Interval Forecasting of Crude Oil Price

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Summary. Uncertainty is the main obstacle in predicting crude oil price. Although there are various models and computational methods on crude oil price forecasting in literature, most of them do not effectively predict the variability of crude oil price due to uncertainty. Very recently, Hu and He [2] reported of using ILS (Interval Least Square) approach to forecast the stock market and obtained much better results than that obtained with traditional point methods, In this paper, we investigate if the ILS approach can forecast the relationship between commodity inventory levels and crude oil spot prices effectively. Our empirical study suggests that both the ILS method and the confidence interval method can produce comparable quality forecasts. While the computational result produced by the ILS method seems slightly worse than the 95% confidence intervals in two quality measurements, the differences are negligible. On a new forecasting quality measurement proposed in this paper, the ILS method produces results better than the 95% confidence intervals. Hence, interval method is a feasible alternative in crude oil price forecasting.

1 Introduction

1.1 Forecasting Crude Oil Price

As a strategic resource, crude oil and its trade have attracted extensive attention for a long time. Since the beginning of this century, international crude oil price has continued to rise rapidly. Therefore, the crude oil price forecasting has become a focus of economists and decision makers.

Recently, numerous studies have focused on the relationship between commodity inventory levels and spot prices. Specifically, in the crude oil market, previous work has explained a relationship between crude oil price and total inventories (crude plus products) of Organization for Economic Cooperation and Development (OECD) countries. For example, Pindyck [5] described the short-run dynamic relationship between commodity prices and inventories by using the petroleum markets for illustrations; Considine and Larson [1] studied the presence of risk premiums on crude oil inventories.

V.-N. Huynh et al. (Eds.): Interval/Probabilistic Uncertainty, ASC 46, pp. 353–363, 2008.
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Ye, Zyren, and Shore [6] developed a more practical simple model for short-term crude oil price forecasting. The model is easily interpreted and accepted, hence, it is intuitively appealing to decision-makers. The model also provides good in-sample and out-of-sample dynamic forecasts for the post-Gulf War time period. In addition to providing good forecasting results, a desirable feature of the model is that it can readily be implemented in a spreadsheet or other software package, with the variables easy to update.

Ye, Zyren, and Shore [7] also provided a nonlinear-term model, which improves upon the model that assumes only linear price adjustments to inventory. The model nonlinearity is based on the feature that inventory has a zero lower bound or a minimum operating requirement. Two nonlinear inventory variables are incorporated: one for the low inventory state and another for the high inventory state of the crude oil market. Compared with the linear forecast model, the nonlinear model demonstrates its superiority in both the model fitness and forecasting ability.

1.2 Interval Computing

Moore [3] proposed interval analysis in the late of 1950's. Since then, thousands of research articles and numerous books have appeared on the subject. The website of Interval Computations [8] provides comprehensive information on interval computations. An increasing amount of software resources for interval computations are available through the Internet. Interval computation, different from classical point arithmetic, has contributed a lot in many researching fields. For illustration, Hales and Ferguson solved the Kepler conjecture about the densest arrangement of spheres in space by using interval computing [8]. This work led them the 2007 Robbins Prize by the American Mathematical Society. In his response to this award, Hales explicitly thanked those who developed the tools of interval computations.

Interval computing does contribute unique merits compared with traditional point methods in the following aspects. First of all, interval computing can bound the errors of finite digit floating-point computing. Interval computations yield a pair of numbers, an upper and a lower bound, which are guaranteed to enclose the exact answer. Maybe we still don't know the truth, but at least we know how much we don't know. For example, interval arithmetic has been used in computers to cope with some of the continuous and infinite aspects of mathematics to fence the round-off errors, which might lead to inaccurate results. Secondly, interval computing plays an important role in dealing with uncertainty in computational modeling. In the real world, the value of data may not be precisely known due to certain forms of uncertainty. Hence, it is unreasonable to use point data for that may lead to inappropriate results especially when the inaccuracy is not negligible. One approach to dealing with such problem is to use intervals, which is not an approximate value of the de-sired data but is guaranteed to contain the unknown value. Thirdly, in reality, many variables are bounded by intervals for a given time. For instance, macroeconomic variables definitely vary within intervals during a given period and the point valued data does not reflect the variability of these variables. Therefore, it makes more sense to use interval inputs instead of the point-based data for that they contain more in-formation. Last but not the least, if the predicted variable possesses the property mentioned above, it is more reasonable to provide interval forecasting outputs to decision makers than the point valued ones.

1.3 Motivation of This Work

In this paper, we propose to apply interval methods in monthly crude oil spot price forecasting for the following reasons.

First, as mentioned above, interval data contains more information than point data. In Ye et al. [7], two new variables were incorporated to capture the non-linear behavior of inventory and the forecasting ability has been improved. In other words, these new variables introduce more information to the former linear model in Ye et al [6]. In this paper, instead of adding new variables to the former model, we directly form interval input data to characterize the volatility of all the variables by which more information were included.

Secondly, variables such as crude oil spot price definitely vary frequently within an interval during a given month. Thus it is much more meaningful to supply interval valued forecasting outputs than the ordinary point valued ones.

Thirdly, Hu and He [2] have developed an interval least squares scheme and ap-plied it in forecasting the S & P 500 index. The annual interval forecasts they obtained are in much better quality than that obtained with point methods. Inspired by their work, we would like to investigate whether interval method is reasonable in forecasting other economical variables, as of crude oil spot price. To evaluate the forecasting quality, we not only apply the same measurements proposed by Hu and He, but also propose a new measurement which checks how many times that the forecasting outputs fail to encompass actual monthly average oil spot prices.

The rest of this paper is organized as follows. We introduce the model equations used in this study and review the interval least squares method (ILS) and rolling forecasting in Section 2. Then we describe the data and software for the empirical study in Section 3. We report the computational results in both numerical tables and graphical charts, and compare them with 95% confidence intervals (CI)[1] of point forecasts in Section 4. We conclude this paper with possible future work in Section 5.

2 The Model and Computational Methods

In this study, we use the oil price forecasting model established by Ye, Zyren and Shore [6]. We use this model mainly because of its broad acceptance in the literature. In this section, we briefly review this model and then introduce the interval least square method reported in [2]. Meanwhile, some computational details are provided.

[1] A confidence interval for a point forecast specifies a range of values within which the actual average monthly oil spot price may lie.

2.1 The Model

In Ye, Zyren and Shore [6], the observed level of petroleum inventory is decomposed into two components: the normal level and the relative level. The former is determined by historical seasonal movements and trends, and reflects the normal market demand and operational requirements. The later represents the difference between the observed and normal levels and reflects short-run market fluctuations. The relative inventory level (denoted by RIN) is determined as

$$RINt = INt - IN^*t; \quad IN^*t = a0 + b1T + \sum^{12} k = 2bkDk \tag{1}$$

where IN is the actual inventory level and IN^* is the normal inventory level. $Dk, k = 2, \cdots, 12$, are 11 seasonal dummy variables[2] and T is time (T begins with 1). $a0, b1$ and $bk(k = 2, 3, \cdots, 12)$ are parameters to be estimated. The crude oil price forecasting model is

$$WTIt = a + \sum_{i=0}^{3} biPINt - i + \sum_{j=0}^{5} cjDj911 + dLAPR99 + eWTIt - 1 + \varepsilon \tag{2}$$

In (2), WTI is the West Texas Intermediate crude oil spot price. Subscript t is for the t^{th} month; subscript i is for i^{th} month prior to the t^{th} month; $Dj911(j = 0, 1, \cdots, 5)$ referring to the 6 months from October 2001 to March 2002) is a set of single monthly variables[3] to account for market disequilibrium following the September 11, 2001 terrorist attacks in the United States; $LAPR99$ is a level-shifting variable[4] corresponding to the effect that OPEC quota tightening had on the petroleum market be-ginning in April 1999; a, bi, cj, d and e are coefficients to be estimated.

Relationships between WTI and the explanatory variables are time-varying. That is to say, the relationship may be valid for a limited time period. The relationships and the forecasts are associated with a time-window. By rolling the time window, one obtains a series of forecasts. The overall forecasting quality can be measured by the average performance of the series. We follow the method of rolling forecasting used by Hu and He [2]. In the paper, it is called an in-sample forecast if using the obtained coefficients in a time window to calculate the WTI for the last period in the time window. It is called an out-of-sample forecast if using the obtained coefficients in a time window to calculate the WTI for the first time period that immediately follows the time window.

2.2 Interval Least Square Method

As many economic variables are time-varying, it is more meaningful to analyze interval data rather than point data. This urges the emergence of interval

[2] Dummy variables are binary variables. For example, when it is February, $D2$ equals 1 and other $Dk(k = 3, 4, \cdots, 12)$ equals 0.

[3] $Dj911$ equals 1 at its corresponding month and equals 0 at all the other months.

[4] $LAPR99$ is also a dummy variable. It equals 0 before April 1999 and equals 1 after the month (including April 1999).

econometrics. This cross field is still at its infancy, facing computational difficulties and lack of theoretical basis. As for the simple linear regression for interval data, Hu and He [2] propose a computational scheme. We summarize this scheme here as Interval Least Square method.

In contrast to OLS, the inputs and outputs in ILS are intervals rather than points, though at this stage it still relies on OLS to estimate coefficients of the regression. The computational scheme is as follows:

(i) Read available interval data.
(ii) Find the midpoint of the intervals.
(iii) Solve the linear system of equations: $A\text{mid}x = b\text{mid}$ for x
(iv) Apply the obtained coefficient vector x to calculate the initial prediction with interval data using interval arithmetic
(v) Perform width adjustment for the initial prediction to make a forecast.

3 Data and Software for the Empirical Study

The major difference between this study and the traditional ones is the data type. In this section, we present the details about the data source, data preprocessing, and software used in our empirical study.

3.1 Data Source

The data we used in this study are monthly average West Texas Intermediate crude spot prices and the monthly OECD total inventory levels from the website of Energy Information Association (EIA). Average is commonly used in forecasting monthly oil price. Ye et al. [6, 7] use average as the dependent variable. The monthly data released by EIA official website is also the average of daily prices in a single month. Thus we use the average as the dependent variable and predictor in point model as well as in one of our error measurements which will be explained later. We substitute the OECD total inventory level for the OECD commercial inventory level used in [6] mainly because of that the OECD commercial inventory level is unavailable to access. The WTI spot price is in nominal dollars per barrel and the inventory level is measured in million barrels.

In [6, 7], the authors limit their study to the period from Jan 1992 to April 2003. We do of the same in this study. In [6], the process of generating out-of-sample forecasts begins by fitting the models for the Jan 1992 to Dec 1999 time period. We use the same length of this period, 96 months, as that of the time window for our rolling forecasting. As noticed, the rolling forecasting is interrupted by the event of September 11, 2001. The crude oil spot price fell and rose dramatically after this event. Therefore, six dummy variables are placed in the model. Consequently, because of the dummy variables, the model cannot produce appropriate forecasts for the six months after the event. Thus, the six months are not forecasted. The period of in-sample forecasts is from Dec 1999 to April 2003, and that of out-of-sample forecasts is from Jan 2000 to April 2003, excluding the period of Oct 2001 to March 2002.

3.2 Data Preprocessing

The monthly spot price interval is formed by the minimum and the maximum daily price in the month. The relative inventory level is calculated by using formula (1). Given the common sense that the inventories vary during a month, we transform the relative level into interval data by adding and subtracting a certain percentage. The percentage is determined by repeatedly experiments. In this study, as the percentage grows, the average Absolute Error and the average Accuracy Ratio also grow. We find that around 40%, the 95% confidence interval forecasts and the ILS interval forecasts are not so different to each other.

The percentage seems high, but that doesn't necessarily mean that the fluctuations are fierce, because it is the percentage of the "relative" inventory level rather than the observed inventory level. A high percentage against the "relative" level may only count for a small percentage against the whole inventory. Using the method of adding and subtracting a percentage of the relative level suggests that the widths of the intervals may vary significantly from each other. The interval is going to be large when the inventory is far away from the normal level, while to be small when the inventory is near the normal level. Thus, the widths are related to the fluctuations of the inventory level. Large input intervals are likely to produce large forecast intervals. When relative inventory level varies around zero, it is not likely to get large forecast intervals, that is, fierce fluctuations. In contrast, when the inventory level is far away from normal level, the forecast will exhibit bigger fluctuations and become more sensitive. This is consistent with common sense, when larger interval is interpreted as bigger fluctuations. Therefore, comparing with intervals with constant width, intervals formed in this way will cast non-linear effect on the widths of the price intervals.

3.3 Software

The software package we used in this study is developed and provided by Professor Chenyi Hu who visited the Academy in summer 2007. The package includes a col-lection of interval linear algebra subroutines (Nooner and Hu [4]) and an application program for interval least squares rolling forecasting. The package is written in C++.

4 Computational Results and Comparisons

Using the method discussed in Section 2 and the data discussed in Section 3, we obtain the empirical results that are presented in this section. Firstly, measurements to access the quality of forecasts are discussed, followed by comparisons of the empirical results that are showed in tables and diagrams.

4.1 The Error Measurements

Two error measurements are used to access the quality of the forecasts. One is the Absolute Error, the sum of absolute forecast errors of lower bound and upper bound forecasts; the other is the Accuracy Ratio, the percentage of the intersection of the forecast interval and the actual interval against the union of the two intervals. The Accuracy Ratio should be between 0% and 100%. For more details, please refer to Hu and He [2].

4.2 Comparison with Actual Monthly Price Interval

Using model (2) and ILS method described above, we obtain the in-sample and the out-of-sample monthly interval forecasts. The forecast quality measurements are shown is Table 1.

Table 1. The forecast quality of ILS method; percentage: 40%

	in-sample	out-of-sample
Average Absolute Error	3.147	3.511
Average Accuracy Ratio	51.932	48.557

In order to compare them with the results of point forecasting, we also calculate the 95% confidence intervals of the point forecasts (using the same model and the same forecasting period). The forecast quality of confidence intervals is summarized in Table 2.

Table 2. The forecast quality of 95% confidence intervals

	in-sample	out-of-sample
Average Absolute Error	3.163	3.482
Average Accuracy Ratio	53.239	50.241

A comparison of the in-sample interval forecasts (ILS method) and the actual intervals are shown in Figure 1, and that of the out-of-sample (ILS method) is shown in Figure 2.

The two tables above show that the forecast capability of confidence interval method and ILS method are comparable. Confidence interval method is slightly better than ILS method. However, the difference is not huge. Although the new method does not outperform the tradition one as used in forecasting annual return of S&P index, it is comparable to the traditional one. Thus, it is reasonable to use interval method in this case as well.

Aside from forecast quality, direct interval forecast is more meaningful. Unlike confidence interval, which stands for the possible interval of the mean, interval out-put from ILS method stands for the lower and upper boundaries of the price. The meaning of interval output is clearer and more direct.

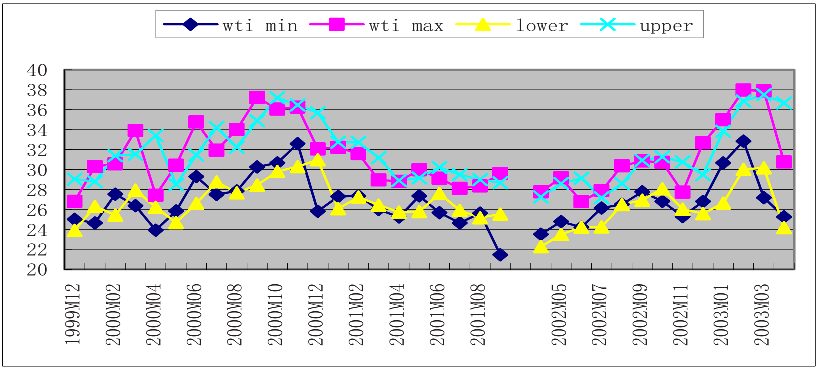

Fig. 1. In-sample forecasts comparing with the actual WTI prices

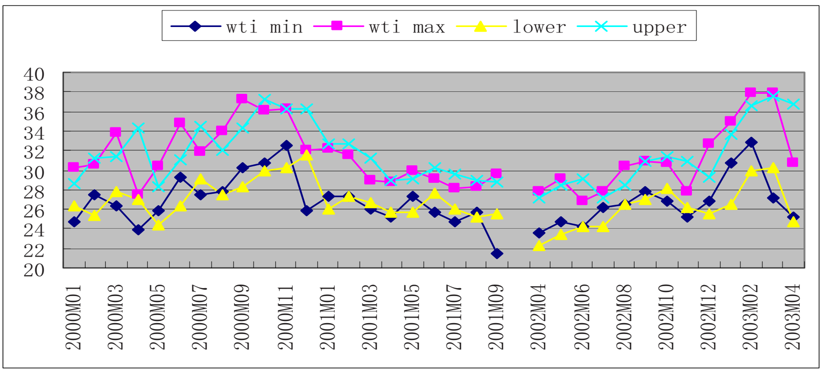

Fig. 2. Out-of-sample forecasts comparing with the actual WTI prices

4.3 Comparison with Actual Monthly Average Price

The following tables show the forecasting quality of the two methods CI and ILS by comparison them with the actual monthly average price of WTI crude oil.

Considering that decision makers who are used to interpret CI results may want to know if the intervals encompass the actual monthly average prices, especially in the case of out-of-sample forecast, we count the numbers of intervals which do not include the actual monthly average price of both methods. The results are represented in Table 3, and Figure 3 and Figure 4 are the related graphs. As is shown in Table 3, the number of fails of ILS method is smaller than that of CI method. The possibility of failure of ILS is 3% lower than that of CI method. That is, in this case, ILS method is slightly better than traditional CI method.

As is shown in Figures 3 and 4, forecasts of the two methods are not so different from each other, because they are based on the same model. Although they use

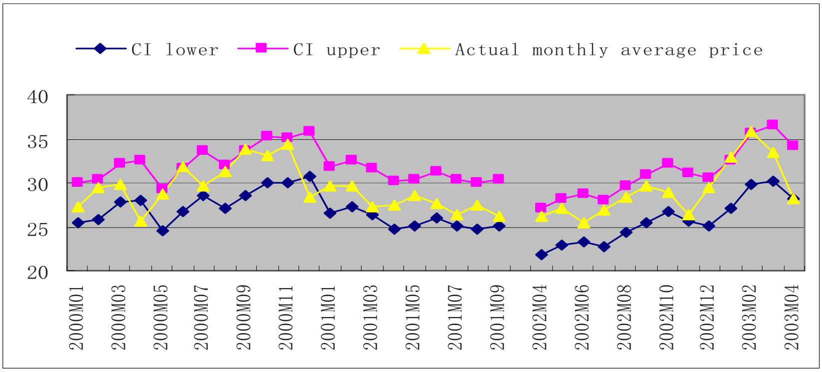

Fig. 3. Out-of-sample forecasts of CI method and monthly average oil spot price

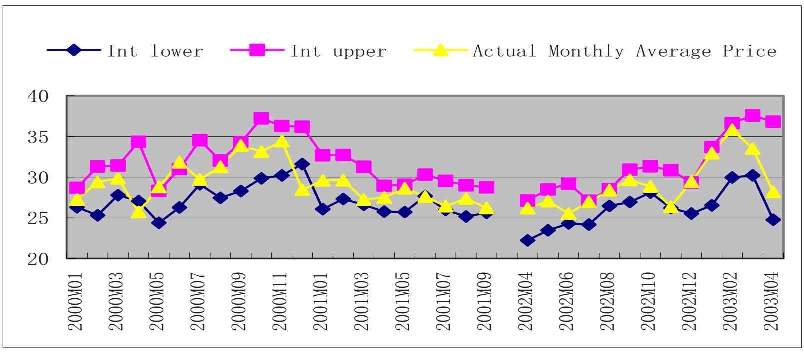

Fig. 4. Out-of-sample forecasts of ILS method and monthly average oil spot price

Table 3. Statistics of out-of-sample forecasts, which fail to encompass actual monthly average oil spot prices, of ILS method and CI method

	Number of fails	Prob.
ILS	6	17.65%
CI	7	20.59%

different approaches to produce the intervals, they share the same explanatory variables and the same lags. The model, due to its simple, linear form, may not perform well when the price fluctuates a lot. Yet, because we mainly focus on comparing the two approaches-ILS and CI, how to improve the ability of the model by, let us say, adding new variables, is not in our scope currently. However, since the model is made for point data, to make a model for interval data may improve overall forecast quality of ILS.

5 Conclusions and Future Work

In this work we study the potentials of interval arithmetic to forecast economic variables which manifest high uncertainty. Interval least square method is used to predict monthly WTI oil spot price. We use data ranging from January 1992 to April 2003 to build the relative stock model (RSTK) based on which we calculate the 95% confidence intervals for the rolling forecasts from December 1999 (in-sample) /January 2000 (out-of-sample) to April 2003. Then we apply the ILS approach to the RSTK model to obtain direct interval forecasts for the same period. The forecast results of traditional confidence interval method and ILS method are compared from two views, the view of researchers of interval arithmetic who emphasize intersections of intervals and the view of traditional decision makers who are familiar with average values. The empirical study shows that for predicting oil prices, the interval method is al-most as good as the tradition one. Thus, it is reasonable to use interval method in this case as well.

We believe that interval method is more meaningful in analyzing constantly changing economic variables and thus it is worth further studies following this one. We mention three here. First, asymmetric intervals of relative inventory level, for example, adding 40% and subtracting 60% to the point data, may improve forecast quality, since there are literatures arguing about asymmetric effects, like Ye et al. [7]. Second, the ILS method has the potential for improvement. For instance, we can use information about the width of intervals to gain better estimation of the coefficients. Since the estimation includes information of both the midpoint and the width, the relationship between variables may become more reliable. Finally, further efforts are needed to establish theoretical foundation for interval economic analysis.

Acknowledgments

Authors of this paper would like to express their appreciations to the anonymous referees for their insightful comments that helped us to improve the quality of this paper. Authors would also like to thank Professor Chenyi Hu for his technical contributions on the software that we used to perform the empirical experiments.

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