Composed Fuzzy Rough Set and Its Applications in Fuzzy RSAR

Weigen Qiu[1,2] and Zhibin Hu[1]

[1] Computer Faculty of GuangDong University of Technology, GuangZhou, GuangDong, 510090, P.R. China
[2] State Key Laboratory of Intelligent Technology & Systems, Department of Computer Science & Technology, Tsinghua University, Beijing 100084, P.R. China

Abstract. Pawlak rough set theory is a powerful mathematic tool to deal with imprecise, uncertainty and incomplete dataset. In this paper, we study the fuzzy rough set attribute reduction (fuzzy RSAR) in fuzzy information systems. Firstly, we present the formal definition of a kind new rough set form-the composed fuzzy rough set. The second, some properties of extension forms of Pawlak rough set are also discussed. Lastly, we illustrate the fuzzy RSAR based on composed fuzzy rough set, and a simple example is given to show this approach can retain less attributes and entailing higher classification accuracy than the crisp RST-based reduction method.

Keywords: Fuzzy information system, Composed fuzzy rough sets, Fuzzy rough set, attribute reduction.

1 Introduction

It is well known many classification problems involve high-dimensional descriptions of input features. However, some existing methods tend to destroy the underlying semantics of the features after reduction or require additional information beyond the given data set. A technique that can reduce the dimensionality by using information contained within the data set and preserving the meaning of the features is clearly desirable. Rough Sets Theory (RST) can be used as such a tool to discover data dependencies and reduce the number of attributes contained in data set by purely structural methods [4].

With more than twenty years development [5], RST has indeed become an expanding research area, recent theoretical developments are collected in papers [7]. However, in traditional Pawlak RST, an equivalence relation seems to be a very stringent condition which limits its applications fields. As well known, the fuzzy set theory and rough set theory represent different aspects of uncertainty and aim to two different purpose, so many attempts have been made to combine these two theories [1,2,9]. Because the values of attributes may often be both crisp and real-valued in more and more application cases, therefore the traditional RST encounters a problem. Based on information entropy, paper [8] presents a discretization algorithm of

M. Xu et al. (Eds.): APPT 2007, LNCS 4847, pp. 753–763, 2007.

real-valued attributes values information system for selecting cut points. Because the discretization process itself often requires some additional information beyond the aimed data sets, the paper [4] introduce a new concepts for fuzzy-rough attribute reduction based on fuzzy rough sets.

In this paper, a kind new rough set concept is presented and the composed fuzzy rough set is formally defined and its properties are discussed. The fuzzy rough attribute reduction based on the composed fuzzy rough set is illustrated with a simple example. This approach can retain less attributes and entail higher classification accuracy than the crisp RST-based reduction method. The rest of this paper is organized as follows. Section 2 discusses some related basic theory with this paper later, such as fuzzy set, Pawlak rough set theory, information system, and so forth. Section 3 mainly explores some extension Pawlak rough set models. Firstly some properties of generalized fuzzy rough set are discussed. The second, the composed fuzzy rough set is initiated and formally defined; its some properties are also discussed in detailed. The four section illustrate the fuzzy RSAR based the composed fuzzy rough set; and an example is given to show its efficiency and accuracy in classification. In section 5 we make a conclusion on the paper.

2 Preliminaries

Let $U=\{u_1, u_2, \ldots, u_n\}$ stands for the finite and nonempty set of objects. The power $P(U)$ can be viewed as a subset of fuzzy power $\mathscr{F}(U)$, $X\in\mathscr{F}(U)$ can be represented as form $X=\{(u, \mu_{X(u)})| u \in U\}$, where for every $u\in U$, the value $\mu_{X(u)}\in[0, 1]$. X is also represented as form $X=(\mu_{X(u1)}, \mu_{X(u2)}, \ldots, \mu_{X(un)})$ when U is finite set, or as $X=\int\mu_{X(u)}/u$ when U is infinite set. For arbitrary $\lambda\in I=[0, 1]$, the λ-level X_λ and the strong λ-level $X_{\lambda+}$ are respectively $X_\lambda=\{u\in U|\mu_{X(u)}\geq\lambda\}$ and $X_{\lambda+}=\{u\in U|\mu_{X(u)}>\lambda\}$, $X=\bigvee_{\lambda\in I}(\lambda\wedge X_\lambda) = \bigvee_{\lambda\in I}(\lambda\wedge X_{\lambda+})$, , $X_0=U$, $X_{1+}=\emptyset$ [7]. A fuzzy binary relation R over U is a function $R: U\times U\rightarrow[0, 1]$, its membership function is represented by $\mu_R(x, y)$. The class of all fuzzy binary relations of U will be denoted as $\mathscr{F}(U\times U)$.

Let R be an ordinary equivalence relation on U, U/R denotes the equivalence classes by R. For every $u\in U$, $[u]_R\in U/R$ denotes the equivalence class of u, the pair (U, R) is called as the Pawlak approximation space. Let $X\subseteq U$,

$$\underline{R}X=\{u|[u]_R\subseteq X\},\ \overline{R}X=\{u|[u]_R\cap X\neq\emptyset\} \tag{2.1}$$

Where $\underline{R}X$ is called the lower approximation of X, while $\overline{R}X$ is the upper approximation of X, $(\underline{R}X, \overline{R}X)$ is called as rough set of X. $\alpha_R(X)=\mathrm{card}(\underline{R}X)/\mathrm{card}(\overline{R}X)$ denotes the accuracy of approximation, $\overline{R}X-\underline{R}X$ as the boundary set of X. The fuzzy set $\tilde{X}$ over U is defined as following [7]:

$$\mu_{\tilde{X}}(u)=\frac{card([u]_R \cap X)}{card([u]_R)}, \text{ for every object } u\in U \qquad (2.2)$$

So, $\mu_{\tilde{X}}(u)=1$ iff $u\in \underline{R}X$; $\mu_{\tilde{X}}(u)=0$ iff $u\in U-\overline{R}X$; otherwise $0<\mu_{\tilde{X}}(u)<1$ iff $u\in \overline{R}X-\underline{R}X$. However, if X is fuzzy rough set or R is fuzzy equivalence relation over U, the above calculation formula of membership function $\mu_{\tilde{X}}(u)$ may need some changes in form.

Information system (IS) is an ordered quadruple S=(U, A, f, V), where $U=\{u_1, u_2, \ldots, u_n\}$ is a non-empty finite objects set, $A=\{a_1, a_2, \ldots, a_m\}$ is a non-empty finite attributes set. $V=\cup_{a\in A} V_a$ is a set of attributes values, where V_a is the domain of attribute $a\in A$. f: $U\times A\rightarrow V$ is an information function, where for all $(u, a)\in U\times A$, $f(u, a)\in V_a$, $Inf(u)=\{(a, f_a(u))|a\in A\}$ is called as an information vector of u.

Let $P\subseteq A$, $Ind(P)=\{(u, v)\in U\times U|$ for all $a\in P$, $f(a, u)=f(a, v)\}$. If $(u, v)\in Ind(P)$, then u and v are indiscernible under attributes subset P. For every $u\in U$, its equivalence class is denoted as $[u]_P=\{v|(u, v)\in Ind(P)\}$ and $U/P=\{[u]_P|u\in U\}$. Let P, $Q\subseteq A$, the positive region $POS_P(Q) = \bigcup_{X\in U/Q} \underline{P}X$ contains all objects of U that can be classified to classes of U/Q by using the knowledge in attributes P. For P, $Q\subseteq A$, we call Q depends on P in a degree k ($0\le k\le 1$), where

$$k=\gamma_P(Q)=\frac{card(POS_P(Q))}{card(U)} \qquad (2.3)$$

If k=1, then call Q depends totally on P; if 0<k<1, then call Q depends partially on P with the degree k, denoted by $P\Rightarrow_k Q$; and if k=0 then call Q does not depend on P.

3 Extensions of Pawlak Rough Set Model

Let $U=\{u_1, u_2, \ldots, u_n\}$ and $W=\{w_1, w_2, \ldots, w_m\}$ be two finite and nonempty sets, $R\in \mathscr{F}(U\times W)$ is fuzzy relation from U to W. When U and W are finite nonempty sets, R can be represented by n*m matrix $R=(r_{ij})_{n\times m}$ where $r_{ij}=\mu_R(u_i, w_j)\in[0, 1]$, for all i=1, 2, …, n, j=1, 2, …, m [3]. For each $\lambda\in[0, 1]$, matrix $(\lambda r_{ij})_{n\times m}$ denotes the cut relation R_λ, where $\lambda r_{ij}=1$ iff $r_{ij}\ge\lambda$, otherwise $\lambda r_{ij}=0$. If $R\in P(U\times W)$, then for all $u\in U$, we call $R_W(u)=\{w\in W|(u, w)\in R\}$ as the successor neighborhood of u.

Definition 1. Let U be two finite and nonempty sets, $R\in\mathscr{F}(U\times W)$. $Y\in\mathscr{F}(W)$, $u\in U$, the generalized fuzzy rough set ($\underline{apr}_R Y$, $\overline{apr}_R Y$) can be defined as following:

$$\underline{apr}_R\ Y(u)=\min_{w\in W}\{\max(1-R(u,w), Y(w))\}$$
$$\overline{apr}_R\ Y(u)=\max_{w\in W}\{\min(R(u,w), Y(w))\} \quad (3.1)$$

Where the triple (U, W, R) is called as generalized fuzzy approximation space.

Theorem 1. Let $R\in\mathscr{F}(U\times W)$, then for all $Y\in\mathscr{F}(W)$ and arbitrary $\alpha\in[0, 1]$,

$$(\underline{apr}_R\ Y)_\alpha=\underline{apr}_{R_{(1-\alpha)+}}\ Y_\alpha,\ (\overline{apr}_R\ Y)_\alpha=\overline{apr}_{R_\alpha}\ Y_\alpha$$
$$(\underline{apr}_R\ X)_{\alpha+}=\underline{apr}_{R_{(1-\alpha)}}\ Y_{\alpha+},\ (\overline{apr}_R\ Y)_{\alpha+}=\overline{apr}_{R_{\alpha+}}\ Y_{\alpha+}$$

Proof. For arbitrary $\alpha\in[0, 1]$,

$$\begin{aligned}(\underline{apr}_R\ Y)_\alpha&=\{u\in U\mid \underline{apr}_R\ Y(u)\geq\alpha\}\\
&=\{u\in U\mid \min_{w\in W}\{\max(1-R(u,w), Y(w))\}\geq\alpha\}\\
&=\{u\in U\mid \text{for all } w\in W, \max(1-R(u,w), Y(w))\geq\alpha\}\\
&=\{u\in U\mid \text{for each } w\in W, 1-R(u,w)\geq\alpha, \text{ or } Y(w)\geq\alpha\}\\
&=\{u\in U\mid \{w\in W\mid 1-R(u,w)\geq\alpha\}\cup\{w\in W\mid Y(w)\geq\alpha\}=W\}\\
&=\{u\in U\mid \{w\in W\mid R(u,w)>1-\alpha\}\subseteq\{w\in W\mid Y(w)\geq\alpha\}\}\\
&=\{u\in U\mid (R_W(u))_{(1-\alpha)+}\subseteq Y_\alpha\}=\underline{apr}_{R_{(1-\alpha)+}}\ Y_\alpha\end{aligned}$$

$$\begin{aligned}(\overline{apr}_R\ Y)_\alpha&=\{u\in U\mid \overline{apr}_R\ Y(u)\geq\alpha\}\\
&=\{u\in U\mid \max_{w\in W}\{\min(R(u,w), Y(w))\}\geq\alpha\}\\
&=\{u\in U\mid \exists w\in W, \min(R(u,w), Y(w))\geq\alpha\}\\
&=\{u\in U\mid \exists w\in W, R(u,w)\geq\alpha \text{ and } Y(w)\geq\alpha\}\\
&=\{u\in U\mid (R_W(u))_\alpha\cap Y_\alpha\neq\emptyset\}=\overline{apr}_{R_\alpha}\ Y_\alpha\end{aligned}$$

$$\begin{aligned}(\underline{apr}_R\ Y)_{\alpha+}&=\{u\in U\mid \underline{apr}_R\ Y(u)>\alpha\}\\
&=\{u\in U\mid \min_{w\in W}\{\max(1-R(u,w), Y(w))\}>\alpha\}\\
&=\{u\in U\mid \text{for each } w\in W, \max(1-R(u,w), Y(w))>\alpha\}\\
&=\{u\in U\mid \text{for each } w\in W, 1-R(u,w)>\alpha, \text{ or } Y(w)>\alpha\}\\
&=\{u\in U\mid \{w\in W\mid 1-R(u,w)>\alpha\}\cup\{w\in W\mid Y(w)>\alpha\}=W\}\\
&=\{u\in U\mid \{w\in W\mid R(u,w)\geq 1-\alpha\}\subseteq\{w\in W\mid Y(w)>\alpha\}=W\}\\
&=\{u\in U\mid (R_W(u))_{1-\alpha}\subseteq Y_{\alpha+}\}=\underline{apr}_{R_{(1-\alpha)}}\ Y_{\alpha+}\end{aligned}$$

$$\begin{aligned}(\overline{apr}_R\ Y)_{\alpha+}&=\{u\in U\mid \overline{apr}_R\ Y(u)>\alpha\}\\
&=\{u\in U\mid \max_{w\in W}\{\min(R(u,w), Y(w))\}>\alpha\}\end{aligned}$$

$= \{ u \in U \mid \exists w \in W, \min(R(u, w), Y(w))>\alpha\}$
$= \{ u \in U \mid \exists w \in W, R(u, w)>\alpha \text{ and } Y(w)>\alpha\}$
$= \{ u \in U \mid (R_W(u))_{\alpha+} \cap Y_{\alpha+} \neq \emptyset\} = \overline{apr}_{R_{\alpha+}} Y_{\alpha+}$

Remark 1. From the theorem 1 and the decompose theory, we can immediately conclude that the following conclusion.

1) $\underline{apr}_R Y = \vee_{\alpha\in[0,1]} (\alpha \wedge (\underline{apr}_{R_{(1-\alpha)+}} Y_{\alpha})) = \vee_{\alpha\in[0,1]} (\alpha \wedge (\underline{apr}_{R_{(1-\alpha)}} Y_{\alpha+}))$

2) $\overline{apr}_R Y = \vee_{\alpha\in[0,1]} (\alpha \wedge (\overline{apr}_{R_{\alpha+}} Y_{\alpha+})) = \vee_{\alpha\in[0,1]} (\alpha \wedge (\overline{apr}_{R_{\alpha}} Y_{\alpha}))$

In paper [6], Wu construct the generalized fuzzy rough sets exactly starting from the above formulas 1) and 2). Our results in the paper show these two approaches are totally equivalence.

Let the fuzzy equivalence classes set $U/R=\{F_1, F_2, \ldots, F_k\}$, we consider the approximations problem for every $X\in\mathscr{F}(U)$, let $\overline{Apr}_R X$ denotes upper approxim -ation and $\underline{Apr}_R X$ lower approximation respectively. Because the membership values of individual object to the approximations are not explicitly available directly, so we need obtain them from another point. Let $F_i\in U/R$ denoted by $F_i=\{(u, \mu_{Fi(u)})|u\in U\}$, consider the following fuzzy set forms:

$$\underline{Apr}_R X = \sum_{i=1,\cdots,k} \mu_{\underline{RX}}(F_i)/F_i \quad \overline{Apr}_R X = \sum_{i=1,\cdots,k} \mu_{\overline{RX}}(F_i)/F_i \tag{3.2}$$

Where $\underline{RX}$ and $\overline{RX}$ are short writing of fuzzy sets $\underline{Apr}_R X$ and $\overline{Apr}_R X$, respectively. For every $F\in U/R= \{F_1, F_2, \ldots, F_k\}$, the membership degree values $\mu_{\underline{RX}}(F)$ and $\mu_{\overline{RX}}(F)$ can be respectively calculated by the following:

$$\mu_{\underline{RX}}(F)=\min_{v\in U} \max(1-\mu_F(v), X(v)),\ \mu_{\overline{RX}}(F)=\max_{v\in U} \min(\mu_F(v), X(v)).$$

On the other hand, for every $u\in U$,

$$\mu_{\underline{RX}}(u)=\max_{F\in U/R} \min(\mu_F(u), \mu_{\underline{RX}}(F)),\ \mu_{\overline{RX}}(u)=\max_{F\in U/R} \min(\mu_F(u), \mu_{\overline{RX}}(F)).$$

If R is a crisp equivalence relation over U and $U/R=\{F_1, F_2, \ldots, F_k\}$. Let $X\in P(U)$, then the $\mu_{\underline{RX}}(F)$ and $\mu_{\overline{RX}}(F)$ can be computed as follows.

$$\mu_{\underline{RX}}(F)=\min_{v\in U} \max(1-\mu_F(v), X(v)) = \min_{v\in F} X(v)$$
$$\mu_{\overline{RX}}(F)=\max_{v\in U} \min(\mu_F(v), X(v)) = \max_{v\in F} X(v)$$

It means that $\mu_{\underline{RX}}$ (F) =1 iff $F \subseteq X$, otherwise $\mu_{\underline{RX}}$ (F)=0; and $\mu_{\overline{RX}}$ (F)=1 iff $F \cap X \neq \emptyset$, other wise $\mu_{\overline{RX}}$ (F)=0. Therefore,

$$\mu_{\underline{RX}}(u) = \max_{F \in U/R} \min(\mu_F(u), \mu_{\underline{RX}}(F)) = \max\{\mu_F(u) \mid u \in F \text{ and } F \subseteq X\}$$

$$\mu_{\overline{RX}}(u) = \max_{F \in U/R} \min(\mu_F(u), \mu_{\overline{RX}}(F)) = \max\{\mu_F(u) \mid u \in F \text{ and } F \cap X \neq \emptyset\}$$

Above statements show that our viewpoint is a natural generalization of Pawlak rough set from crisp case to the fuzzy circumstance. Below the paper, we will present the more general statement of above extension. Let $U=\{u_1, u_2, \ldots, u_n\}$ and $W=\{w_1, w_2, \ldots, w_m\}$ be two finite and nonempty universe,

$$R = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{pmatrix} = (r_{ij})_{n \times m} = \begin{pmatrix} r_1 \\ r_2 \\ \cdots \\ r_n \end{pmatrix} = (r'_1, r'_2, \ldots, r'_m) \tag{3.3}$$

Where for all i=1, 2, …, n, j=1, 2, …, m, $r_{ij}=\mu_R(u_i, w_j)$, $R_W(u_i)=r_i=(r_{i1}, r_{i2}, \ldots, r_{im})$, $R_U(w_j)=r'_j=(r_{1j}, r_{2j}, \ldots, r_{nj})^T$, $R'=(r'_{ij})_{m\times n}$□$\mathcal{F}(W\times U)$, where $r'_{ji}=r_{ij}$.

Definition 2. Let U and W be two finite and nonempty sets, $R\in\mathcal{F}(U\times W)$. $X\in\mathcal{F}(W)$, then $\overline{Apr}_R X$, $\underline{Apr}_R X\in\mathcal{F}(W)$ can be defined as following. For all $v\in W$,

$$\underline{Apr}_R X(v) = \max_{u\in U} \min(R'(v, u), \min_{w\in W} \max(1-R(u, w), X(w))),$$

$$\overline{Apr}_R X(v) = \max_{u\in U} \min(R'(v, u), \max_{w\in W} \min(R(u, w), X(w))) \tag{3.4}$$

The pair ($\underline{Apr}_R X$, $\overline{Apr}_R X$) is called as the composed fuzzy rough set of X, and the triple (U, W, R) as the composed fuzzy approximation space.

Proposition 4. Let U and W be two finite and nonempty universes, $R\in\mathcal{F}(U\times W)$. Then for all $X\in\mathcal{F}(W)$,

$$\underline{Apr}_R X = \overline{apr}_{R'}(\underline{apr}_R X), \quad \overline{Apr}_R X = \overline{apr}_{R'}(\overline{apr}_R X).$$

Where $\underline{apr}_R$ and $\overline{apr}_R$ are respectively generalized fuzzy rough lower and upper approximation operators, $\overline{apr}_{R'}$ is upper approximation operator related with R'.

If U=W and R is a fuzzy equivalence relation, then R=R' and R' (v, u) =R (u, v). For each $u \in U$, $F_u = \sum_{v \in U} R(u, v)/v$. Furthermore,

$$\underline{Apr}_R X = \sum_{u \in U} (\min_{v \in U} \max(1-R(u, v), X(v)))/F_u$$
$$\overline{Apr}_R X = \sum_{u \in U} (\max_{v \in U} \min(R(u, v), X(v)))/F_u \tag{3.5}$$

Therefore, from above definition 2,

$$\underline{Apr}_R X(v) = \max_{u \in U} \min(R'(v, u), \min_{w \in W} \max(1-R(u, w), X(w)))$$
$$= \max_{u \in U} \min(\mu_{Fu}(v), \mu_{\underline{RX}}(F_u))$$
$$\overline{Apr}_R X(v) = \max_{u \in U} \min(R'(v, u), \max_{w \in W} \min(R(u, w), X(w)))$$
$$= \max_{u \in U} \min(\mu_{Fu}(v), \mu_{\overline{RX}}(F))$$

Proposition 5. Let U be finite and nonempty set, $R \in P(U \times U)$, then for all $X \in P(U)$,

$$\underline{Apr}_R X = \underline{R} X, \quad \overline{Apr}_R X = \overline{R} X$$

Proof. Since $R \in P(U \times U)$, $X \in P(U)$, then for arbitrary $u \in U$, there exists a unique equivalence class $[u]_R = \sum_{v \in U} R(u, v)/v$. Therefore, for all $X \in P(U)$ and $u \in U$,

$$\underline{Apr}_R X(v) = \max_{u \in U} \min(R'(v, u), \min_{w \in W} \max(1-R(u, w), X(w)))$$
$$= \max_{u \in U} \min(R(v, u), \min(\min_{w \notin [u]_R} \max(1-R(u, w), X(w)), \min_{w \in [u]_R} \max(1-R(u, w), X(w))))$$
$$= \max_{u \in U} \min(R(v, u), \min_{w \in [u]_R} X(w))$$
$$= \max(\max_{u \in [v]_R} \min(R(v, u), \min_{w \in [u]_R} X(w)), \max_{u \notin [v]_R} \min(R(v, u), \min_{w \in [u]_R} X(w)))$$
$$= \min_{w \in [v]_R} X(w) = \underline{R} X(v)$$

$$\overline{Apr}_R X(v) = \max_{u \in U} \min(R'(v, u), \max_{w \in W} \min(R(u, w), X(w)))$$
$$= \max_{u \in U} \min(R(v, u), \max(\max_{w \in [u]_R} \min(R(u, w), X(w)), \max_{w \notin [u]_R} \min(R(u, w), X(w))))$$
$$= \max_{u \in U} \min(R(v, u), \max_{w \in [u]_R} X(w))$$
$$= \max(\max_{u \in [v]_R} \min(R(v, u), \max_{w \in [u]_R} X(w)), \max_{u \notin [v]_R} \min(R(v, u), \max_{w \in [u]_R} X(w)))$$
$$= \max_{w \in [v]_R} X(w) = \overline{R} X(v).$$

That is $\underline{Apr}_R X = \underline{R} X$, $\overline{Apr}_R X = \overline{R} X$.

Theorem 2. Let $R \in \mathscr{F}(U \times W)$, then for all $X \in \mathscr{F}(W)$, and arbitrary $\alpha \in [0, 1]$,

1) $(\underline{Apr}_R X)_\alpha = \overline{apr}_{(R')_\alpha} \underline{apr}_{R_{(1-\alpha)+}} X_\alpha$, 2) $(\overline{Apr}_R X)_\alpha = \overline{apr}_{(R')_\alpha} \overline{apr}_{R_\alpha} X_\alpha$,

3) $(\underline{Apr}_R X)_{\alpha+} = \overline{apr}_{(R')_\alpha} \underline{apr}_{R_{(1-\alpha)}} X_{\alpha+}$, 4) $(\overline{Apr}_R X)_{\alpha+} = \overline{apr}_{(R')_{\alpha+}} \overline{apr}_{R_{\alpha+}} X_{\alpha+}$.

Proof. For arbitrary $\alpha \in [0, 1]$,

1) $(\underline{Apr}_R X)_\alpha = \{v \in U \mid \underline{Apr}_R X(v) \geq \alpha\}$

$= \{v \in U \mid \max_{u \in U} \min(R'(v, u), \min_{w \in W} \max(1-R(u, w), X(w))) \geq \alpha\}$

$= \{v \in U \mid \exists u \in U, R'(v, u) \geq \alpha \text{ and } \min_{w \in W} \max(1-R(u, w), X(w)) \geq \alpha\}$

$= \{v \in U \mid (R'_v)_\alpha \cap (\underline{apr}_R X)_\alpha \neq \emptyset\} = \overline{apr}_{(R')_\alpha} (\underline{apr}_R X)_\alpha = \overline{apr}_{(R')_\alpha} \underline{apr}_{R_{(1-\alpha)+}} X_\alpha$

2) $(\overline{Apr}_R X)_\alpha = \{v \in U \mid \overline{Apr}_R X(v) \geq \alpha\}$

$= \{v \in U \mid \max_{u \in U} \min(R'(v, u), \max_{w \in W} \min(R(u, w), X(w))) \geq \alpha\}$

$= \{v \in U \mid \exists u \in U, R'(v, u) \geq \alpha \text{ and } \max_{w \in W} \min(R(u, w), X(w)) \geq \alpha\}$

$= \{v \in U \mid (R'_v)_\alpha \cap (\overline{apr}_R X)_\alpha \neq \emptyset\} = \overline{apr}_{(R')_\alpha} (\overline{apr}_R X)_\alpha = \overline{apr}_{(R')_\alpha} \overline{apr}_{R_\alpha} X_\alpha$

3) $(\underline{Apr}_R X)_{\alpha+} = \{v \in U \mid \underline{Apr}_R X(u) > \alpha\}$

$= \{v \in U \mid \max_{u \in U} \min(R'(v, u), \min_{w \in W} \max(1-R(u, w), X(w))) > \alpha\}$

$= \{v \in U \mid \exists u \in U, R'(v, u) > \alpha \text{ and } \min_{w \in W} \max(1-R(u, w), X(w)) > \alpha\}$

$= \{v \in U \mid (R'_v)_{\alpha+} \cap (\underline{apr}_R X)_{\alpha+} \neq \emptyset\} = \overline{apr}_{(R')_{\alpha+}} (\underline{apr}_R X)_{\alpha+} = \overline{apr}_{(R')_\alpha} \underline{apr}_{R_{(1-\alpha)}} X_{\alpha+}$

4) $(\overline{Apr}_R X)_{\alpha+} = \{v \in U \mid \overline{Apr}_R X(v) > \alpha\}$

$= \{v \in U \mid \max_{u \in U} \min(R'(v, u), \max_{w \in W} \min(R(u, w), X(w))) > \alpha\}$

$= \{v \in U \mid \exists u \in U, (R'(v, u) > \alpha \text{ and } \max_{w \in W} \min(R(u, w), X(w)) > \alpha)$

$= \{v \in U \mid (R'_v)_{\alpha+} \cap (\overline{apr}_R X)_{\alpha+} \neq \emptyset\} = \overline{apr}_{(R')_{\alpha+}} (\overline{apr}_R X)_{\alpha+} = \overline{apr}_{(R')_{\alpha+}} \overline{apr}_{R_{\alpha+}} X_{\alpha+}$

Remark 2. From above theorem 2, then

1) $\underline{Apr}_R Y = \bigvee_{\alpha \in [0,1]} (\alpha \bigwedge (\overline{apr}_{(R')_\alpha} \underline{apr}_{R_{(1-\alpha)+}} Y_\alpha)) = \bigvee_{\alpha \in [0,1]} (\alpha \bigwedge (\overline{apr}_{(R')_\alpha} \underline{apr}_{R_{(1-\alpha)}} X_{\alpha+}))$,

2) $\overline{Apr}_R Y = \bigvee_{\alpha \in [0,1]} (\alpha \bigwedge (\overline{apr}_{(R')_\alpha} \overline{apr}_{R_\alpha} X_\alpha)) = \bigvee_{\alpha \in [0,1]} (\alpha \bigwedge (\overline{apr}_{(R')_{\alpha+}} \overline{apr}_{R_{\alpha+}} X_{\alpha+}))$.

4 Attribute Reduction Based on Composed Fuzzy-Rough Set

An information system S=(U, A, f, V) is called as decision table system, if $A=C\cup D$ and $C\cap D=\varnothing$, where C is the conditional attributes set and $D\neq\varnothing$ is the set of decision attributes. The issue of decision table mainly focuses on how to obtain whole rules by as less rules and attributes as possible from the information table. The main approach is attribute reduction including the reduction of attributes-values and deletion of redundant rules. In fuzzy case, fuzzy rough set attributes reduction (Fuzzy RSAR) should be built on the notion of the composed fuzzy lower approximation. Let fuzzy decision table system $S=(U, C\cup\{d\}, f, V)$, for arbitrary $P\subseteq C$, the fuzzy positive region $POS_P(\{d\})=\bigcup_{F\in U/\{d\}} \underline{Apr}_P F$, where $\mu_{POS_P(\{d\})}(u) = \sup_{F\in U/\{d\}} \mu_{\underline{PF}}(u)$, Then the dependency function $\gamma_P(\{d\})$ can be calculated by the following:

$$\gamma_P(\{d\})=\frac{|POS_P(\{d\})|}{|U|}=\frac{\sum_{u\in U}\mu_{POS_P(\{d\})}(u)}{|U|} \tag{4.1}$$

In fuzzy case, we use the fuzzy positive region $POS_C(\{d\})$ rather than |U| as the denominator of normalization, then

$$\gamma(\{d\})=\frac{|POS_P(\{d\})|}{|POS_C(\{d\})|}=\frac{\sum_{u\in U}\mu_{POS_P(\{d\})}(u)}{\sum_{u\in U}\mu_{POS_C(\{d\})}(u)} \tag{4.2}$$

An data set example from stock market [10] is given to illustrate the operation of fuzzy RSAR, in which $U=\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$, two real-valued attributes are feature a (profit ratio of per stock) and feature b (harvest ratio of per capital), decision 2-valued attribute is d (representing invest or not), fuzzy equivalence class over U are (H_a, L_a) partitioned by attribute a and (H_b, L_b) partitioned by attribute b, respectively.

Table 1. Stock Information Table

U	a: profit ratio		b: harvest ratio of per capital		d: investment
	L_a	H_a	L_b	H_b	
1	1	0	1	0	Y
2	0.7	0.3	0.2	0.8	N
3	0.8	0.2	0.9	0.1	Y
4	0.9	0.1	1	0	Y
5	0.1	0.9	0.2	0.8	N
6	0.8	0.2	1	0	Y
7	0.1	0.9	0.2	0.8	N
8	0.8	0.2	0.2	0.8	Y

Setting A={a}, B={b}, C={a, b} and Q={d}, then the following equivalence classes are obtained from the above decision table.

$$U/Q=\{X, Y\}=\{\{u_1, u_3, u_4, u_6, u_8\}, \{u_2, u_5, u_7\}\}.$$

$$U/A= \{L_a=(1.0,0.7,0.8,0.9,0.1,0.8,0.1,0.8), H_a= (0.0,0.3,0.2,0.1,0.9,0.2,0.9,0.2)\},$$

$$U/B= \{L_b= (1.0,0.2,0.9,1.0,0.2,1.0,0.2,0.2), H_b= (0.0,0.8,0.1,0.0,0.8,0.0,0.8,0.8)\},$$

$$U/C= \{L_a \cap L_b, L_a \cap H_b, H_a \cap L_b, H_a \cap H_b\}$$

$$= \{(1.0, 0.2, 0.8, 0.9, 0.1, 0.8, 0.1, 0.2), (0.0, 0.7, 0.1, 0.0, 0.1, 0.0, 0.1, 0.8),$$

$$(0.0, 0.2, 0.2, 0.1, 0.2, 0.2, 0.2, 0.2), (0.0, 0.3, 0.1, 0.0, 0.8, 0.0, 0.8, 0.2)\}.$$

The first step is to calculate the lower approximations of the sets A, B and C. For simplicity, only A will be considered here. For object u_1 and decision equivalence class X= $\{u_1, u_3, u_4, u_6, u_8\}$ and Y=$\{u_2, u_5, u_7\}$,

$$\mu_{\underline{AX}}(u_1)=\max_{F\in U/A} \min(\mu_F(u_1), \min_{y\in U} \max\{1-\mu_F(y), \mu_X(y)\})=0.3$$

$$\mu_{\underline{AY}}(u_1)=\max_{F\in U/A} \min(\mu_F(u_1), \min_{y\in U} \max\{1-\mu_F(y), \mu_X(y)\})=0.0$$

Hence, $\mu_{POS_A(Q)}(u_1)=0.3$. For the other objects, $\mu_{POS_A(Q)}(u_2)=0.3$, $\mu_{POS_A(Q)}(u_3)=0.3$, $\mu_{POS_A(Q)}(u_4)=0.3$, $\mu_{POS_A(Q)}(u_5)=0.8$, $\mu_{POS_A(Q)}(u_6)=0.2$, $\mu_{POS_A(Q)}(u_7)=0.8$, $\mu_{POS_A(Q)}(u_8)=0.3$. Then $r_A(Q)=\frac{\sum_{u\in U}\mu_{POS_A(Q)}(u)}{|U|}=3.3/8=0.4125$. Calculating for B and C gives $r_B(Q)=5/8=0.625$, $r_C(Q)=5.4/8=0.675$. Because there are only two condition attributes in this example, so its core and reduction are set {a, b}. The result is exactly in accordance with that of come from by the method of fuzzy cluster in paper [10].

5 Conclusions

As a suitable mathematical model to handle partial knowledge in data set, traditional RSAR encounters some critical problems when the noise and real-valued attributes value is included in the information system. The fuzzy RSAR method can alleviate these important problems and has been applied in more than one field with very promising results. In this paper, we study the fuzzy RSAR methods used in fuzzy information systems. We extend the rough set model to fuzzy case and present the formal definition of the composed fuzzy rough set. We also illustrate the fuzzy RSAR method and give a simple example to show its higher efficiency and accuracy.

References

1. Dubois, D., Prade, H.: Rough fuzzy sets and fuzzy rough sets. International Journal General Systems 17(2-3), 191–209 (1990)
2. Jensen, R., Shen, Q.: Fuzzy rough attribute reduction with application to web categorization. Fuzzy Sets and Systems 141, 469–485 (2004)

3. Liu, G.L.: The Axiomatic Systems of Rough Fuzzy Sets on Fuzzy Approximation Space (in Chinese). Chinese Journal Of Computers 27(9), 1187–1191 (2004)
4. Pawlak, Z.: Rough sets. In: Lin, T.Y., Cercone, N. (eds.) Rough Sets and Data Mining, pp. 3–8. Kluwer Academic Publishers, Boston (1997)
5. Slowinski, R.: Intelligent Decision Support: Handbook of Applications and Advances of the Rough Sets Theory, pp. 287–304. Kluwer Academic Publishers, Dordrecht (1992)
6. Wu, W.-Z., Mi, J.-S., Zhang, W.-X.: Generalized fuzzy rough sets. Information Sciences 151, 263–282 (2003)
7. Zhang, W., Wu, W., Liang, J., Li, D.: Theory and Methods of the Rough Sets (in Chinese). Sciences Publisher Press, Beijing (2001)
8. Hong, X., Zhong, C.H., Xiao, N.D.: Discretization of Continuous Attributes in Rough Set Theory Based on Information Entropy(in Chinese). Chinese Journal of Computers 28(9), 1570–1573 (2005)
9. Yao, Y.Y.: A comparative study of fuzzy sets and rough sets. J. of Information Science 109(1-4), 227–242 (1998)
10. Zhang, S., Sun, J., Zhang, J.: A Study on the Reducing Method of Rough Set Decision Table Based on Fuzzy Cluster (in Chinese). Computer Engineering and Applications 15, 175–177 (2004)