s)$, is given by

$$B_0(s) = G(s)\big(X_1(s) - G(s)A_0(s)\big) \tag{4}$$

with

$$A_0(s) = X_0(s) - G(s)\big(X_1(s) - G(s)A_0(s)\big) \tag{5}$$

where $G(s)$ is a second order mass-spring-damper system, with mass and spring corresponding the beginning of the lumped flexible system, and with viscous damping at half critical [Fig. 2].

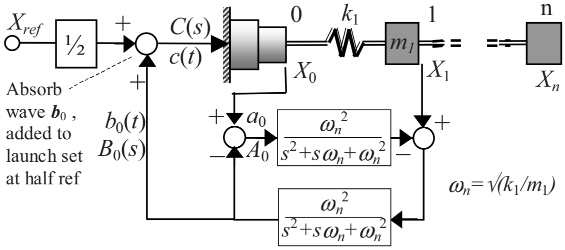

Fig. 2 A wave-based control implementation using (4) and (5) to determine return wave, b_0 from x_0 and x_1, with G as shown.

4 The Control Strategy

For rest-to-rest motion to a target position, the strategy is as follows. The input to the actuator is set as the sum of two components. The first component is set directly by the controlling computer, the only essential requirement being that its time profile ends at half the target displacement and holds there. The second component is the measured return wave, b_0, calculated by measuring two variables, such as x_0 and x_1, or x_0 and f, and calculated using for example (3) or (4).

Adding the second input component, which has the form of a positive feedback signal, provides active vibration damping, by making the actuator appear as a matched viscous impedance to "waves" returning from the flexible system towards the actuator. See for example (3). This causes the actuator to act as a very efficient, one-way, active vibration absorber, yet without impeding the action of simultaneously setting a launch wave.

A second effect of the absorbing component is to cause the total steady-state actuator displacement to become exactly double the launch component. This can be seen most clearly in the case of (2) and (3). For rest to test motion, the force integral terms must become zero, so that at rest a_0 and b_0 must become equal to each other and equal to $\frac{1}{2}x_0$. Thus, if the launch displacement component is set to settle at half the target displacement, adding the absorbing component ensures that (a) the system vibrations are absorbed, and (b) in absorbing them, the system arrives exactly at target. Thus, the main control problem has been solved, in a simple and elegant manner.

The launch component of the actuator motion can be considered as *pushing* the system half the distance to the target, while the absorbing component acts as if the reaction of the system were *pulling* the actuator the other half displacement, but in such a way that all momentum and energy return to zero precisely on completion of the process, just when the system arrives to target.

4.1 Launch Wave Profile

The launch waveform (time profile) is to a great extent arbitrary. It can arrive at the half-target displacement in many ways (step, ramp, constant acceleration, or using a pre-determined motion plan), limited only by the actuator dynamics. The control strategy works very well for all such choices.

There is one choice that is particularly neat [11]. The absorb wave motion is added throughout the entire manoeuvre. If the absorb wave is recorded from the beginning, the initial part of it can be used to determine a very good way to complete the launch wave. The actuator gets half way to target before the launch wave has reached its steady (half-target) value (Fig. 3). At this point, the launch wave can be set to complete its trajectory using an inverted and time-reversed version of the wave that has been absorbed out of the system up to that point. Thus, the "echo" that was

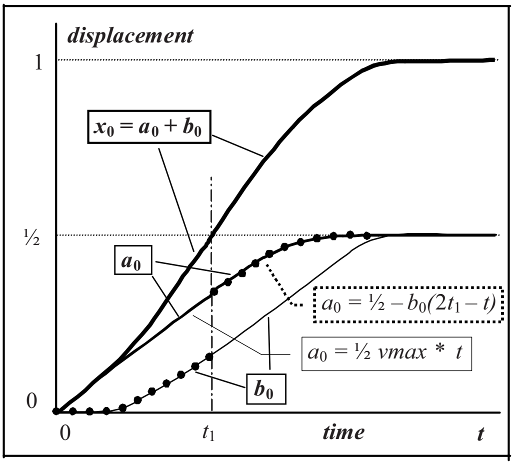

Fig. 3 Wave-echo control to determine actuator motion x_0. The target distance is 1. The a_0 component is set by the controller as a ramp until $t=t_1$ (when $x_0=\frac{1}{2}$), then as a reverse replay of recorded b_0, but inverted (shown dotted). At all stages b_0 is determined from the measured system response.

absorbed from the system in the first half of the manoeuvre is played back into the system in reverse to complete the manoeuvre. This has the effect of bringing the tip to rest in a time-and-space mirror-image of the start-up motion. In other words, the load stops dead, rapidly, precisely at the target, while the actuator continues to move so that the rest of the system then relaxes in just the right way to leave everything motionless in the correct position.

5 Sample Results

As an example, Fig. 4 shows the control of a uniform, lumped three-mass system. The actuator and end-mass positions are shown against time expressed in units of the period, T, or $2\pi/\omega_n$, where $\omega_n = \sqrt{(k/m)}$. The target displacement is 1m. Also shown are $a_0(t)$ and $b_0(t)$.

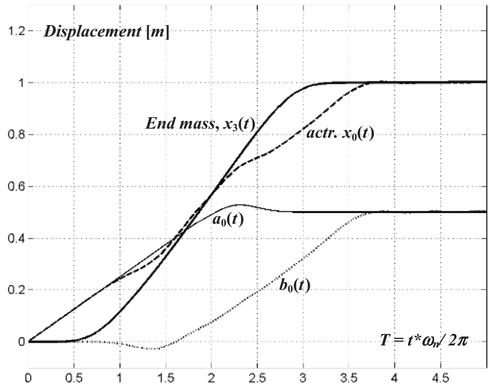

Fig. 4 End point of a uniform, three-mass system, moved 1 m.

As can be seen, the response is remarkable. Without a control strategy, the position of the end mass ("load") would, of course, oscillate somewhere between zero and two, with three frequency components superposed. Instead the load is translated from rest to rest, in a single, controlled movement, with almost no overshoot and with negligible oscillations (and so little or no settling time). The total manoeuvre time is excellent. Depending how strictly one defines the settling time, it is between 3 and 3.5 "periods" of ω_n, (This corresponds to about only 1.5 periods of the fundamental mode of the 3-mass system.)

The end mass (load) comes to rest exactly at target. It does so sooner than its actuator. The actuator's movements are smooth and easily achievable. Around mid-manoeuvre, the speed of the end-mass (the slope in Fig. 4) is close to that of the actuator: the flexible system is then behaving as if rigid, or almost so: vibration is under control.

Similarly impressive results are obtained whether the system is long or short, uniform or not, with linear or hardening or softening springs (other than the first), with or without internal damping, with ideal or realistic actuator, and with or without precise values for all the terms in (3) or (4). Illustrating a mixture of such added complexities, Fig.5 shows the response of a 5 DOF system with non-linear (hardening) springs; variations of the masses of 1, 0.5, 1, 2, 1; damping between the masses of 0, 0.25, 0.1, 0.25, 0, 0.05 times critical damping; an actuator modelled as a first order system of time constant $1/3\omega_n$; and $b_0(t)$ approximated by (3). These parameter values and system size were chosen almost at random: a similar result is obtained for almost arbitrary choices of these system variables.

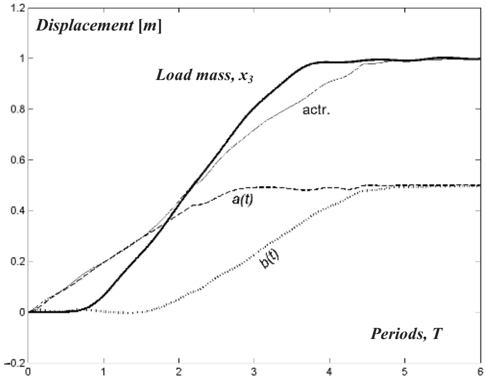

Fig. 5 Response of end mass of 5 DOF system, with non uniform masses, various dampers between the masses, non-linear springs, a first order actuator, and $b_0(t)$ determined simply by (3).

As can be seen, the non-uniformities, actuator limitations, and so on, make things less smooth, but despite everything, the controller gets the load exactly to target, rapidly, and it works very effectively to remove vibratory energy from the system during the motion and on arrival at target.

The presence of system non-uniformities requires no adjustment to the control strategy. The actuator's action is restricted to either launching or absorbing waves. To absorb returning waves the actuator must await their arrival. The non-uniformities will delay, and stretch out, their arrival and therefore delay the absorbing process. Also the non-ideal actuator response will slightly prolong the final tidying up. But even though the strategy and controller settings were not changed, their effectiveness in meeting the much more difficult challenge is almost undiminished.

Figure 6 gives a trolley crane example, moving a load 2 m, with a 4-m cable of significant mass [10]. The launch waveform is set to correspond to half the maximum trolley velocity, to which is added the return wave, b_0. For a long manoeuvre, this causes the trolley velocity to approach the maximum for the middle part of the transit, with the swing angle approaching zero. In other words, the system is then moving at top speed and as if it were rigid, with the load displacement tracking the trolley displacement. After the halfway point for the trolley (1 m), the launch displacement a_0 is based on the previous b_0. This causes the trolley to decelerate in precisely the way needed to get the load to land at target (2 m) and stop dead. The load arrives before the trolley, which continues to move in just the right way to allow the cable to straighten up as all wave energy flows out of the system.

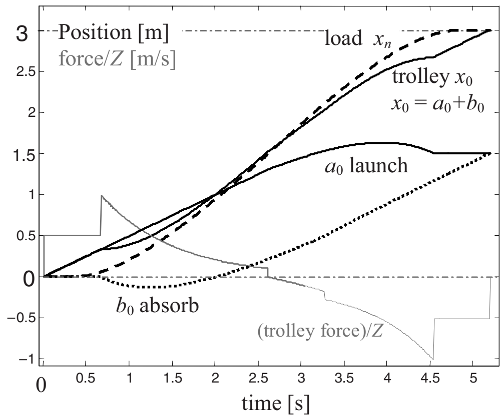

Fig. 6. Load arrives and stops at target before trolley. Crane target distance 3 m. Cable length=4 m, ρ=0.1 kg/m, m=2 kg, T=23.54 N, Z=1.534 Ns/m. Note time symmetry of load & trolley motions, due to a_0 and b_0 following wave-echo scheme of Fig. 3.

6 Discussion & Conclusions

Aspects of the problem of controlling flexible systems have been presented in new and fruitful ways, leading to new control algorithms that perform remarakbly well. They easily move a load from point to point, rapidly, yet with negligible residual vibration and negligible overshoot and zero steady-state error. They move the load at close to the actuator velocity (the ideal), in one controlled motion, without exciting load or system vibrations unnecessarily. The control strategies are very robust; they are applicable to a wide variety of problems; they require minimal system information, little computational overhead, and are very tolerant of limitations in the actuator dynamics. Sensing requirements are also minimal. Other than the actuator's own motion, only one other sensed input is needed, and the second sensor supplying this information is located conveniently close to the actuator, where sensing is generally easiest and safest in practice.

Modelling errors hardly feature. System changes are automatically accommodated. The order of the controller automatically matches that of the system, and explicit information, for example, about locations of poles (or natural frequencies and damping ratios of modes) is not needed.

The control approach can be considered a combination of "command shaping" *and* feedback control, the launch wave being a simple, shaped input, and the absorb part the feedback contribution.

With the wave-echo idea, the returning waveform, b_0, reveals to the controller the entire system dynamics in just the form the controller needs to achieve ideal system deceleration to rest. In a sense, the system itself serves as the system model, which is therefore always accurate, up to date, and of the correct order. The system itself also serves as the model's computer. To put it another way, all the required system identification is done in real time, as part of the controlled motion, with minimal computational overhead. This partly explains the control system's robustness to system changes.

The interface between the actuator and the flexible system is seen as a wave gateway, controlled and managed by the actuator's motion. Energy and momentum enter and leave the flexible system at the interface. They propagate in two directions within the system, from actuator to end-mass, and back again, albeit in ways that are faltering, complex, and highly dynamic. Rest-to-rest motion corresponds to getting the energy and momentum into, and then out of, the system in just the right way to ensure that the entire system comes to rest at the target.

The actuator is the sole agent for all this. But the actuator interacts directly only with the part of the system dynamics to which it is directly connected, namely the interface. All the control can be done, and must be done, at this interface and through this interface. The wave approach shows just how this can be done.

A very comprehensive study of the field [6] has observed that "to date a *general solution* to the control problem [of flexible mechanical systems] has *yet to be found*. One important reason is that computationally efficient (real-time) mathematical methods do not exist for solving the extremely complex sets of partial differential equations and incorporating the associated boundary conditions that most accurately model flexible structures." (*Emphasis added.*)

It is here contended that wave-based methods provide just such a "general solution" for a wide class of problems, with many additional attractive features.

Acknowledgements

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References

1. Book, W.J., 1993, "Controlled Motion in an Elastic World", *ASME Journal Dynamic Systems, Measurement, and Control*, 115, pp. 252–261, June.
2. Jayasuriya, S., Choura, S., 1991, "On the Finite Settling Time and Residual Vibration Control of Flexible Structures", *Journal of Sound and Vibration*, 148, No. 1, pp. 117–136.
3. Meckl, P., Seering, W., 1985 "Active Vibration Damping in a Three-Axis Robotic Manipulator", *ASME Journal of Vibration, Acoustics, Stress and Reliability in Design*, 107, pp. 38–46, January.
4. Meirovitch, L., 1989, *Dynamics and Control of Structures*, Wiley.
5. Pao, L.Y. 1996, "Minimum-time Control Characteristics of Flexible Structures", AIAA Journal of Guidance, Control and Dynamics, 1, pp. 123–129, January–February.
6. Robinett, R.D. III, Dohrmann, C.R., Eisler, G.R., Feddema, J.T., Parker, G.G., Wilson, D.G., Stokes, D., 2002, *Flexible Robot Dynamics and Controls*, Kluwer Academic/Plenum, New York, p. 165 & passim.
7. O'Connor, W.J., Lang D., 1998, "Position Control of Flexible Robot Arms using Mechanical Waves", *ASME Journal Dynamic Systems, Measurement, and Contro*l, 120, No. 3, pp. 334–339, September.
8. O'Connor, W.J., 2002, "Gantry Crane Control: a Novel Solution Explored and Extended", *Proceedings American Control Conference 02*, Alaska, 8 May.

9. O'Connor, W.J., Hu, C. 2002, "A Simple, Effective Position Control Strategy for Flexible Systems", *2nd IFAC Conference on Mechatronic Systems*, Berkeley, California, pp. 153–158, December.
10. O'Connor, W.J. 2003, "A gantry crane problem solved", *ASME Journal Dynamic Systems, Measurement, and Control*, 125, No. 4, pp. 569–576, December.
11. O'Connor, W.J., 2006, "Wave-Echo Control of Lumped Flexible Systems", *Journal of Sound and Vibration*, 298, Nos. 4–5, pp. 1001–1018, December.
12. O'Connor, W.J. 2007, "Wave-Based Analysis and Control of Lump-Modeled Flexible Robots", *Robotics, IEEE Transactions on*, 23, pp. 342–352.