# Design of Fuzzy Sliding-Mode Controller for Chaos Synchronization

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**Abstract.** This paper presents a fuzzy controller to solve a master-slave chaos synchronization problem. At first, the method of traditional sliding mode control is considered, which utilizes the discontinuous sign function to make the system state reaching a sliding surface. Next, fuzzy rules are determined according to the Lyapunov theorem, and the fuzzy controller is designed for chaos synchronization. Finally, an example of chaos synchronization for an uncertain Duffing-Holmes system is presented to illustrate the validity and feasibility of the proposed controller.

## 1 Introduction

A chaos synchronization problem means making both chaotic oscillators behave exactly the same. Generally two chaotic systems in synchronization are called a drive system and a response system, respectively. Chaos synchronization can be applied in many areas such as in chemical reactions, power converters, signal process, communication, and biological systems [1, 3, 4, 8, 12]. There are many methods for synchronization of a chaotic system such as adaptive control method [5, 6, 14], back-stepping control method [7], $H^{\infty}$ control method [10], sliding mode (variable structure) control method [2, 13, 16], and fuzzy control method [15].

Zadeh [17, 18] initiated a fuzzy set theory. The fuzzy logic control schemes have been widely developed for almost 40 years, and have been successfully applied to many applications [13, 15]. We can easily apply the fuzzy logic control to control an ill-modeled system by experiments of skilled operators. Although there have been some successful applications of fuzzy logic control, it still has some drawbacks in the design procedure. For example, the fuzzy control rules are often experience-oriented and suitable membership functions should be given by time-consuming trial-and-error procedures. Besides, the dynamic behavior of control system cannot be specified precisely. In recent years, some chaos synchronization based on fuzzy system has been proposed [15].

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In this paper, we are devoted to the research of design fuzzy controller for synchronizing the state trajectories of two Duffing-Holmes systems with differential initial conditions, system uncertainties and external disturbances. This paper is organized as follows: Section 2 described the dynamics of mater-slave chaos synchronization system. In section 3 described the design approaches of fuzzy controller. Numerical simulations that confirm the validity and feasibility of proposed method are shown in Section 4. Finally, conclusions are given in Section 5.

## 2 System Description and Problem Formulation

Consider the following two n-dimensional chaotic systems,

$$\begin{cases} \dot{x}_i = x_{i+1}, & 1 \le i \le n-1 \\ \dot{x}_n = f(x,t) \end{cases}, \quad x \in R^n \tag{1}$$

$$\begin{cases} \dot{y}_i = y_{i+1}, & 1 \le i \le n-1 \\ \dot{y}_n = f(y,t) + \Delta f(y) + d(t) + u \end{cases}, \quad y \in R^n \tag{2}$$

where $u \in R$ is a control input, $f$ is a given nonlinear function of $x$ and $t$, $\Delta f(y)$ is an uncertain term representing the unmodeled dynamics or structural variation of the system (2) and $d(t)$ is the disturbance of system (2). In general, the uncertain term $\Delta f(y)$ and disturbance term are assumed bounded, i.e.

$$|\Delta f(y) < \alpha| \text{ and } |d(t) < \beta|, \tag{3}$$

where $\alpha$ and $\beta$ are positive.

It is assumed that $f(x,t)$, $f(y,t)$ and $\Delta(y,t)$ satisfy all the necessary conditions, such that system (1) and (2) have a unique solution in the time interval $[t_0, +\infty)$, $t_0 > 0$, for any given initial condition $x_0 = x(t_0)$ and $y_0 = y(t_0)$. The dynamics of system (1) display a chaotic motion without control input $(u = 0)$.

The control problem considered in this paper is that for different initial conditions of systems (1) and (2), the two coupled system, i.e. the master system (1) and the slave system (2), to be synchronized by designing an appropriate control $u(t)$ which is attached to the slave system (2) such that

$$\lim_{t \to \infty} \| x(t) - y(t) \| \to 0, \tag{4}$$

where $\| \cdot \|$ is the Euclidean norm of a vector.

## 3 Fuzzy Controller for Design Methodology

In this section, we want to address the sliding-mode control and design procedures of a fuzzy controller for chaos synchronization.

### 3.1 Sliding-Mode Control

Let the error state be $e_i = y_i - x_i$, $i = 1,2,\ldots,n$, and $g(e,t) = f(e+x,t) - f(x,t)$, the error dynamic equations are

$$\begin{aligned} \dot{e}_i &= e_{i+1} \quad ; \ 1 \le i \le n-1 \\ \dot{e}_n &= g(e,t) + \Delta f(e+x) + d(t) + u. \end{aligned} \tag{5}$$

Using the concept of extended systems, the standardized state space equations of the error states can be obtained as

$$\begin{aligned} \dot{e}_i &= e_{i+1} \quad ; \ 1 \le i \le n-1 \\ \dot{e}_n &= g(e,t) + \Delta f(e+x) + d(t) + u = e_{n+1} \\ \dot{e}_{n+1} &= \frac{d}{dt}(g(e,t) + \Delta f(e+x) + d(t)) + \dot{u} \end{aligned} \tag{6}$$

System (6) is of the controllable canonical form. In such a case, there are no internal dynamics [11]. Based on the control law proposed by [2], the sliding surface can be defined as

$$s = e_{n+1} - e_{n+1}(0) + \int_0^t \sum_{j=1}^{n+1} c_j e_j \, dt = 0 \tag{7}$$

where $e_{n+1}(0)$ denotes the initial state of $e_{n+1}$. Eq. (7) can also be formulated as

$$\dot{e}_{n+1} = -\sum_{j=1}^{n+1} c_j e_j \tag{8}$$

with the initial condition $e_{n+1}(0) = e_{0(n+1)}$, and the sliding mode dynamics can be described by the following system of equations:

$$\begin{aligned} \dot{e}_i &= e_{i+1} \quad ; \ 1 \le i \le n \\ \dot{e}_{n+1} &= -\sum_{j=1}^{n+1} c_j e_j \end{aligned} \tag{9}$$

or in a matrix equation form as

$$\dot{e}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ -c_1 & -c_2 & \cdots & \cdots & \cdots & -c_{n+1} \end{bmatrix} e_i = A_i e_i \quad ; \ 1 \le i \le n+1 \tag{10}$$

with the initial states being $e_i(0) = [e_{0(1)} \ e_{0(2)} \ \cdots e_{0(n+1)}]^T$. The design parameters $c_j$ can be determined by choosing the eigenvalues of $A_i$ such that the corresponding characteristic polynomial

$$P(e) = \dot{e}_{n+1} + \sum_{j=1}^{n+1} c_j e_j \tag{11}$$

is Hurwitz. These eigenvalues are also relative to the speed of system response.

### 3.2 Fuzzy Controller Designs

The fuzzy logic control arose from the desire to describe complex control with linguistic descriptions. The fuzzy logic control is easy to understand and simple to implement, because fuzzy logic emulates human control. The fuzzy controller and expert systems have been successfully applied in many complex industrial processes. In this section, we utilize the fuzzy logic and fuzzy propositions to design the controller. The block diagram of fuzzy controller with chaos system is illustrated in Fig. 1. We have the error state from the master and slave systems. The fuzzy controller can determine $u(t)$ by the error and system states.

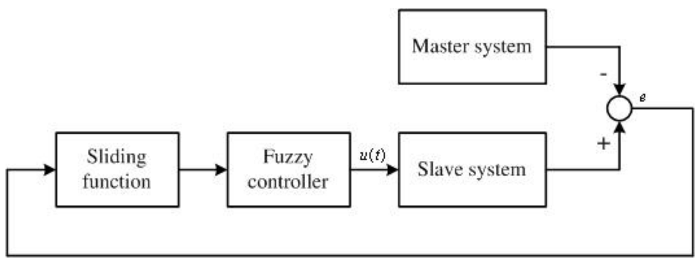


**Fig. 1.** Diagram of the fuzzy controller

The fuzzy controller has a two-input and a single-output. Input variables are the normalized sliding function (7) and the derivative of sliding function $\dot{s}$. The overall control output is chosen as

$$u_f(t) = F(s, \dot{s}) . \tag{12}$$

where $F(s, \dot{s})$ denotes the functional characteristics of the fuzzy linguistic decision schemes. The membership function of the input linguistic variables $s$ and $\dot{s}$, and the membership functions of the output linguistic variable $u_f(t)$ are shown in Fig. 2, respectively. They are decomposed into seven fuzzy partitions expressed as negative big (NB), negative small (NS), zero (ZE), positive small (PS), and positive big (PB). The fuzzy rule table is designed in Table 1.

The reaching law can be chosen as

$$\dot{s} = -k_f F(s, \dot{s}) . \tag{13}$$

where $k_f$ is a positive constant value. From Eqs. (7) and (13), we can obtain

$$\dot{s} = \dot{e}_{n+1} + \sum_{k=1}^{n+1} c_k e_k = -k_f F(s, \dot{s}) . \tag{14}$$

The differential equation of control input $u(t)$ is

$$\dot{u} = -\frac{d}{dt}\left[g(e,t) + \Delta f(y) + d(t)\right] - \sum_{k=1}^{n+1} c_k e_k - k_f F(s, \dot{s}) . \tag{15}$$

In the real world, the external disturbance $d(t)$ and the system uncertainty $\Delta f(y)$ are unknown. So, the implemented control input is described as

$$\dot{u} = -\frac{d}{dt} g(e,t) - \sum_{k=1}^{n+1} c_k e_k - k_f F(s, \dot{s}) . \tag{16}$$

After integration of Eq. (16), we can obtain the control input of the slave system

$$u = \int_0^t \left[ -\frac{d}{dt} g(e,t) - \sum_{k=1}^{n+1} c_k e_k - k_f F(s, \dot{s}) \right] dt . \tag{17}$$

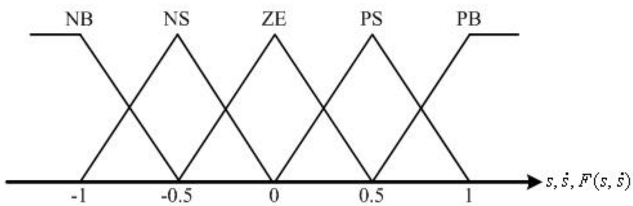


**Fig. 2.** The membership functions of input variables and output variable

**Table 1.** Rules-table of fuzzy controller

| $F(s,\dot{s})$ | | $s$ | | | | |
|---|---|---|---|---|---|---|
| | | NB | NS | ZE | PS | PB |
| | NB | NB | NB | NB | NS | ZE |
| | NS | NB | NB | NS | ZE | PS |
| $\dot{s}$ | ZE | NB | NS | ZE | PS | PB |
| | PS | NS | ZE | PS | PB | PB |
| | PB | ZE | PS | PB | PB | PB |

**Theorem 1.** Consider the master-slave system (1) and (2), the two systems are synchronized by the controller $u(t)$ (17) for the slave system. Then the error state trajectory converges to the sliding surface $s(t) = 0$ .

**Proof.** We define a Lyapunov function as

$$V = \tfrac{1}{2} s^2 . \tag{18}$$

Taking the time derivative of Eq. (16), we have

$$\dot{V} = s\dot{s} = s\left\{ \frac{d}{dt}\left[\Delta f(y) + d(t)\right] - k_f F(s, \dot{s}) \right\} . \tag{19}$$

Let $\left| \frac{d}{dt}\left[\Delta f(y) + d(t)\right] \right|$ is bounded, and $\left| \frac{d}{dt}\left[\Delta f(y) + d(t)\right] \right| \le k_f$ . There, we have

$$\dot{V} \le -k_f |s| . \tag{20}$$

The reaching condition $s\dot{s} < 0$ is maintained and $\lim_{t \to \infty} s(t) \to 0$ . This completes the proof.

## 4 Numerical Example

In this section, simulation results are presented to demonstrate the effectiveness of the proposed fuzzy sliding-mode controller for chaos synchronization problem. Consider two coupled Duffing-Holmes systems as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -p_1 x_1 - p_2 x_2 - x_1^{\;3} + q\cos(\omega t) , \end{aligned} \tag{21}$$

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -p_1 y_1 - p_2 y_2 - y_1^{\;3} + \Delta f(y) + d(t) + q\cos(\omega t) + u(t) . \end{aligned} \tag{22}$$

The second equation of the slave system (22) is perturbed by an uncertainty term $\Delta f(y)$ and interfered with a disturbance $d(t)$ and the control input $u(t)$ is attached to the slave system. Let us define the synchronization errors between the master system and slaver system as $e_1 = y_1 - x_1$ and $e_2 = y_2 - x_2$ . Subtracting (22) from (21), we have the synchronization error dynamics as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -p_1 e_1 - p_2 e_2 - y_1^{\;3} + x_1^{\;3} + \Delta f(e + x) + d(t) + u(t) . \end{aligned} \tag{23}$$

Then, the standardized state space equations can be described as

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= -p_1 e_2 - p_2 e_3 - 3y_1^{\;2} y_2 + 3x_1^{\;2} x_2 + \frac{d}{dt}\left(\Delta f(e + x) + d(t)\right) + \dot{u}(t) \end{aligned} \tag{24}$$

Let the sliding surface be defined as

$$s = e_3 - e_3(0) + \int_0^t (c_3 e_3 + c_2 e_2 + c_1 e_1) dt . \tag{25}$$

The eigenvalues corresponding to the sliding surface can be decided by $[c_3 \quad c_2 \quad c_1]$ and these eigenvalues dominate the converging rate of the error dynamics. They can arbitrarily be assigned. Choose the reaching law as in Eq. (17). The control input is determined as

$$u = \int_0^t \{p_1 e_2 + p_2 e_3 + 3{y_1}^2 y_2 - 3{x_1}^2 x_2 - (c_3 e_3 + c_2 e_2 + c_1 e_1) - k_f F(s, \dot{s})\} dt , \tag{26}$$

with the initial condition $u(0) = 0$ .

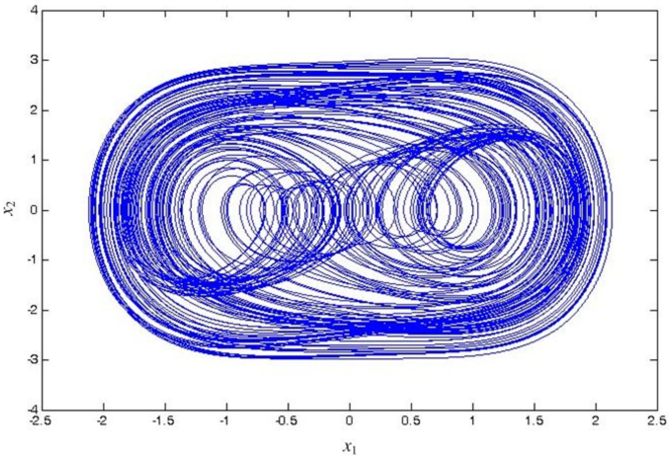


**Fig. 3.** Hyperchaotic behavior of the Duffing-Holmes oscillator in $x_1 - x_2$ plane

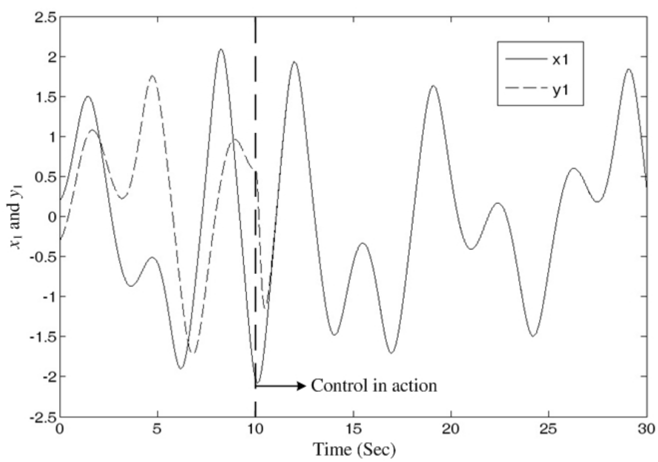


**Fig. 4.** The time response of $x_1$ and $y_1$

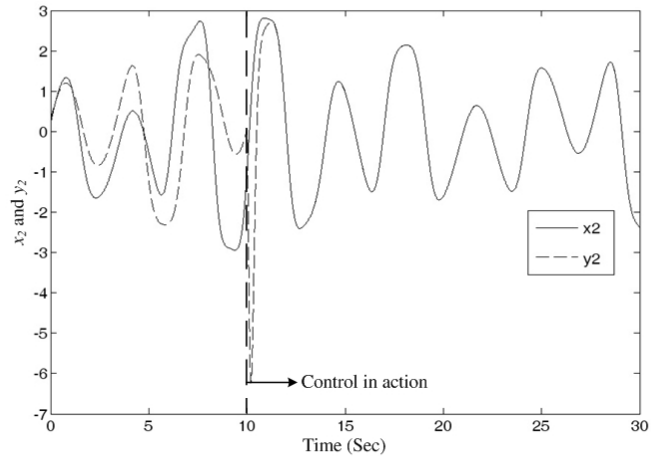


**Fig. 5.** The time response of $x_2$ and $y_2$

For the master-slave synchronization control systems (21) and (22), the parameters are $p_1 = -1.1$, $p_2 = 0.4$, $q = 2.1$ and $\omega = 1.8$, the master system (21) displaces chaotic behavior [9]. The master system (21) will exhibit a hyperchaotic behavior, as shown in Fig. 3, where the attractor is shown on the $x_1 - x_2$ plane by giving initial conditions $x_1(0) = 0.1$ and $x_2(0) = 0.1$. It is supposed that the disturbance $d(t) = 0.2\cos(\pi t)$ and the uncertainty term $\Delta f(y) = -0.05 y_1$ in the slaver system (22). The eigenvalues corresponding to the sliding surface are chosen as $[c_1 \quad c_2 \quad c_3] = [600 \quad 200 \quad 20]$, and coefficient of the sliding-mode controller is chosen as $k_f = 50$. The simulation step size was $0.001$ sec.

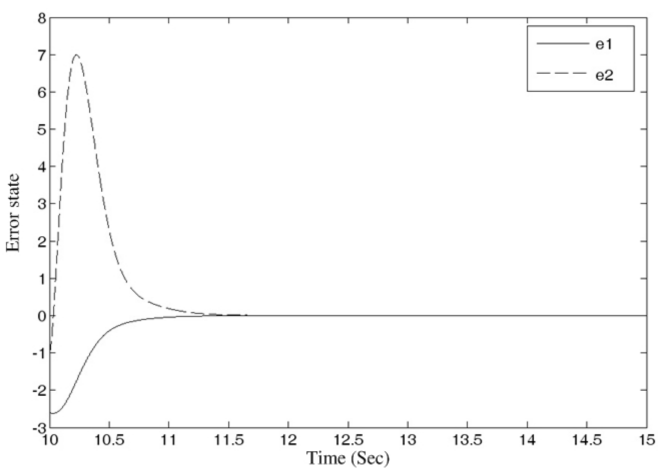


**Fig. 6.** The error state responses

The simulation results with initial conditions $x_1(0) = 0.2$ , $x_2(0) = 0.2$ , $y_1(0) = -0.3$ , and $y_2(0) = 0.3$ are shown in Fig. 4 and Fig. 5. Those show that the slave and the master systems can reach synchronization when control operation in the slave system at $t = 10$ secs. Fig. 6 shows the error state responses. The control input of slave system was demonstrated in Fig. 7.

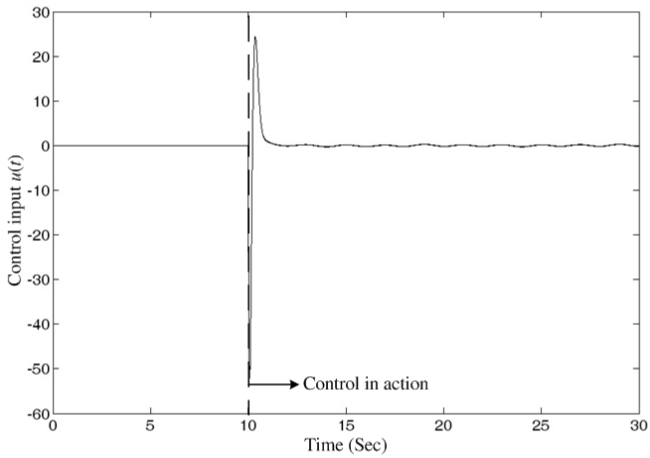


**Fig. 7.** The control input of slave system

## 5 Conclusions

In this paper, fuzzy sliding-mode control for chaos synchronization has been proposed. The fuzzy controller based on the sliding-mode and Lyapunov stability theory, which is designed for the regulation of the error state vector to a desired point in the state space. Numerical simulation results demonstrate that the proposed method can be successfully applied to synchronization problems of a Duffing-Holmes system. The derived controllers are robust so that the closed-loop system is stable in the presence of uncertainties and disturbance. The chattering phenomenon of conventional switching type sliding controls does not occur in this study.

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