Tactical Operating Theatre Scheduling: Efficient Appointment Assignment

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Summary. Finding an appointment for elective surgeries in hospitals is a task that has a direct impact on the optimization potential for offline and online daily surgery scheduling. A novel approach based on bin packing which takes into account limited resource availability (e.g. staff, equipment), its utilization, clinical priority, hospital bed distribution and surgery difficulty is proposed for this planning level. A solution procedure is presented that explores the specific structure of the model to find appointments for elective surgeries in real time. Tests performed with randomly generated data motivated by a mid size hospital suggest that the new approach yields high quality solutions.

1 Introduction and Problem Description

Scheduling elective surgeries at a tactical level deals with finding an appointment for a surgery over a planning horizon of several weeks, while booking individual or generic resources for the particular appointment day. Its output is used as input for the operational planning level problem, which consists in assigning an estimated start and end time to each surgery, as well as solving the corresponding rostering problem.

The importance of an efficient management of operating theatre (OT) resources has been widely documented (refer e.g. to [1]). An important part of a hospital's budget is spent in the OT. Guaranteeing a reduction in idle times of equipment and operating rooms, which in turn results in a more efficient use of the staff's available time, is motivation enough to study this problem in depth.

In practice, the tactical operating theatre scheduling problem (TOTSP) is hardly ever considered as surgery appointments are given on a "first come, first served" basis. The goal of this paper is to present a mathematical model which provides a decision support for finding not only an available and feasible appointment, but also one that will achieve desired criteria. In this sense, the TOTSP will be modelled as a multi-dimensional online packing problem with time windows. Sect. 2 describes the mathematical formulation for the TOTSP. Sect. 3 presents the exact solution method. Sect. 4 reports the computational experience and compares the performance of the proposed approach with the "first fit" (FF) method often used in practice. Finally, Sect. 5 presents some conclusions and directions for further research.

2 Mathematical Formulation of the TOTSP

The problem of finding an appointment for a surgery is considered over a desired time window which describes the patients' preferences for the surgery day and the medical requirements predefined by the surgeon. It is assumed that all departments of the hospital have information regarding resource availabilities and that staff qualifications have been identified. The required notation is introduced as follows:

Index sets T = Set of days resulting from the discretization of the planning horizon (e.g. a year is divided into 365 days); S = Set of elective surgeries, of which the first s_{i-1} already have a fixed appointment and surgery s_i is the surgery that needs to be assigned to $t \in T$; R = Set of resources (e.g. rooms, equipment, staff, beds); R_s = Set of resources required to carry out surgery s; $I = [\underline{I}, \overline{I}]$ = Set of days during which surgery s_i is desired to take place ($I \subset T$).

Parameters c_1, c_2 = penalty weights awarded to scheduling a surgery with overtime or outside the time window I, respectively; ℓ_{rs_k} = amount of time required of resource r for surgery s_k; u_{rs_k} = number of days in which surgery s_k requires resource r; q_{rt} = regular capacity of resource r on day t; v_{rt} = additional capacity of resource r on day t; $M = \max_{r \in R, t \in T}\{q_{rt}\}$.

Decision variables $x_{s_i t} = 1$ if surgery s_i is assigned to day t, and 0 otherwise; $y_{rt} = 1$ if resource r does not require additional capacity on day t, and 0 otherwise.

The following packing formulation is proposed to model the TOTSP, given that surgery s_i has a desired time window $[\underline{I}, \overline{I}]$ during which the surgery should be scheduled and that all previous surgeries $s_1, \ldots, s_{i-1}$ already have a fixed appointment.

$$\text{Min} \;\; c_1 \sum_{t \in T \setminus I} x_{s_i t} \;+\; c_2 \;\left[\, 1 - \left(\sum_{t \in T} (x_{s_i t} \cdot \prod_{r \in R_{s_i}} y_{rt}) \,\right) \,\right] \tag{1}$$

$$\text{subject to} \qquad \sum_{t=\underline{I}}^{|T|} x_{s_i t} = 1 \tag{2}$$

$$\sum_{t=1}^{\underline{I}-1} x_{s_i t} = 0 \tag{3}$$

$$\sum_{k=1}^{i} \ell_{r s_k} [x_{s_k t} + \sum_{j=2}^{u_{r s_k}} x_{s_k t-j+1}] \leq q_{rt} + M(1 - y_{rt}), \forall r \in R_{s_i}, t \in T \tag{4}$$

$$\sum_{k=1}^{i} \ell_{r s_k} [x_{s_k t} + \sum_{j=2}^{u_{r s_k}} x_{s_k t}] \leq M y_{rt} + q_{rt} + v_{rt}, \forall r \in R_{s_i}, t \in T \tag{5}$$

$$x_{s_i t} \in \{0, 1\}, \qquad \forall t \in T \tag{6}$$

$$y_{rt} \in \{0, 1\}, \qquad \forall r \in R_{s_i}, t \in T \tag{7}$$

The objective function (1) consists of two terms, namely the penalty factor c_1 when time window I is not satisfied and the penalty factor c_2 when at least one resource r required by surgery s_i uses additional capacity. Equation (2) ensures that upon the existence of a feasible solution, the surgery is appointed to a day of the planning horizon T. Equation (3) ensures that a surgery is not scheduled before the start of the time window $\underline{I}$. Inequalities (4) and (5) represent the common notation for disjunctive constraints and describe whether the surgery's requirement for resources $r \in R_{s_i}$ can be satisfied under consideration of the previously reserved capacities for surgeries $s_1, \ldots, s_{i-1}$. In particular, Inequalities (4) are soft constraints since the capacity may be expanded by the parameter v_{rt} as in Inequalities (5), where the auxiliary variable y_{rt} indicates which of the two inequalities is binding for a particular r and t. Finally, Relations (6) and (7) define the domain of all decision variables as being binary.

The use of the above formulation presents a close relation to practice relevant criteria when determining an appointment for a surgery. Due to medical and patient preferences, an appointment is searched within the desired time window I. Such an appointment will be selected taking into account that a low over- and under-utilization of resources is desired, that hospitalization bed use is levelled, that no staff overtime is incurred and that such a solution can be found in real time. Based on field work carried out in several German hospitals, satisfying the desired time window is more important than incurring permissible overtime (i.e. $c_1 > c_2 > 0$). The proposed solution method described in the next section makes use of this fact and of the structure of the feasible space of the TOTSP to find an optimal and practice relevant solution using a hybrid algorithm based on simple bin packing rules.

3 Solving the TOTSP

The feasible space of the TOTSP can be divided into equivalence classes according to their objective values. All feasible appointment days within the time window I that can be assigned to a surgery without any resources incurring overtime have the same objective function value, namely zero. Furthermore, all feasible appointment days within the time window and where at least one resource incurs overtime, have the same objective function value, namely c_2. Likewise, feasible appointment days lying outside I and that do not require overtime for any of the resources, have the same objective function value, namely c_1. Finally, feasible appointment days that lie outside I and which require overtime for at least one of the surgery's required resources, have an objective function value of $c_1 + c_2$. Based on this structure of the feasible space, finding an optimal solution for the TOTSP consists in selecting one solution amongst those solutions in the best equivalence class. Such a selection will be carried out according to the practical situation arising in each instance of the TOTSP.

Finding a solution within the first equivalence class will be done either with a FF or with a "best fit" (BF) strategy. The parameter that triggers either strategy is the desired time window I. Instances vary depending on its size and how immediate the start of the interval is. Since unused capacity corresponding to immediate days is lost when these become part of the past, it is important that days in the immediate future are filled to capacity. The FF strategy thus looks for the first appointment day that does not require overtime for any of the resources within candidate days belonging to the set $\mathcal{FF} = I \cap T_{FF}$, assuming that $\mathcal{FF}$ is non-empty and where T_{FF} is a hospital dependent and predefined number of days in the future that are desired to be filled. If such a solution is found, it will be optimal as it belongs to the first equivalence class. Otherwise, a BF strategy will be applied to the set of candidate appointment days $\mathcal{BF} = I \cap {T_{FF}}^C$. The BF strategy searches for each day in the set $\mathcal{BF}$, the resource $r \in R_s$ with the tightest fit. For the day to be eligible, no overtime is incurred for any of the resources. The BF strategy then selects the appointment day as the day $t \in \mathcal{BF}$ that has the resource with tightest fit. If there are several candidate days with the same tightest fit, a tie breaking rule is applied according to the sum of squares (SSq) rule. The SSq rule analyzes the bed distribution for a portion of the days of the planning horizon T, say $\tilde{T} = [\underline{I} - p, \overline{I} + q]$, with p, q parameters selected in such a way that $\tilde{T} \subseteq T$. The optimal solution is then obtained by finding the solution to the following expression (and assuming that β_t represents the number of available beds on day t):

$$t^* = argmin_{t \in \tilde{T}} \{ \sum_{t \in T} \beta_t^2 + \sum_{t \in \tilde{T} \setminus \{\overline{I}+q\}} (\beta_{t+1} - \beta_t)^2 \} \tag{8}$$

If no feasible solution is found using either the FF or the BF strategy, then the first equivalence class is empty and solutions belonging to the second equivalence class (time window I is fulfilled and overtime is incurred for at least one of the required resources) will be optimal. Assuming that there exist feasible solutions, the optimal solution is thus found by selecting the day in time window I with the lowest incurred overtime as a result of the soft capacity constraints in the TOTSP. Should the equivalence class be empty, then the optimal solution will belong to the third equivalence class, which includes those solutions where the time window I is violated and no overtime is incurred. For this, a FF strategy is employed to find the first day after $\overline{I}$ for which an appointment can be found and all resources do not require the use of overtime. Only in the case that this third equivalence class is empty, will a FF strategy be required to find an appointment after $\overline{I}$.

4 Computational Experience

The proposed hybrid method was implemented in C++ and solved on a Pentium 4 PC with a 1.7 GHz processor and 512 MB RAM. Randomly generated instances were created based on a pool of 18 frequent surgeries in a mid size hospital in Germany. These instances consisted of patient arrival over a course of $11 - 36$ weeks and requiring a surgery appointment within a time horizon of six months. Four surgery rooms, 21 staff members (surgeons, anesthesiologists), one nursing ward with $21 - 35$ beds and one intensive care unit with nine beds were modelled. A surgery team consisted of at least two members and at most four. Finally, the set T_{FF} corresponded to the next week.

The results of the proposed hybrid approach are compared in Table 1 with the FF strategy commonly used in practice. It can be seen that the proposed solution approach performs better than the classical FF approach. Moreover, on average, an appointment was found within 0.3 seconds with the new method.

Table 1. Results of the hybrid and FF strategy to solve the TOTSP

Instance	Satisfaction of I (%)		Avg. deviation from I (days/surgery)		Overbooking of ORs (total hours)	
	Hybrid	FF	Hybrid	FF	Hybrid	FF
I-1	93	89	0,48	0,77	174	235
I-2	79	76	2,24	2,43	224	266
I-3	53	48	8,69	10,65	262	303
I-4	86	83	1,45	1,77	43	59
I-5	81	76	2,08	2,56	42	65
I-6	87	84	1,34	1,70	42	59
I-7	96	93	0,38	0,59	17	31
I-8	93	91	0,70	0,67	22	31
I-9	77	69	1,53	2,82	130	133
I-10	69	62	2,46	3,98	118	115
I-11	86	83	1,57	1,95	27	27

5 Conclusions and Outlook

The TOTSP formulation and solution method proposed in this paper support the process of finding an appointment for a surgery and yield a solution in real time. The model considers practice relevant aspects like a desired time interval during which the surgery has to be carried out, thus keeping waiting times within grasp. It utilizes resources efficiently which allows downstream levels of planning (next-day surgery scheduling and online scheduling) to return larger overall improvements in comparison to common practices in hospitals. Possible extensions to the model include allowing previously fixed appointments to be rescheduled to other days or to collect all incoming appointment requests during the course of a certain time period (e.g. one day) and then assign the appointments accordingly.

References

1. Macario A, Vitez TS, Dunn B, McDonald T (1995) Where are the cost in perioperative care? Aneshtesiology 83(6): 2–4