Clustering-Based Minimum Energy Wireless m-Connected k-Covered Sensor Networks

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Abstract. Duty-cycling is an appealing solution for energy savings in densely deployed, energy-constrained *wireless sensor networks* (WSNs). Indeed, several applications, such as intruder detection and tracking, require the design of k-covered WSNs, which are densely in nature and where each location in a monitored field is *covered* (or *sensed*) by at least k active sensors. With duty-cycling, sensors can be turned *on* or *off* according to a scheduling protocol, thus reducing the number of active sensors required to k-cover a field and helping all sensors deplete their energy slowly and uniformly. In this paper, we propose a duty-cycling framework, called *clustered randomized m-connected k-coverage* (CRACC_{mk}), for k-coverage of a sensor field. We present two protocols using CRACC_{mk}, namely T-CRACC_{mk} and D-CRACC_{mk}, which differ by their degree of granularity of network clustering. We prove that the CRACC_{mk} protocols are minimum energy m-connected k-coverage protocols in that each deploys a minimum number of active sensors to k-cover a sensor field and that k-coverage implies m-connectivity between all active sensors, with m being larger than k. We enhance the practicality of the CRACC_{mk} protocols by relaxing some widely used assumptions for k-coverage. Simulation results show that the CRACC_{mk} protocols outperform existing k-coverage protocols for WSNs.

Keywords: WSNs, Duty-cycling, Clustering, Coverage, Connectivity.

1 Introduction

Coverage and *connectivity* have been jointly addressed in *wireless sensor networks* (WSNs). While coverage is a metric that measures the quality of surveillance provided by a WSN, connectivity provides a means for *source sensors* (or simply *sources*) to report their sensed data to the *sink*. In particular, several real-world applications, such as intruder detection and tracking, require high degree of coverage. Hence, the first challenge is determining the number of active sensors required to achieve a certain degree of coverage requested by an application. Also, for such densely deployed WSNs, where sensors have limited battery power (or *energy*), the second challenge is designing an energy-efficient duty-cycling protocol that turns sensors *on* or *off* during the network operational lifetime. This mechanism helps sensors save energy and extend their lifetime.

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1.1 Motivations and Problem Statement

In this paper, we focus on m-connected k-coverage in highly dense deployed WSNs, where each location in a sensor field (SF) is *covered* (or *sensed*) by at least k active (or awake) sensors while maintaining m-connectivity between *all* active sensors. For some real-world applications, such as intruder detection and tracking, the design of this type of over-deployed WSN (i.e., m-connected k-covered WSNs) is necessary. Indeed, the limited energy of sensors and the difficulty of replacing and/or recharging their batteries in hostile environments require that sensors be deployed with high density [14] in order to extend the network lifetime. Also, to cope with the problem of sensor failures due to low energy and to achieve high data accuracy, redundant coverage is an effective solution. Moreover, connectivity between sources and sink should also be guaranteed so data originated from the former could reach the latter for further analysis. Thus, coverage and connectivity should be ensured for the correct operation of WSNs. Finally, for such densely and energy-constrained WSNs, it is important that sensors be duty-cycled to save energy. With duty-cycling, sensors are turned *on* or *off* according to a scheduling protocol, thus reducing the number of active sensors required for k-coverage so all sensors deplete their energy slowly and uniformly. Our study is motivated by three main questions:

1. What is a necessary and sufficient condition of the sensor spatial density for complete k-coverage of a SF?
2. What is a relationship between the sensing and communication ranges of sensors to k-cover a SF while ensuring m-connectivity between active sensors?
3. How can we design a duty-cycling protocol for densely deployed WSNs to k-cover a SF with a minimum number of active and m-connected sensors?

1.2 Contributions and Organization

The major contributions of this paper can be summarized as follows:

1. We compute the minimum sensor density required to k-cover a SF. We find that this density depends only on k and the sensing range of sensors.
2. We prove that all active sensors in a k-covered WSN are m-connected if the communication range of sensors is at least equal to their sensing range.
3. We propose a duty-cycling framework, called *clustered randomized m-connected k-coverage* (CRACC$_{mk}$), for k-coverage of a SF while ensuring m-connectivity between all active sensors. Then, we present two minimum-energy configuration protocols using CRACC$_{mk}$, namely T-CRACC$_{mk}$ and D-CRACC$_{mk}$, which differ by their degree of network clustering granularity. Then, we relax some widely used assumptions for coverage in WSNs to enhance the practicality of T-CRACC$_{mk}$ and D-CRACC$_{mk}$. Simulations show that D-CRACC$_{mk}$ outperforms other existing k-coverage protocols for WSNs.

The remainder of this paper is organized as follows. Section 2 presents some assumptions and definitions while Section 3 reviews related work. Section 4 discusses the CRACC$_{mk}$ framework for m-connected k-coverage in dense WSNs and Section 5 describes T-CRACC$_{mk}$ and D-CRACC$_{mk}$ protocols using CRACC$_{mk}$. Section 6 presents simulations of T-CRACC$_{mk}$ and D-CRACC$_{mk}$ while Section 7 concludes the paper.

2 Assumptions and Definitions

In this section, we present our assumptions and key definitions. Relaxation of some widely used assumptions in WSN coverage will be discussed in Section 5.

Assumption 1 (Static and location-aware WSN). *All sensors and a single sink are static and aware of their locations via some localization technique [7].*

Assumption 2 (Sensing and communication disk model). *The sensing range of a sensor s_i is a disk of radius r_i, centered at ξ_i (the location of s_i) and defined by the point set $SD(\xi_i, r_i) = \{\xi \in \mathrm{IR}^2 : \ |\xi_i - \xi| \ \leq r_i\}$ (also called sensing disk of s_i), where $|\xi_i - \xi|$ is the Euclidean distance between ξ_i and ξ. Also, the communication range of a sensor s_i is a disk of radius R_i, centered at ξ_i and defined by the point set $CD(\xi_i, R_i) = \{\xi \in \mathrm{IR}^2 : \ |\xi_i - \xi| \ \leq R_i\}$ (also called communication disk of s_i).*

Assumption 3 (Homogeneous sensors). *All sensors have the same sensing range and same communication range.*

Assumption 4 (Random and uniform deployment). *All sensors are randomly and uniformly deployed in a square sensor field.*

Definition 1 (Sensing neighbor set). *The sensing neighbor set of a sensor s_i, denoted by $SN(s_i)$, consists of all sensors in the sensing disk of s_i.*

Definition 2 (Communication neighbor set). *The communication neighbor set of a sensor s_i, denoted by $CN(s_i)$, is a set of all sensors located in the communication disk of s_i.*

Definition 3 (k-Coverage, m-connectivity, and degree of coverage). *A point p in a region A is said to be k-covered if it belongs to the intersection of sensing disks of at least k sensors. A region A is said to be k-covered if every point $p \in \mathrm{A}$ is k-covered. A k-covered WSN is a WSN that k-cover a SF. We call degree of coverage provided by a WSN the maximum value of k such that a SF is k-covered. An m-connected WSN is a WSN in which each pair of sensors is connected by at least m paths.*

Definition 4 (Width of a closed convex area). *The width of closed convex area A is the maximum distance between parallel lines that bound A.*

Definition 5 (Largest enclosed disk). *The largest enclosed disk of a closed convex area A is a disk that lays inside A and whose diameter is equal to the minimum distance between any pair of points on A's boundary.*

3 Related Work

Adlakha and Srivastava [1] proposed an exposure-based model to find the sensor density required to achieve full coverage of a desired region based on the physical characteristics of sensors and the properties of the target. Bai *et al.* [3] proposed an optimal deployment strategy to achieve full coverage and 2-connectivity regardless of the relationship between R and r. Huang *et al.* [6] studied the relationship between sensing coverage and communication connectivity of WSNs and proposed distributed protocols to guarantee both coverage and connectivity of WSNs. Kumar *et al.* [8] showed that the minimum number of sensors needed to achieve k-coverage with high probability is approximately the same regardless of whether sensors are deployed deterministically or randomly, if sensors fail or sleep independently with equal probability. Lazos and Poovendran [9] formulated the coverage problem in heterogeneous WSNs as a set intersection problem and derived analytical expressions, which quantify the coverage achieved by stochastic coverage. Li *et al.* [10] proposed efficient distributed algorithms to optimally solve the best-coverage problem with the least energy consumption. Megerian *et al.* [12] proposed optimal polynomial time worst and average case algorithm for coverage calculation based on the Voronoi diagram and graph search algorithms. Shakkottai, *et al.* [13] gave necessary and sufficient conditions for 1-covered, 1-connected wireless sensor grid network. A variety of algorithms have been proposed to maintain connectivity and coverage in large WSNs. Xing *et al.* [16] proved that if the radius R of the communication range of sensors is at least double the radius r of their sensing range, the network is connected provided that coverage is guaranteed. They also proposed a k-coverage configuration protocol regardless of the relationship between R and r. Zhang and Hou [20] proposed a distributed algorithm, called *Optimal Geographical Density Control*, to keep a small number of active sensors in a WSN regardless of the relationship between sensing and communication ranges. Zhou *et al.* [21] discussed the problem of selecting a minimum size connected k-cover. They proposed a greedy algorithm to achieve k-coverage with a minimum set of connected sensors. Tian and Georganas [15] improved on the work in [16], [20] by proving that if the original network is connected and the identified active nodes can cover the same region as all the original nodes, then the network formed by the active nodes is connected when the communication range is at least twice the sensing range.

Although all these approaches on coverage and connectivity are promising, none of them provided an exact value on the minimum density of active sensors required to achieve k-coverage. Moreover, all of them were based on the claim that k-coverage implies k-connectivity when the radius of the communication disks of sensors is at least double the radius of their sensing disks [16]. Our work is complementary to these approaches in the two following ways: first, we compute the minimum sensor spatial density necessary for complete k-coverage of a sensor field. Second, we derive a tighter bound on network connectivity of k-covered WSNs, where the radius of the communication disks of sensors only needs to be at least equal to the radius of their sensing disks.

4 Our Framework for m-Connected k-Coverage

In this section, we first model the m-connected k-coverage problem in WSNs. Then, we present our duty-cycling framework, called *clustered randomized m-connected k-coverage* (CRACC_{mk}), to k-cover a SF while maintaining m-connectivity between all active sensors.

4.1 m-Connected k-Coverage Problem Modeling

Solving the m-connected k-coverage problem in WSNs requires finding a sensor deployment strategy such that each location in a SF is covered by at least k active sensors while ensuring m-connectivity between all active sensors at any time during the WSN operation. Our approach solution to the k-coverage problem in WSNs consists of decomposing it into two sub-problems, namely *sensor field slicing* and *sensor selection*, and solving them. The *sensor field slicing* problem is to slice a SF into small regions of particular shape (which will be defined later), each of which is guaranteed to be k-covered provided that at least k sensors are randomly deployed in it. The *sensor selection* problem is to select a minimum subset of sensors to remain active such that each location in a SF is guaranteed to be k-covered. Thus, our solution to the k-coverage problem is to find out how to achieve at least k-coverage of a SF and select an appropriate subset of active sensors so that each location in a SF is k-covered. Besides selecting a *minimum* number of active sensors, for energy efficiency, all selected sensors should have the *maximum* remaining energy. Hence, the m-connected k-coverage problem that we deal with is called *min-max m-connected k-coverage* and is described as follows:

Problem: *min-max m-connected k-coverage*
Instance: A SF, a set S of sensors, and a positive integer k.
Question: Select a minimum subset $S_{\min} \subset S$ of sensors such that each location in CF is k-covered, the network induced by all sensors in $S_{\min}$ is m-connected, and $\sum_{s_i \in S_{\min}} E_{\text{rem}}(s_i)$ is maximized.

The problem of selecting a minimum subset of sensors to remain active for k-coverage of a sensor field is NP-hard [21], and so is *min-max m-connected k-coverage*. Hence, we propose efficient approximation algorithms to solve it.

4.2 Network Slicing-Based m-Connected k-Coverage

This section provides our solution to the sensor field slicing problem, where all sensors have the same sensing and communication disks whose radii are r and R, respectively. First, we provide a characterization of k-coverage of a SF. To this end, we need to compute the maximum size of a convex area A that is guaranteed to be k-covered when exactly k sensors are deployed in it. Lemma 1 gives an upper bound on the width of such a k-covered area.

Lemma 1. *Let r be the radius of the sensing disk of sensors and $k \geq 3$. A convex area* A *is guaranteed to be k-covered when k homogeneous sensors are deployed in it, if the width of* A *does not exceed r.*

Proof. Each point $p \in A$ is k-covered if $|\xi_i - p| \leq r$, for all $1 \leq i \leq k$. In particular, this should be true for the locations of sensors. Thus, for any pair of sensors s_i and s_j covering A the maximum distance between s_i and s_j is r so that any location in A is covered by k sensors. Otherwise, there must be a pair of sensors s_i and s_j such that $|\xi_i - \xi_j| > r$, meaning that the locations of the two sensors are not being covered by both sensors at the same time. This contradicts the hypothesis that all $p \in$ A, including the locations of sensors, are k-covered by all sensors s_l, for all $1 \leq l \leq k$, and in particular s_i and s_j. Thus, the width of region A cannot exceed r.

Lemma 2 (instance of Helly's Theorem [4]) will help us compute the minimum sensor spatial density required to guarantee k-coverage of a SF. More specifically, this lemma together with a nice geometric structure, called *Reuleaux triangle* [23], will be used to characterize k-covered WSN, i.e., how a WSN can guarantee k-coverage of a SF.

Lemma 2. *The intersection of k sensing disks is not empty if and only if the intersection of any three of those k sensing disks is not empty, where $k \geq 3$.*

Theorem 1, which exploits the results of Lemma 1 and Lemma 2, computes the minimum sensor spatial density necessary for complete k-coverage of a SF.

Theorem 1. *Let $k \geq 3$. The minimum sensor spatial density required to guarantee k-coverage of a SF is computed as $\lambda(r,k) = \frac{2\,k}{(\pi - \sqrt{3})\,r^2}$, where r is the radius of the sensing disks of sensors.*

Proof. First, we compute the maximum area that is guaranteed to be k-covered provided that k sensors are deployed in it. Let A be the intersection area of the

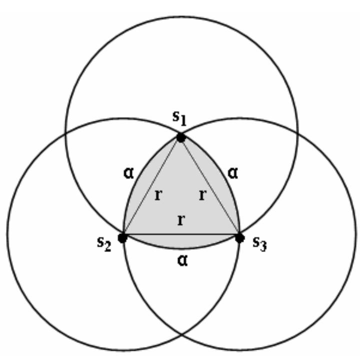

Fig. 1. Intersection of three disks

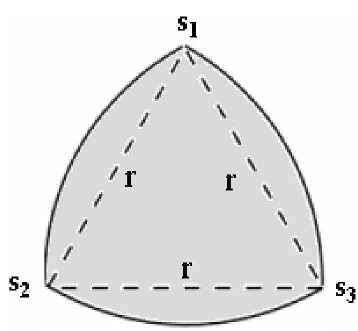

Fig. 2. Reuleaux triangle

sensing disks of k sensors. From Lemma 1, it is clear that the width of A should be upper-bounded by r so that any location in A is k-covered by these k sensors. Using the Venn diagram given in Figure 1, the maximum size of the intersection of the sensing disks of sensors s_1, s_2, and s_3, called *Reuleaux triangle* [23] and denoted by $RT(r)$, is obtained when s_1, s_2, and s_3, are symmetrically located from each other so that the distance between any pair of sensors is equal to r (Figure 2). We refer to this model as the *Reuleaux Triangle* model. As can be seen from Figure 1, a WSN is connected if each active sensor senses the location of at least another active sensor. Thus, the maximum size of A denoted by $A_{\max}(r)$ is upper-bounded by the area of $RT(r)$, which is given by $A_{\max}(r) = A_1 + 3A_2$, where $A_1 = \sqrt{3}\, r^2/4$ is the area of the central equilateral triangle of side r and $A_2 = (\pi/6 - \sqrt{3}/4)\, r^2$ is the area of each of the three curved regions α. Hence, to achieve k-coverage of a SF, k sensors should be deployed in an $RT(r)$ area. Thus, the *minimum sensor spatial density* that guarantees k-coverage of SF is equal to $\lambda(r, k) = k/\mathrm{A}_{\max}(r) = 2\, k/(\pi - \sqrt{3})\, r^2$.

Notice that $\lambda(r, k)$ depends only on r and k, and decreases as r increases, thus reflecting the expected behavior. Adlakha and Srivastava [1] also showed that the number of sensors required to cover an area of size A is in the order of $\mathrm{O}\ (A/\hat{r}^2_2)$, where $\hat{r}_2$ is a good estimate of the radius r of the sensing disk of sensors. Specifically, r lies between $\hat{r}_1$ and $\hat{r}_2$, where $\hat{r}_1$ overestimates the number of sensors required to cover A, while $\hat{r}_2$ underestimates it.

Theorem 2, which follows from Theorem 1, states a necessary and sufficient condition for complete k-coverage of a SF.

Theorem 2. *Let $k \geq 3$. A SF is guaranteed to be k-covered if and only if any Reuleaux triangle region in the SF contains at least k active sensors.*

Theorem 3, which follows from the proof of Theorem 1, states that k-coverage implies connectivity only if $R \geq r$.

Theorem 3. *Let $k \geq 3$. A k-covered WSN is guaranteed to be connected if the radius R of the communication range of sensors is at least equal to the radius r of their sensing range, i.e., $R \geq r$.*

Theorem 4 computes the network connectivity of k-covered WSNs.

Theorem 4. *Assume a uniformly random distribution of sensor in a square sensor field and let r and R be the radii of the sensing and communication disks of sensors, respectively, $\alpha = R/r$ and $k \geq 3$. The connectivity m of a k-covered WSN is given by $m = \pi\, \alpha^2\, k/2\, (\pi - \sqrt{3})$.*

Proof. Consider a *boundary sensor* s_b (i.e., sensor located at one corner of a square field that has the least communication neighbor set). Although it has been proved that the optimum location of the sink in terms of energy-efficient data gathering is the center of the field [11], the sink could be located anywhere in the field. Thus, s_b can be either a sensor or the sink itself. Following the same approach used by Xing *et al.* [16], sensor s_b can be isolated by removing all of its communication neighbors. In other words, at least $\lambda(r, k) \times \pi\, R^2/4$ sensors should be removed. Thus, the network connectivity of k-covered WSNs is equal to $m = \pi\, \alpha^2\, k/2\, (\pi - \sqrt{3})$.

Given that $\alpha = R/r \geq 1$, it is easy to check that $m \geq 1.11k > k$. However, Xing *et al.* proved in [16] that the connectivity of k-covered WSNs is equal to k provided that $R \geq 2\, r$. Moreover, Xing *et al.* [16] assumed in their analysis that there are k coinciding sensors at some location. Our measure of network connectivity of k-covered WSNs, however, is based on the minimum sensor spatial density necessary for complete k-coverage of a SF. Thus, our network connectivity measure is more realistic and tighter. Furthermore, we only require that $R \geq r$ for a k-covered WSN to be m-connected, where $m \geq 1.11k$. It is worth noting that m-connectivity implies m disjoint paths between any pair of sensors although the proof of Theorem 4 considers the number of communication neighbors a sensor has. Indeed, under the assumption of uniform sensor distribution, each sensor has at least m communication neighbors, where $m \geq 1.11k$ since $R \geq r$.

Previous Work on k-Coverage Characterization. According to [16] ([20], respectively), a SF is k-covered if all *intersection points* (*crossing points*, respectively) between the boundaries of sensing disks of sensors and all the *intersection points* between the boundaries of sensing disks of sensors and the boundary of a SF are k-covered. This is a generalization of the result for 1-coverage [5]. Hence, if two sensing disks intersect, at least one more sensing disk needs to cover their intersection/crossing point. In case of 1-coverage, a location that coincides with an intersection/crossing point would be 3-covered instead of 1-covered. Thus, both approaches [16], [20] require more than enough sensors to k-cover a SF. In addition to characterizing k-coverage, our approach *quantifies* the minimum sensor density $\lambda(r, k)$ required to k-cover a SF.

Slicing Approach. Let SF be a square sensor field and $k \geq 3$. Based on the minimum sensor spatial density $\lambda(r, k)$, it is easy to check whether a given WSN can k-cover SF. For this purpose, we propose a *slicing* scheme of CF by dividing it into overlapping Reuleaux triangles of width r, called *slices*, such that two adjacent slices intersect in a region shaped as a *lens* (also known as the *fish bladder*) as shown in Figure 3. This implies that SF is sliced into regular triangles of side r. The result of this slicing operation is called *slicing grid*. Figure 4 shows a slicing grid of SF.

4.3 Impact of Network Slicing on Sensor Selection

Slicing a WSN can be *static* or *dynamic*. Next, we show the problems caused by a static slicing approach and propose a dynamic one as a remedy to the former.

Static Network Slicing. Our sensor selection scheme exploits the overlap between adjacent slices to select a minimum number of active sensors in each round for complete k-coverage of a SF. As can be seen from Figure 3, sensors

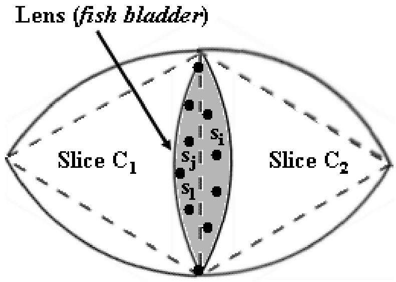

Fig. 3. Intersection of adjacent slices

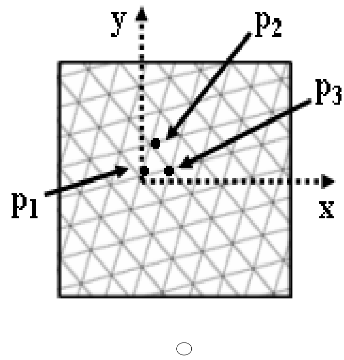

Fig. 4. Random slicing grid of a field

located in the lens of two adjacent slices participate in the k-coverage of the area associated with the union of these two slices. Lemma 3 states this result.

Lemma 3. *Sensors located in a lens participate to k-cover its adjacent slices.*

Notice that each slice overlaps with at most three others. By Lemma 3, sensors located in the three lenses of a given slice should be selected first in each round. This process is repeated until all slices in a SF are k-covered. We assume that each slice has a unique *id*.

The sensor selection scheme described earlier generates only one subset of active sensors to k-cover a SF. If this scheme is executed in each round on the same slicing grid, such as the one given in Figure 4, sensors located in the lenses would suffer from a severe energy depletion problem. Thus, it would be more efficient if in each round a different subset of sensors is selected for k-coverage of a SF. Next, we describe a strategy based on dynamic network slicing in order to achieve this goal.

Dynamic Network Slicing. Our goal is to select different subsets of sensors $S_i,\ i \geq 1$ such that each subset S_i is selected to remain active in the i^{th} round to k-cover a SF. Notice that in order to achieve a better load balancing among the sensors, we could add a restriction that the selected subsets are mutually disjoint. However, the disjointness constraint yields a small number of mutually disjoint subsets of sensors. Thus, we only require that those selected subsets of sensors be *partially disjoint*.

The first question that we want to address now is: *How would partially disjoint minimum subsets of sensors be selected to k-cover a SF*? To address this question, we consider the *dynamics of slicing grid* from one round to another. Since our scheme for selecting active sensors highly prioritizes the ones located in the lenses of all slices, it is important that those lenses be able to scan the entire SF, and hence include distinct subsets of sensors in different rounds. Thus, the slicing grid undergoes some dynamics to achieve balanced load among sensors during the operation of T-CRACC$_{mk}$ and D-CRACC$_{mk}$.

The second question that we want to address now is: *How would a slicing grid of a SF be randomly generated?* First, we randomly generate one point p_1 in a SF, which is temporarily considered as the center of the Euclidean plane. To randomly determine a second point p_2, we generate a random angle $0 \leq \theta \leq 2\,\pi$ so that the segment $\overline{p_1 p_2}$ forms an angle θ with the x-axis centered at p_1 and the length of $\overline{p_1 p_2}$ is r. Then, we deterministically find a third point p_3 to form the first regular triangle (p_1, p_2, p_3), called *reference triangle*, as shown in Figure 4. All other regular triangles are computed based on the reference triangle.

5 m-Connected k-Coverage Protocol Design

In this section, we describe our T-CRACC$_{mk}$ and D-CRACC$_{mk}$ protocols for m-connected k-coverage in WSNs based on their network clustering granularity. Then, we relax some widely used assumptions to enhance their practicality.

In general, the sink is connected to an infinite source of energy, such as a wall outlet, and thus has no energy constraint. In both T-CRACC$_{mk}$ and D-CRACC$_{mk}$, the sink is responsible for randomly generating a slicing grid of a SF and selecting a cluster-head for each cluster in each round. Each cluster-head is physically located within its cluster and is in charge of selecting some of its sensing neighbors to k-cover it. To this end, the sink should be aware of all sensors' locations. Moreover, we do not assume any strict ordering of the cluster-heads that determines the order in which cluster-heads select their active sensors. However, neighboring cluster-heads need to coordinate between themselves through message exchanges in order to select a minimum number of sensors to k-cover their clusters. The slicing grid generation and cluster-head selection could be assigned to each sensor in a *round-robin* fashion. However, this solution would be costly for sensors in terms of energy and space.

5.1 The T-CRACC$_{mk}$ Protocol

In T-CRACC$_{mk}$, a cluster is a *slice* ("T" for Reuleaux *triangle*) in a slicing grid and a cluster-head is called *slice-head*. Given that each slice has at most three adjacent slices (Figure 5), the T-CRACC$_{mk}$ protocol requires that each slice-head coordinates its activity with its adjacent slice-heads in order to select a minimum total number of sensors to k-cover a SF. Figure 5 shows slice-head sh_0 sharing three lenses with slice-heads sh_1, sh_2, and sh_3. For instance, sh_0 could k-cover its slice by selecting sensors located in its three lenses. Then, it communicates the numbers n_1, n_2, and n_3 of sensors selected from lenses *Lens* 1, *Lens* 2, and *Lens* 3, respectively, to its adjacent slice-heads sh_1, sh_2, and sh_3, respectively. Slice-head sh_1 would need to select $k - n_1$ more sensors from its lenses to k-cover its slice. It would definitely coordinate with its adjacent slice-heads to k-cover its slice and so does each slice-head. Theorem 5 states that T-CRACC$_{mk}$ is a minimum-energy protocol.

Theorem 5. *T-CRACC$_{mk}$ is a minimum energy-consuming protocol.*

Proof. Each slice-head ensures that each slice of a SF is k-covered by exactly k sensors. Thus, by Theorem 2, T-CRACC$_{mk}$ guarantees that a SF is k-covered with a minimum number of active sensors, and hence consumes a minimum amount of energy in each round.

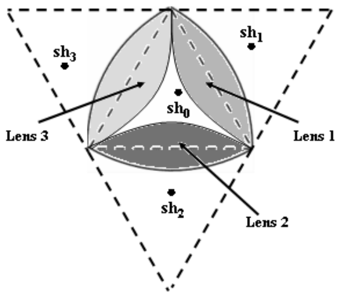

Fig. 5. Slice-heads for T-CRACC$_{mk}$

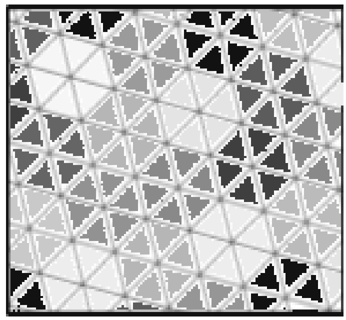

Fig. 6. Clustering for D-CRACC$_{mk}$

5.2 The D-CRACC$_{mk}$ Protocol

D-CRACC$_{mk}$ ("D" for *disk*) has higher network clustering granularity than T-CRACC$_{mk}$. Precisely, each cluster consists of six adjacent slices forming a *disk* (Figure 6). In each round, the sink selects for each cluster a sensor, called *disk-head*, which is located nearer the center of its disk to k-cover it. Similarly, each disk-head needs to coordinate with at most six adjacent disk-heads to k-cover its disk with a minimum number of sensors. Each disk-head manages at most six *interior lenses* (i.e., lenses between adjacent slices of the same disk) and at most six *boundary lenses* (i.e., lenses between adjacent slices of two adjacent disks). Hence, a disk-head should select sensors from its interior lenses with no coordination with other disk-heads but should coordinate with its adjacent disk-heads to select sensors from its boundary lenses. Theorem 6, which is similar to Theorem 5, states that D-CRACC$_{mk}$ is a minimum-energy protocol.

Theorem 6. *D-CRACC$_{mk}$ is a minimum energy-consuming protocol.*

5.3 Promoting T-CRACC$_{mk}$ and D-CRACC$_{mk}$

In this section, we relax the sensing and communication disk (*Assumption 2*) and homogeneous sensor (*Assumption 3*) models. Our goal is to promote the use of T-CRACC$_{mk}$ and D-CRACC$_{mk}$ in real-world scenarios.

Relaxing the Unit Sensing and Communication Disk Models. Zhou et al. [22] found that the communication range of radios is highly probabilistic and irregular. In this section, for tractability of the problem, we consider convex sensing and communication models, where sensors have the same sensing and communication ranges, which are convex but not necessarily circular.

The following results correspond to Lemma 1 and Theorem 1, respectively. Their proof is literally the same as that in Section 4.2 by using the notion of largest enclosed disk of the sensing ranges of sensors instead of their sensing disk.

Corollary 1. *Let $k \geq 3$. A convex area* A *is guaranteed to be k-covered when exactly k homogeneous sensors are deployed in it, if the width of* A *does not exceed r_{led}, where r_{led} is the radius of the largest enclosed disk of the sensing range of sensors.*

Corollary 2. *Let r_{led} be the radius of the largest enclosed disk of the sensing range of sensors and $k \geq 3$. The minimum sensor spatial density required to k-cover a SF by homogeneous convex sensing ranges is given by $\lambda(r_{\text{led}}, k) = 2\,k/(\pi - \sqrt{3})\, r_{\text{led}}^2$.*

To implement T-CRACC$_{mk}$ and D-CRACC$_{mk}$ with the above convex models, the sink should slice a SF into triangles of side r_{led}. Assumption 2 can thus be relaxed using the largest enclosed disk of the sensing ranges of sensors. It is worth noting that even if the sensing and communication ranges of sensors do not have the same convex shape, our results about coverage implying connectivity still hold as long as the communication range of sensors is larger than their sensing range, i.e., the sensing range is entirely included in the communication range. This assumption is realistic and conforming to previous work [20] reporting that the communication range of Berkeley motes is much higher than the sensing range of several typical sensors.

Relaxing the Homogeneous Sensor Model. Real-world applications may require heterogeneous sensors in terms of sensing and communication capabilities in order to enhance network reliability and extend its lifetime [18]. In this section, we consider heterogeneous sensors with different yet convex sensing and communication ranges.

The following results correspond to Lemmae 1 and 2, and Theorem 1, respectively. They can be proved using the concept of largest enclosed disk instead of sensing disk.

Corollary 3. *Let $k \geq 3$. A convex area* A *is guaranteed to be k-covered when exactly k heterogeneous sensors whose sensing ranges are convex but not necessarily circular are deployed in it, if the width of* A *does not exceed $r_{led}^{\min}$, where $r_{led}^{\min}$ is the smallest radius of the largest enclosed disks of the sensing ranges of sensors.*

Corollary 4. *Let $k \geq 3$. The intersection of k heterogeneous convex sensing ranges is not empty if and only if the intersection of any smallest three largest enclosed disks of these k heterogeneous convex sensing ranges is not empty.*

Corollary 5. *The minimum sensor spatial density required to k-cover a SF by heterogeneous sensors whose sensing ranges are convex but not circular is given by* $\lambda(r_{led}^{\min}, k) = 2\,k \Big/ (\pi - \sqrt{3})\, {r_{led}^{\min}}^2$, *where* $r_{led}^{\min}$ *is the minimum radius of the largest enclosed disks of the sensing ranges of heterogeneous sensors and* $k \geq 3$.

In this case, the sink slices a SF into regular triangles of side $r_{led}^{\min}$ and applies the same processing as in Section 4.2. Therefore, the assumption of homogeneous sensors can also be relaxed with slight updates to T-CRACC$_{mk}$ and D-CRACC$_{mk}$. Notice that while these corollaries hold, they may greatly overestimate the sensor spatial density required for guaranteeing full k-coverage of a sensor field. For instance, even if a single sensor with a very small sensing range is deployed, the entire network would be required to have a large sensor spatial density. In this case, it is important that the CRACC$_{mk}$ protocols adapt the sensor spatial density to the sensing ranges of sensors in the area. Due to space limitations, we will address this issue in our future work.

6 Performance Evaluation

In this section, we present the simulation results of T-CRACC$_{mk}$ and C-CRACC$_{mk}$ using a high-level simulator written in the C programming language. We consider a square field of side length 1000 m. We use the energy model given in [19], where the sensor energy consumption in transmission, reception, idle, and sleep modes are 60 mW, 12 mW, 12 mW, and 0.03 mW, respectively. Following [20], *one unit of energy* is defined as the energy necessary for a sensor to stay idle for 1 second. We assume that the initial energy of each sensor is 60 Joules enabling a sensor to operate about 5000 seconds in reception/idle modes [19]. All simulations are repeated 20 times and the results are averaged.

Figure 7 plots $\lambda(r, k)$ versus k, where $r = 30$ m. Figure 8 plots $\lambda(r, k)$ versus the r, where $k = 3$. We observe a perfect match between simulation and analytical results in both experiments. As expected, $\lambda(r, k)$ decreases with r for a fixed k, and increases with k for a fixed r. As can be observed from Figures 7 and 8,

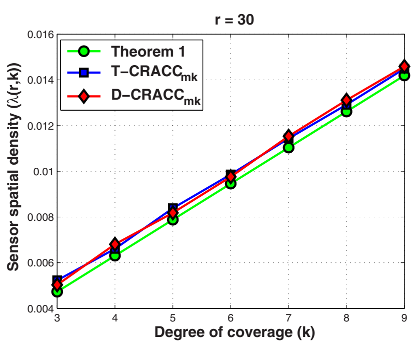

Fig. 7. $\lambda(r, k)$ vs. k

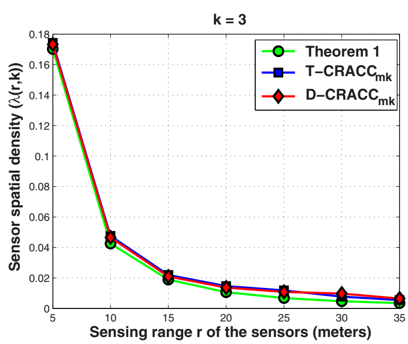

Fig. 8. $\lambda(r, k)$ vs. r

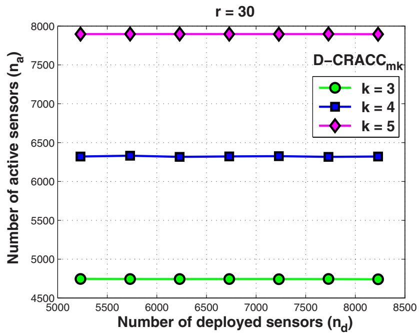

Fig. 9. n_a vs. n_d while varying k

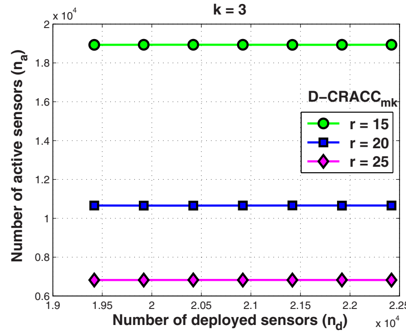

Fig. 10. n_a vs. n_d while varying r

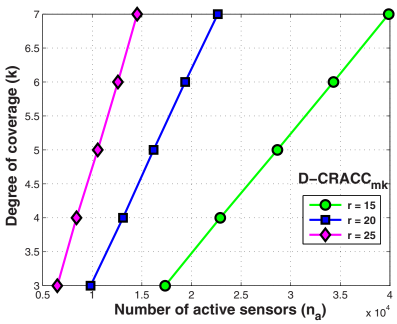

Fig. 11. k vs. n_a

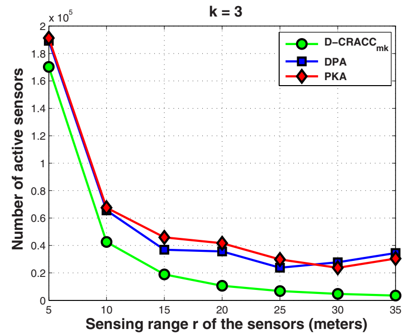

Fig. 12. Performance comparison

both T-CRACC$_{mk}$ and D-CRACC$_{mk}$ require the same number of active sensors. From now on, we focus only on the performance of D-CRACC$_{mk}$ protocol.

Figures 9 and 10 show the number of active sensors versus the total number of deployed sensors in the field for the D-CRACC$_{mk}$ protocol. In Figure 9, we consider different values of k, while in Figure 10, we consider different values of r. For higher values of k, more sensors need to be active to achieve the required coverage. However, for higher values of r, less number of sensors is needed for k-coverage. However, the number of active sensors for a given k does not depend on the number of deployed sensors. It depends only on k and r.

Figure 11 plots k versus the number n_a of active sensors for D-CRACC$_{mk}$. As can be seen, k increases with n_a. Also, k increases with r for fixed n_a. There is also a perfect match between our simulation and theoretical results.

We have also compared our D-CRACC$_{mk}$ protocol with two other distributed k-coverage protocols, namely PKA [17] and DPA [21], which are close to ours. Figure 12 shows that D-CRACC$_{mk}$ uses less number of sensors than PKA [17] and DPA [21] to achieve the same degree k of coverage, thus yielding more energy savings.

7 Conclusion

We have addressed the problem of energy-efficient m-connected k-coverage configuration in WSNs. We have characterized k-coverage in WSNs based on the intersection of sensing disks of k sensors. We have also computed the minimum sensor spatial density required to k-cover a SF. We have proved that k-coverage of a SF implies m-connectivity with $m \geq 1.11k$ when the radius R of the communication disks of sensors is at least equal to the radius r of their sensing disks, i.e., $R \geq r$. Since it is based on the minimum sensor density necessary to achieve full k-coverage of a sensor field, our bound on connectivity of k-covered WSNs is tighter than the one provided by Xing *et al.* [16] and adopted by all subsequent approaches for coverage and connectivity in WSNs. We have proposed two minimum energy-consuming protocols, called T-CRACC$_{mk}$ and D-CRACC$_{mk}$, for complete k-coverage of a SF while all active sensors remain m-connected. Finally, we have extended our analysis by relaxing several assumptions to promote the use of our CRACC$_{mk}$ protocols in real scenarios. Simulation results have showed perfect match with our theoretical ones and that our CRACC$_{mk}$ protocols outperform other existing k-coverage protocols.

Our future work is four-fold. First, we plan to conduct more simulations to compare our protocols with existing ones with respect to energy savings. Second, we also plan to extend T-CRACC$_{mk}$ and D-CRACC$_{mk}$ to three-dimensional (3D) WSNs. For instance, underwater WSNs [2] require design in 3D rather than 2D space. Third, we focus on joint m-connected k-coverage and routing in WSNs. Indeed, most of the routing protocols for WSNs assume that *all* sensors are always *on* during data forwarding. This assumption, however, is not valid in real-world scenarios, where sensors are turned *on* or *off* to save energy. Fourth, we intend to study m-connected k-coverage in WSNs using stochastic models of sensing and communication ranges, and considering shadowing.

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