# SmartVolumes - Adaptive Voronoi Power Diagramming for Real-Time Volumetric Design Exploration

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**Abstract.** Voronoi Diagrams and Delaunay Triangulations are two concepts fundamental to computational geometry, which have been applied in the most varied disciplines. In recent years, they are increasingly used in architectural design. In this paper, a novel method for volumetric design exploration based on three-dimensional (additively weighted) Voronoi power diagrams is described. The method combines fast calculation of three-dimensional weighted Voronoi Power Diagrams with a volume-dependent feedback loop, resulting in a real-time interactive modeling tool. This tool, named SmartVolumes, has been integrated into the modeling environment BehaviourLinks, where the interaction between parametric volumes and other entities can be further elaborated through behavioral linkages. Applications of SmartVolumes in urban design and architectural design are described, implications of the use of Voronoi diagrams for architectural modeling and environments are discussed and directions of consecutive developments are indicated.

**Keywords:** Design Exploration, Interactive Architecture, Design Environments, Computational Design, Voronoi diagram, Delaunay triangulation

## 1 Introduction

The Voronoi diagram resembles many different kinds of natural structures and thus may be a concept of considerable antiquity. One of the first known appearances resembling a Voronoi diagram is an illustration showing the disposition of matter in the solar system published by Descartes in 1644. Though the first comprehensive representations date back to the mid-nineteenth and early twentieth century, given by respectively Dirichlet and Voronoï, Voronoi diagrams were repeatedly rediscovered in various disciplines, even up to the 1980's [1]. During the first decade of the third millennium, Voronoi diagrams are increasingly applied in urban and architectural design. Two-dimensional Voronoi diagrams have been used in interactive environments for the dynamic assignment of regions of augmented space [2], in urban design stakeholder games for dividing an urban area into parcels, iteratively optimizing plots [3], and structural surface tessellations [4]. Three-dimensional Voronoi diagrams have been used for space-filling structural design [5], and functional optimization of various environmental aspects in architectural design [6].

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In all architectural applications Voronoi diagrams were arguably also used for their inherent aesthetics.

In this paper, a novel method for volumetric design exploration based on three-dimensional (additively weighted) Voronoi power diagrams is described. The method combines fast calculation of three-dimensional weighted Voronoi Power Diagrams with a volume-dependent feedback loop, resulting in a real-time interactive modeling tool. This tool, named SmartVolumes, has been integrated into the modeling environment BehaviourLinks, where the interaction between parametric volumes and other entities can be further elaborated through behavioral linkages. Applications of SmartVolumes in urban design and architectural design are described, implications of the use of Voronoi diagrams for architectural modeling and environments are discussed and directions of consecutive developments are indicated.

## 2 Voronoi Diagrams and Delaunay Triangulations

Manual approaches to architectural geometric design are generally based on the construction of drawings with the use of points, lines, circles, metrics, and various types of grids and structures derived from these. The application of computers as drawing medium adds a third or even more dimensions, and an undo stack. Also, it makes architectural geometric design techniques enter the realm of computational geometry, where the focus is not only on how to solve a geometric problem for one or several objects, but primarily on cases where large sets of 'n' objects are involved. The application of computers to architectural geometric design lets the designer address entire populations of objects, within the limits of computability. In the exploration of the affordances of computational design media, many techniques have been developed [7].

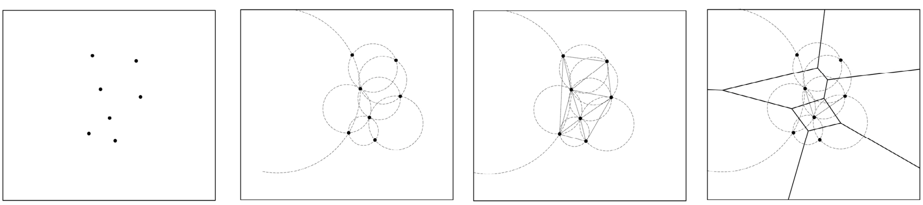

**Fig. 1.** Far Left: a set of points. Center Left: circles touching three points but not containing other points. Center Right: the circles are circumcircles to the triangles of the Delaunay triangulation. Far Right: the Voronoi Diagram is the dual diagram of the Delaunay triangulation, and has the circumcircle centers as vertices of the boundaries of the Voronoi regions.

Many of these approaches can be utilized in specialized design applications or as feature present in most contemporary design suites: for example parametric design approaches like solid modeling, or splines, which are curves and surfaces generated by interpolating over the elements of ordered sets of points [8]. Next to the common application of off-the-shelf tools, custom made algorithmic techniques are only marginally used in architectural design. Examples would be neural networks, cellular automata, genetic algorithms, shape grammars, swarm-solvers, particle-spring systems.

Common to all these approaches is that they are algorithmic geometric constructions, and as such are based on sets of rules, and sets of features like points which define a line, grid or other kind of structure. In contrast, Voronoi diagrams are not constructions but properties of a set of points in regard to a given metric. Voronoi diagrams enable design directly on a population of points, without the need to first define a structure according to which the geometric construction has to be laid out.

The Voronoi diagram and Delaunay triangulation of a point set can be understood as a dual map to the point-set's topology. The Voronoi Diagram maps for each point of the cloud the region of space which it dominates according to a given metric. The Delaunay triangulation maps the relationships of neighboring Voronoi regions of a given point set. Both Voronoi diagram and Delaunay triangulations are not geometric constructions, they are implicit properties of a point set under a chosen metric. They are maps to the spaces a point set generates, representing them as a set of locations and providing insight into the relationships between these locations.

They provide notions of space given in connections and separations, primordial architectural characteristics as described by Georg Simmel in the essay *Bridge and Door*:

"Only to humanity, in contrast to nature, has the right to connect and separate been granted, and in the distinctive manner that one of these activities is always the presupposition of the other. […] The forms that dominate the dynamics of our lives are thus transferred by bridge and door into the fixed permanence of visible action."[9]

In different context, a notion of *clearing* space by defining relationships and establishing regions can be found the in Martin Heidegger's essay *Building, Dwelling, Thinking*¸ in the description of a bridge gathering landscape:

"The bridge[…] does not just connect banks that are already there. The banks emerge as banks only as the bridge crosses the stream. The bridge designedly causes them to lie across from each other. One side is set off against the other by the bridge. Nor do the banks stretch along the stream as indifferent border strips of the dry land. With the banks, the bridge brings to the stream the one and the other expanse of the landscape lying behind them. It brings stream and bank and land into each other's neighborhood. The bridge gathers the earth as landscape around the stream." [10]

Given the fundamental geometric nature of Voronoi diagrams, and its possible theoretical implications, it may seem surprising they have found use in architecture only in recent years. This may be due to the fact that they take a long time to manually construct, and that they generally lead to non-standard elements which used to be very time-consuming and expensive to produce and assemble. Several developments of the near past may have made Voronoi diagrams more applicable in architectural design:

- The changes to architectural praxis following the introduction of digital technologies,
- The off-the-self availability of Voronoi solvers in commercial CAD software, e.g. the 'Point set reconstruction tool' for Rhino

- The availability of open source of computational graphics libraries containing algorithms for the construction of 2D and 3D Voronoi diagrams and Delaunay triangulations, e.g. VTK[11], CGAL[12] and Qhull[13].
- The growing number of architects literate in the use of computational tools and programming,
- The advances of non-standard design and production in architecture

# 3 Smartvolumes

The complex interrelationships between the location of points, their weight, and the geometry of the Delaunay triangulation and its Voronoi diagram demand for an efficient tool for design exploration. SmartVolumes is a design tool developed to meet this demand. It is based on real-time interaction between the set of weighted points, properties of its Voronoi diagram, a user and possible parametric relationships the users have set.

SmartVolumes as described here is realized in the game-development environment Virtools [14], in conjunction with C++ coding and utilizing the Computational Graphics Algorithms Library CGAL [12]. The adaptation of the parametric plan is taking place in real-time, several times per second, in parallel with the users' interaction with the diagram that results from the rules and constrains they have set. These adaptations and interactions are set up in a game loop. In each execution of the loop, the user interfaces are read, the parameters they control are updated, the SmartVolumes are computed, the point set is adapted so it approaches the user demand, and eventually the frame is drawn to screen. In the real-time design exploration, weights and positions of points so the volumes of the individual Voronoi cells are adapted to comply with the designer's demands. Simultaneously, the designer is given opportunity to adjust his aims and explore design space.

The geometry of the SmartVolume cells is executed during each game loop as follows:

1. Determine the boundary condition

The boundary for the volumetric Voronoi diagram is derived from a set of 'boundary' points as their three-dimensional convex hull. It is found by making a Delaunay Triangulation of the point set and collecting all facets incident to infinite cells of the Delaunay triangulation.

2. Weighted Delaunay triangulation

Temporarily eight points are added to the set of SmartVolume points, forming a relatively very large cube around the original set of points. A weighted Delaunay triangulation of the SmartVolume points plus the eight additional points is computed using the CGAL [12] library. The weighted Delaunay triangulation in CGAL is based on the power product for determining distance between points:

Let $S^{(w)}$ be a set of weighted points in $R^3$. Let $p_i^{(w)}$ and $p_j^{(w)}$ be two weighted points with positions $p_i, p_j \in R^3$ and weights $w_i, w_j \in R$. The weighted points can be seen as spheres of centers $p_i, p_j$ and radii $w_i, w_j$. Then the power product between $p_i^{(w)}$ and $p_j^{(w)}$ is defined as

$$\prod(p_i^{(w)}, p_i^{(w)}) = \left\| p_i - p_j \right\|^2 - w_i - w_j \ ,$$

where $\left\| p_i - p_j \right\|^2$ is the Euclidean distance between $p_i$ and $z_i$ [12].

The result of this triangulation is equivalent to the triangulations in the center-left column of Figure 2.

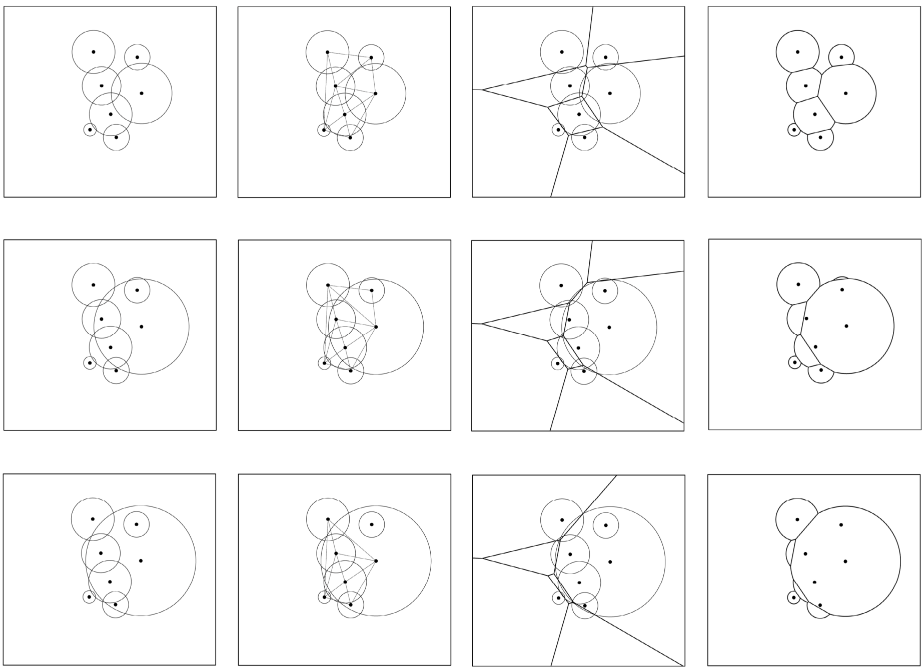

**Fig. 2.** Far Left Column: Sets of weighted points, weights are indicated as circles. Center-Left Column: Weighted Delaunay Triangulation. Center-Right Column: (additively weighted) Voronoi power Diagram. Far Right Column: the resulting SmartVolumes volumes.

3. Determine Voronoi cells

The weighted centers of the circumspheres of the tetrahedrons which are the outcome of the weighted Delaunay triangulation are calculated. For each SmartVolume point, the set of circumsphere centers of tetrahedrons to which the point is incident are collected. From these sets of circumsphere centers the Voronoi Cells of the SmartVolume points are determined by computing their convex hulls.

4. Intersect cells with the boundary

The intersections of the Voronoi cells with the boundary volume are determined. The result is equivalent to the diagrams in the center-right column of Figure 2: bounded weighted Voronoi diagrams. This diagram is the Power Diagram, with the Voronoi generation distance

$$d(p_i, p_j) = \|\mathbf{x} - \mathbf{x}_i\|^2 - w_i \text{ [1].}$$

5. Tessellate

Each cell of the bounded Voronoi diagram is tessellated up to a chosen depth.

6. Shrink-wrap

All vertices of the tessellated cells, whose distance from their generator point is larger than the square of its weight, are translated towards the point so the distance equals the square of its weight. The result of this ‘shrinking’ is equivalent to the diagrams in the far right column of Figure 2.

7. Adapt

The volume of each SmartVolume is calculated from the resulting geometry. The weight of each SmartVolumes point is adapted according to the difference between the volume of its SmartVolume and the user demand.

# 4 Applications

## 4.1 Urban Design: BehaviourLinks :: Urban Mode

*SmartVolumes* is developed as spatial design extension for the design environment *BehaviourLinks :: Urban Mode*. The agenda of BehaviourLinks is to reshape the way architects employ digital techniques by proposing a dynamic, open, collaborative design praxis. A *BehaviourLink* is a piece of programming code that is executed in real-time and defines interactions between conceptual entities, by manipulating data contained in one entity based on data contained in another entity. The nature of these manipulations is dependent on the type of link chosen by the user. The conceptual entities can represent architectural concepts but also digital interfaces to sensors, users and exchange data.

By defining conceptual nodes and laying behavioral links, users can grow a parametric diagram of the urban plan. Its shape, structure and visualization originate from the behavioral design rules and decisions made by the users as well as the feedback negotiation between interacting nodes. The interactive diagram lets a group of stakeholders make fast and well-informed urban design decisions.

In BehaviourLinks::Urban mode, all data is subject to real-time interactive, iterative adaptation by the behavioral links the user laid between entities. Functional extensions of BehaviourLinks consist of node typifications, which can be affected by additional link types. For BehaviourLinks :: Urban Mode, extensions have been implemented that let the users simultaneously explore how the program of demands, volumetric plan, traffic models, parametric spatial relationships, shadow volumes, façade styles, plan boundaries and urban data affect each other, with direct visual feedback and embedded media of the actual site.

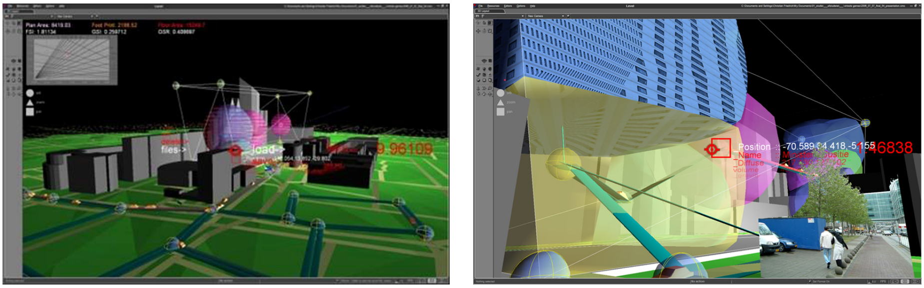


**Fig. 3.** Left: An actual in-game screenshot of BehaviourLinks :: Urban Mode. Visible within the urban scene are SmartVolume building proposals, traffic modeling, statistical data derived from the proposal, the user interface. Right: A closer view, showing embedded media, facade styles and contextual node data displayed next to the node which is closest to the cursor. Some nodes typified as SmartVolumes are also integrated into the path-finding graph of the traffic modeler.

SmartVolumes has been integrated into BehaviourLinks as a possible typification of nodes, which makes behavioral nodes act as parametric volumes. The resulting volumes can then be analyzed for their properties. Volumes, floor area estimates and surface area of the SmartVolumes are displayed on-screen and can be used as input for behavioral links that affect the entire set of nodes. In this way, for example, a node typified as SmartVolume can request traffic from a path-finding graph modeled on the street pattern. The effects of making different access routes to the proposed building volumes and the overall impact of their position and spatial relations can then be explored.

### 4.2 Architectural Design: DP Korea Project

A first application of SmartVolumes in architectural praxis was the Digital Pavilion Korea project at ONL, office of Kas Oosterhuis and Ilona Lénárd [15]. For this project, a design environment for a technology exhibition was developed based on SmartVolumes and BehaviourLinks. The application would allow exhibitors to specify their demands in area and location, and to collaboratively explore spatial layout that would meet their demands. A chosen solution could be written file to formats for CAD/CAM, graphic design and web navigation, as well as to spreadsheets containing lists and statistics of the needed elements.

For the DP Korea project, the SmartVolumes modeler was modified to directly connect a spatial layout of an exhibition space to the partitioning and detailing of the space. In the game, several algorithmic tools for generating a point cloud are available, so the chance of having a point cloud with a desirable Voronoi solution is enhanced. Next to placing points manually the following algorithms can be used for placing points:

- Groups of points can be placed in elements of the buildings construction (columns, walls) ensure these elements can be entirely hidden in spatial partitions, which do not differ in form or structure from their surroundings.

- Groups of points can be placed with a radial distribution relative to a specified center, This results in a three-dimensional density gradient in the point set, and its Voronoi diagram.
- Groups of points can be placed along paths defined by splines, to create passages.

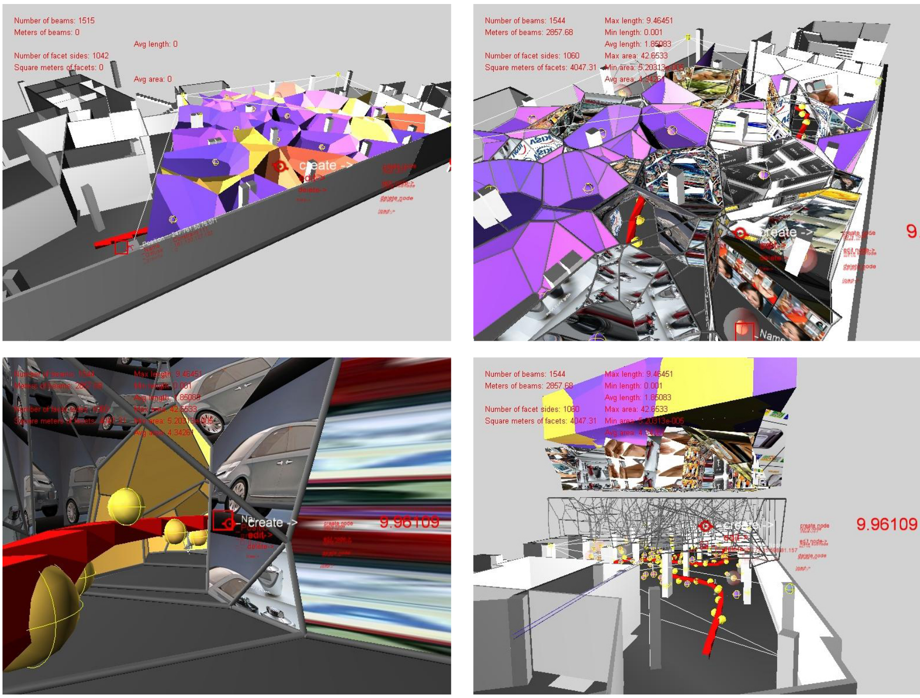


**Fig. 4.** Top Left: In-game screenshot of an exhibition floor design based on SmartVolumes within the real-time BehaviourLinks environment, Top Right: structural solution with numerical data, Bottom Left: inside the exhibition space, viewing generator points and details, Bottom Right: algorithmically generated point set, struts, facets and volumes in extruded view, ready for export.

In the generation of the Voronoi solution, the three-dimensional volumes of the cells are generated for each point, together with the facets that lie between two cells and edges of the facets. Cells, edges and facets can be looked up for each point individually. In this way it is possible to directly remove facets which are between specific neighboring cells. Voronoi cells belonging to a group of points forming a path can be opened to each other, making a passage, and the cells of one exhibition stand can be combined to form a greater, more complex volume.

Due to the additional computation involved in building a multi-relational database of cells, facets and edges, the changes in the point set could not result in direct visual feedback of the entire structure. Once the complete structure is generated however, it is ready to be exported as annotated production drawing in DXF and SVG format, and tables containing the metric data of a solution for cost calculation and production planning.

# 5 Discussion

## 5.1 Necessity

Whereas the traditional architect's approach is to fit demands into a chosen structure, the SmartVolumes technique offers a tool for finding structures and generating geometries based on volumetric and behavioral demands.

It also supports designers in the exploration of the complexity of geometries based on Voronoi diagrams: In a three-dimensional Voronoi diagram the location of points, the volumes of cells, the faces of cell surfaces, the edges of these facets and the endpoints of these edges are all implicitly related to each other. Each change to the generative point cloud simultaneously affects structure, building physics, details, aesthetics and other performances of the design. These complex interrelations demand for a tool to efficiently explore possible solution spaces. SmartVolumes is intended to meet these demands.

## 5.2 Precedents

SmartVolumes makes use of adapting (additively weighted) Voronoi power diagrams in real-time. A similar iterative, adaptive Voronoi diagramming method has been developed R. Reitsma, S. Trubin and S. Sethia [16], for regionalizing an information space based on a set of information items with given locations and predetermined area. Their regionalization method is an application of multiplicatively weighted Voronoi diagrams, and intended for visualization optimization, not interactive design exploration during an open-ended adaptation process. Applications of multiplicatively weighted Voronoi diagrams and other generalizations of Voronoi diagrams for explorative design techniques like SmartVolumes yet have to be investigated.

## 5.3 Possible Improvements

SmartVolumes enables the designer to apply algorithms that explicitly shape the point set, and to directly see the implicit effect on the point set's Voronoi diagram. However, since the resulting geometries are always implicitly defined, in order to individually relate further parametric geometric operations to individual elements of the geometry derived from the Voronoi diagram, the original point set has to be fixed. This makes it difficult to further develop Voronoi-based designs without dismissing their parametric, malleable nature. Possible ways of relating explicit descriptions of desirable shape manipulations of Voronoi diagrams, without sacrificing malleability, form a wide field of research which has not yet been investigated from an architectural point of view.

### 5.4 Application Perspectives

SmartVolumes is a generic spatial modeling tool, with a wide range of possible uses in architectural design and environments. It can be used in the initial, conceptual phase of the building process. From there it can directly, or after further manipulations, lead to specific data for computer aided manufacturing of the design. It could also be used for spatial analysis needed to control distributed participators in interactive architectural environments, be it humans or elements of the architectural object itself.

### 5.5 Outlook

In its current implementation, on contemporary workstations, the application of SmartVolumes in real-time is limited to several dozen volumes. With more elements, the geometry becomes too complex not only to compute within time, but also to transfer to the graphics processor. Within near future, similar real-time design exploration techniques should be expected to be not only commonly available but also more applicable in office situations as the computational power of desktop workstation increases.

## 6 Conclusions

With SmartVolumes, a novel use for Voronoi diagrams in computer aided architectural and urban design has been developed. Two practical applications have been made which show the high potential of the technique, in conceptual urban design and in architectural design, bridging from the conceptual phase directly to the planning phase. Other applications, for example in interactive architectural environments, structural and environmental design, have been described in theory and should be investigated in the near future.

## Acknowledgments

This article presents a technique developed for the author's Master of Science final thesis project BehaviourLinks :: Urban Mode which was made at Hyperbody [17], the educational program and research group at TU Delft. Hyperbody is directed by Prof. Ir. Kas Oosterhuis, who has proven to be a very insightful and supportive mentor. The thesis is created as part of Hyperbody's efforts to realize the group design environment Protospace, which is installed in the iWeb building at the department of architecture of Delft University of Technology.

This article also describes an application of this technique in the Digital Pavilion Korea project of ONL [15], office of Kas Oosterhuis and Ilona Lénárd.

The basic principle of the BehaviourLinks environment was inspired by a data exchange course given by Bige Tunçer of the Building Technology department of the Faculty of Architecture.

The SmartVolumes technique, and the BehaviourLinks design environment were to equal parts implemented in the Virtools game development environment and Visual

C++. The fast computation of three-dimensional Delaunay Triangulation and Voronoi Diagrams was originally tested using VTK, the open source Visualization ToolKit C++ library [11]. Eventually CGAL, the Computational Graphics Library [12], was chosen for its wider range of features and robustness. Sylvain Pion and Monique Teillaud of INRIA Sophia Antipolis have been very helpful hosts on the CGAL User Mailing List.

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