Optimal Routing Algorithm and Diameter in Hexagonal Torus Networks*

Zhen Zhang [1,2], Wenjun Xiao[1], and Mingxin He[1,2]

[1] Dept. of Computer Science, South China University of Technology
Guangzhou, China 510641
[2] Dept. of Computer Science, Jinan University, Guangzhou, China 510632
zhang2003174@yahoo.com.cn,
wjxiao@scut.edu.cn, mx.he@yeah.net

Abstract. Nodes in the hexagonal mesh and torus network are placed at the vertices of a regular triangular tessellation, so that each node has up to six neighbors. The routing algorithm for the Hexagonal Torus is very complicated, and it is an open problem by now. Hexagonal mesh and torus are known to belong to the class of Cayley digraphs. In this paper, we use Cayley-formulations for the hexagonal torus, along with some result on subgraphs and Coset graphs, to develop the optimal routing algorithm for the Hexagonal Torus, and then we draw conclusions to the network diameter of the Hexagonal Torus.

1 Introduction

Hexagonal networks belong to the family of networks modeled by planar graphs. These networks are based on triangular plane tessellation, or the partition of a plane into equilateral triangles. The closest networks are those based on regular hexagonal, called honeycomb networks, and those based on regular square partitions, called mesh networks. Hexagonal networks and honeycomb have been studied in a variety of contexts. The Honeycomb architecture was proposed in [12], where a suitable addressing scheme together with routing and broadcasting algorithms were investigated, higher dimensional hexagonal networks have been defined in [5] and [4] as a generalization of the plane hexagonal networks. Addressing scheme, routing and broadcasting algorithms have been also proposed. An addressing scheme for the processors, and the corresponding routing and broadcasting algorithms for a hexagonal interconnection network has been proposed in [2]. The performance of hexagonal networks has been further studied in [3] and [11]. Hexagonal networks has been used in tracking mobile users and connection rerouting in Cellular networks[9]. The 2D hexagonal torus has been used in the HARTS project[13]. But the routing algorithm for the hexagonal torus has been an open problem.

* This research is supported by the Natural Science Foundation of Guangdong Province, China (No. 04020130).

M. Xu et al. (Eds.): APPT 2007, LNCS 4847, pp. 241–250, 2007.

Hexagonal mesh and torus, as well as honeycomb and certain other pruned torus networks, are known to belong to the class of Cayley graphs which are node symmetric and possess other interesting mathematical properties[15,16,17,19,7]. In this paper we use Cayley-graph formulations of hexagonal torus to develop an optimal routing algorithm, and then discuss the network diameter.

2 Knowledge of Cayley Graph

Before proceeding further, we introduce some definitions and notations related to digraphs Cayley digraphs in particular, and interconnection networks. For more definitions and mathematical results on graphs and groups we refer the reader to [1] and [6], for instance, and on interconnection networks to [8] and [10].A digraph $\Gamma=(V, E)$ is defined by a set V of vertices and a set E of directed edges. The set E is subset of elements (u, v) of If the subset E is symmetric, that is, $(u,v)\in E$, implies $(v,u)\in E$, we identify two opposite arcs (u,v) and (v,u) by the undirected edge (u,v). Let G be a group and S a subset of G. The subset S is said to be a generating set of G, and the elements of S are called generators of G, if every element of G can be expressed as a finite product of their powers. We also say that G is generated by S. The Cayley digraph of the group G and the subset S, denoted by $Cay(G,S)$, has vertices that are elements of G and arcs that are ordered pairs (g,gs) for $g\in E,s\in S$. If S is a generating set of G, we say that $Cay(G,S)$ is the Cayley digraph of G generated by S. When $1\notin S$ and $S=S^{-1}$, the graph $Cay(G,S)$ is a simple graph. Assume that Γ and Σ if for and $(u,v)\in E(\Gamma)$ we have $(\phi(u), \phi(v))\in E(\Sigma)$. In particular, if ϕ is bijection such that both ϕ and the inverse of ϕ are homeomorphisms, then ϕ is called an isomorphism of Γ to Σ. Let G be a group and S a subset of G. Assume that K is a subgroup of G, denoted as $K\leq G$. Let G/K denote the set of the right cosets of K in G. The right coset graph of G with respect to subgroup K and subset S, denoted by $Cos(G,K,S)$set of the right cosets of K in G. is the digraph with vertex set G/K such that there exists an edge (Kg, Kg') if and only if there exists $s\in S$ and $Kgs=Kg'$.

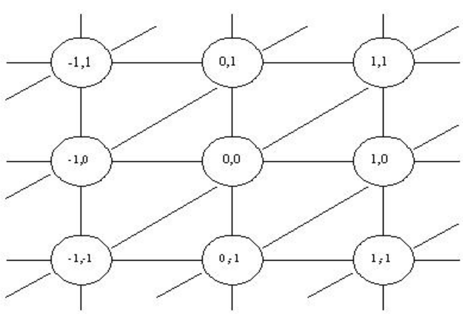

Fig. 1. Connectivity pattern for hexagonal mesh network

3 Hexagonal Mesh and Torus

3.1 Hexagonal Mesh

Let $G=Z\times Z$ where Z is the infinite cyclic group of integers, and consider $\Gamma=Cay(G,S)$ with $S=\{(\pm1,0),(0, \pm1),(1,1),(-1,-1)\}$. It is evident that Γ is isomorphic to the hexagonal mesh network [9][12]. Fig.1 shows a small part of an infinite hexagonal mesh in which the six neighbors of the "center" node (0,0) are depicted.

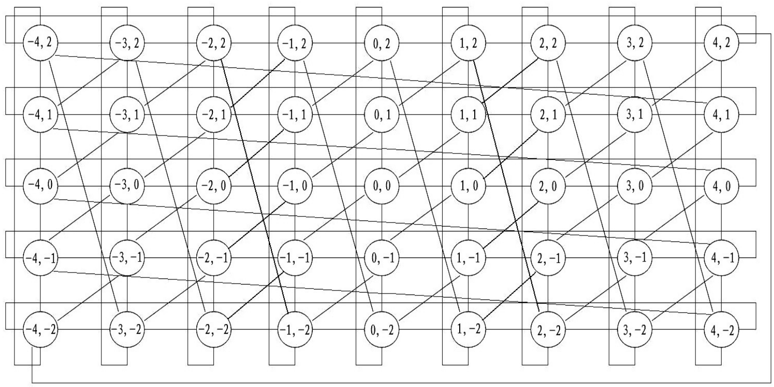

Fig. 2. Hexagonal torus with order 9×5

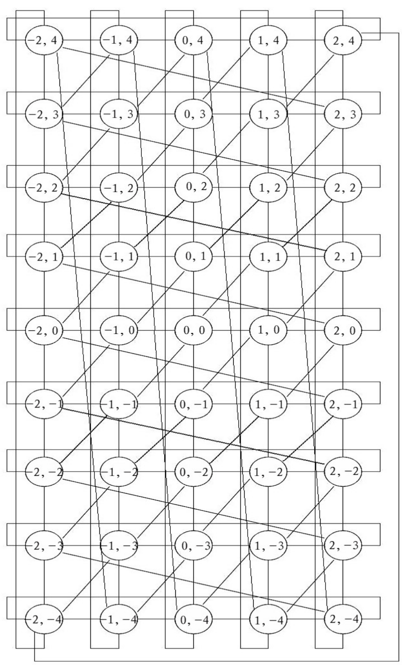

Fig. 3. Hexagonal torus with order 5×9

Using the Cayley-graph formulation of hexagonal networks, we can easily derive the distance *dis*((*a,b*),(*c,d*)) between the vertices (*a,b*) and (*c,d*) in such networks[18].

The routing algorithm of hexagonal mesh has been developed in [16] as the follow proposition.

Proposition 1. In the hexagonal mesh *Γ*, *dis*((0,0),(*a*,*b*)) equals *max*(|*a*|,|*b*|) if a and b have the same sign and |*a*|+|*b*| otherwise.

Proof. See [16].

By symmetry of Cayley graphs, we can easily obtain the distance between any two vertices in the graph *Γ* from Proposition 1, using *dis*((*a*,*b*),(*c*,*d*))=*dis*((0,0),(*c-a*,*d-b*)). This observation and the preceding discussion lead to a simple distributed routing algorithm for *Γ*.

3.2 Hexagonal Torus

Let $G=Z_l \times Z_k$, where Z_l and Z_k are cyclic groups of orders l and k respectively, $l>0,k>0$. Assume that *S* is defined as in the preceding paragraph. Then $\triangle=Cay(H,S)$ is the hexagonal torus of order *lk*. Fig.2 shows hexagonal torus with order 9×5 and Fig.3 shows hexagonal torus with order 5×9.

Using the results obtained for hexagonal meshes according to Proposition 1, we can deal with problems on Hexagonal torus which are, in general, more difficult. Let $\triangle$ be defined as above. Then we have the following result.

Proposition 2. For the hexagonal torus $\triangle$ of order *lk* and integers *a* and *b*, $l>a\geq 0, k>b\geq 0$,we have $dist((0,0),(a,b))=min(max(a,b),max(l-a,k-b),l-a+b,k+a-b)$.

According to the Proposition 2, we can develop a routing algorithm of the hexagonal torus.

4 Optimal Routing Algorithm for Hexagonal Torus

The hexagonal torus $\triangle=Cay(Z_l \times Z_k,S)$ with $S=\{(\pm 1,0),(0, \pm 1),(1,1),(-1,-1)\}$, is vertex transitive. The routing of any two nodes can transform to the routing of (0,0) to (*a,b*), *a* and *b* are integers. Let $a+ml \rightarrow a$ and $b+nk \rightarrow b$, with the choice of integer m and n, we can get

$$-\lfloor l/2 \rfloor \leq a \leq \lceil l/2 \rceil -1 \qquad -\lfloor k/2 \rfloor \leq b \leq \lceil k/2 \rceil -1$$

Now we discuss the routing from (0,0) to (*a*,*b*).

Definition 1. For any node (*a,b*) in the hexagonal torus, *a* is called the *x-dimension* coordinate and *b* is called the *y-dimension* coordinate. When $a+0 \rightarrow a$ or $a+1 \rightarrow a$ is called *x-dimension* increase, and when $x-0 \rightarrow x$ or $x-1 \rightarrow x$ is called *x-dimension* decrease. Also we can define *y-dimension* increase and decrease.

Proposition 3. The Optimal routing from (0,0) to (*a,b*) must keep *x-dimension* and *y -dimension* increase or decrease.

Proof. We only consider the case of first increase then decrease.

Case 1. *x-dimension* first increase then decrease.

Assume the routing from (0,0) to (a,b) is: (0,0) →...→(a_i,b_j) →(a_i+1,b_k) →(a_i,b_m) →…→(a,b), and $dis((0,0),(a,b))=D$. Because the generator $S=\{(\pm1,0),(0,\pm1),(1,1),(-1,-1)\}$, $b_k=b_j$ or b_j+1 and $b_m=b_k$ or b_k-1. We can get $b_m=b_j$ or b_j+1 or b_j-1, (a_i,b_m) may be (a_i,b_j) or (a_i, b_j+1) or (a_i, b_j-1).

If $(a_i,b_m)=(a_i,b_j)$, the (a_i,b_j) →(a_i+1,b_k) →(a_i,b_m) is a circle, the routing can change to (0,0) →...→(a_i,b_j) →(a_i+1,b_k) →(a_i,b_m) →…→(a,b), the distance change to D-2.

If $(a_i,b_m)=(a_i,b_j+1)$, the routing can change to (0,0) →...→(a_i,b_j) →(a_i,b_j+1) →…→(a,b), the distance change to D-1, this means the routing keeps *x-dimension* increase.

If $(a_i,b_m)=(a_i,b_j-1)$, the routing can change to (0,0) →...→(a_i,b_j) →(a_i,b_j-1) →…→(a,b), the distance change to D-1, this means the routing keep *x-dimension* decrease.

Case 2. The case of *y-dimension* first increase then decrease is similar to Case 1.
According to the Proposition 3, the routing between any two nodes can be divided into *x-dimension* routing and *y-dimension* routing. □

Definition 2. We define the functions *dist*( ) to denote the distance of two nodes, $dist_x$( ) to denote the *x-dimension* distance, $dist_y$ ( ) denote the *y- dimension* distance.

Now, we discuss the routing from (0,0) to (a,b) in five cases:

Case 1. One of a or b is zero
Case 2. $a>0$ and $b>0$.
Case 3. $a<0$ and $b<0$.
Case 4. $a>0$ and $b<0$.
Case 5. $a<0$ and $b>0$.

4.1 One of a or b is Zero

If $a=0$ and $b>0$, *x-dimension* and *y-dimension* keep increase to get the routing from (0,0) to (0,b): (0,0) →(0,1) →...→$(0,b)$.

If $a=0$ and $b<0$, *x-dimension* and *y-dimension* keep decrease to get the routing from (0,0) to $(0,b)$ the routing from (0,0) to $(0,b)$ keep *x-dimension* and *y-dimension* decrease : (0,0) →(0,-1) →...→$(0,b)$.

Similarly, we can get the routing in the case of $b=0$.

4.2 *a*>0 and *b*>0

For *x-dimension*, it has two ways to establish the routing from 0 to a: the increase way or the decrease way. According to the increase way, $dist_x((0,0),(a,b))=a$, while according to the decrease way, $dist_x((0,0),(a,b))=l-a$. Because $0 < a \le \lceil l/2 \rceil - 1$, we can get $\lfloor l/2 \rfloor + 1 \le l - a < l$, it is obviously that $l-a>a$. So the *x-dimension* routing must be increased. Because $b>0$, the *y-dimension* routing must be increased too. Synthetically, the routing between (0,0) to (a,b) must be *x-dimension* increased and *y-dimension* increased.

If $a \ge b$, the routing is:
(0,0) →(1,1) →...→(b,b) →$(b+1,b)$ →...→$(a-1,b)$ →(a,b).
If $a<b$, the routing is:
(0,0) →(1,1) →...→(a,a) →$(a,a+1)$ →...→$(a,b-1)$ →(a,b).

4.3 *a*<0 and *b*<0

The discussion of this part is similar to part 3.2, we can draw the conclusion that the routing between (0,0) to (*a,b*) is *x-dimension* decrease and *y-dimension* decrease.

If $|a|\ge|b|$, the routing is:

$(0,0) \to(-1,-1) \to\ldots\to(b,b) \to(b-1,b) \to\ldots\to(a+1,b) \to(a,b)$.

If $|a|<|b|$, the routing is:

$(0,0) \to(-1,-1) \to\ldots\to(a,a) \to(a-1,a) \to\ldots\to(a,b+1) \to(a,b)$.

4.4 *a*>0 and *b*<0

Because $-\lfloor k/2 \rfloor \le b < 0$, we can get $k > k+b \ge k-\lfloor k/2 \rfloor = \lceil k/2 \rceil$, it is obviously that $k+b>|b|$. We have already known that $l-a>a$.

The routing between (0,0) to (*a,b*) can be implemented in four ways:

1. *x-dimension* increase and *y-dimension* decrease, the distance is $a+|b|$.
2. *x-dimension* increase and *y-dimension* increase, the distance is $max(a, k+b)$.
3. *x-dimension* decrease and *y-dimension* decrease, the distance is $max(l-a, -b)$.
4. *x-dimension* decrease and *y-dimension* increase, the distance is $l-a+k+b$.

According to the Proposition 2, we know that $dist((0,0),(a,b))=min(a+|b|, max(a, k+b), max(l-a, -b),l-a+k+b)$, then we discuss the routing problem in two cases:

(1) *l-a>k+b*

Because $l-a>k+b$ and $k+b>|b|$, we can get $max(l-a,-b)=l-a$, $l-a+k+b>l-a$, and $l-a>max(a, k+b)$, then $dist((0,0),(a,b))=min(a+|b|, max(a, k+b))$.

If $a\ge k+b$, $dist((0,0),(a,b))=min(a+|b|, a)=a$. So the routing is *x-dimension* increase and *y-dimension* increase:

$$(0,0) \to (1,1) \to \ldots \to (\lceil k/2 \rceil -1, \lceil k/2 \rceil -1) \to (k/2, -\lfloor k/2 \rfloor) \to (k/2+1, -\lfloor k/2 \rfloor +1)$$
$$\to \ldots \to (k+b,b) \to (k+b+1,b) \to \ldots \to (a,b)$$

Example 1. The routing from (0,0) to (4,-2) in Fig.2 is as follows:

$(0,0) \to(1,1) \to(2,2)\to(3,-2) \to(4,-2)$.

If $a<k+b$, $dist((0,0),(a,b))=min(a+|b|, k+b)$.

When $a+|b|\ge k+b$, , that is $a\ge k+2b$, we have $dist((0,0),(a,b))=k+b$. So the routing is *x-dimension* increase and *y-dimension* increase:

$$(0,0) \to (1,1) \to \ldots \to (a,a) \to (a,a+1) \to \ldots \to$$
$$(a, \lfloor k/2 \rfloor -1) \to (a, -\lfloor k/2 \rfloor) \to (a, -k/2+1) \to \ldots \to (a,b)$$

Example 2. The routing from (0,0) to (1,-2) in Fig.2 is as follows:

$(0,0) \to(1,1) \to(1,2)\to(1,-2)$

When $a+|b|<k+b$, that is $a<k+2b$, we have $dist((0,0),(a,b))=a+|b|$. So the routing is *x-dimension* increase and *y-dimension* decrease:

$(0,0) \to(1,0) \to\ldots\to(a,0)\to(a,-1) \to\ldots\to(a,b)$.

Example 3. The routing from (0,0) to (2,-1) in Fig.2 is as follows:

$(0,0) \to(1,0) \to(2,0)\to(2,-1)$.

(2) *l-a≤k+b*

Because $k+b>l-a>a$, $max(a, k+b)=k+b$, $l-a+k+b>k+b$, and $k+b\geq max(l-a,|b|)$, we get $dist((0,0),(a,b))=min(a+|b|, max(l-a, |b|))$.

If $l-a\geq -b$, $dist((0,0),(a,b))=min(a+|b|, l-a)$.

When $a+|b|\geq l-a$, that is $b\leq 2a-l$, we have $dist((0,0),(a,b))=l-a$. So the routing is *x-dimension* decrease and *y-dimension* decrease:

$$(0,0) \rightarrow (-1,-1) \rightarrow ... \rightarrow (b,b) \rightarrow (b-1,b) \rightarrow ... \rightarrow (-\lfloor l/2 \rfloor, b) \rightarrow (\lceil l/2 \rceil - 1, b) \rightarrow (\lceil l/2 \rceil - 2, b) \rightarrow ... \rightarrow (a,b).$$

Example 4. The routing from (0,0) to (2,-3) in Fig.3 is as follows:

(0,0) →(-1,-1) →(-2,-2)→(2,-3).

When $a+|b|<l-a$ that is $b>2a-l$, we have $dist((0,0),(a,b))=a+|b|$. So the routing is *x-dimension* increase and *y-dimension* decrease:

$(0,0) \rightarrow (1,0) \rightarrow \ldots \rightarrow (a,0) \rightarrow (a,-1) \rightarrow \ldots \rightarrow (a,b)$.

Example 5. The routing from (0,0) to (1,-2) in Fig.3 is as follows:

(0,0) →(1,0) →(1,-1)→(1,-2).

If $l-a<-b$, $k+b>-b>l-a$, we have $dist((0,0),(a,b))=min(a+|b|, |b|)=|b|$. So the routing is *x-dimension* decrease and *y-dimension* decrease:

$$(0,0) \rightarrow (-1,-1) \rightarrow ... \rightarrow (-\lfloor l/2 \rfloor, -\lfloor l/2 \rfloor) \rightarrow (\lceil l/2 \rceil - 1, -\lfloor l/2 \rfloor - 1) \rightarrow (\lceil l/2 \rceil - 2, -\lfloor l/2 \rfloor - 2) \rightarrow ... \rightarrow (a, a-l) \rightarrow (a, a-l-1) \rightarrow ... \rightarrow (a,b).$$

Example 6. The routing from (0,0) to (2,-4) in Fig.3 is as follows:

(0,0) →(-1,-1) →(-2,-2)→(2,-3) →(2,-4).

4.5 a<0 and b>0

Because $0<b\leq\lceil k/2\rceil-1$ and $-\lfloor l/2 \rfloor \leq a < 0$, similar to part 4.5 we can get $k-b\geq b$ and $l+a\geq|a|$.

The routing between (0,0) to (a,b) can be implemented in four ways:

1. *x-dimension* decrease and *y-dimension* increase, the distance is $|a|+b$.
2. *x-dimension* increase and *y-dimension* increase, the distance is $max(l+a, b)$.
3. *x-dimension* increase and *y-dimension* decrease, the distance is $l+a+k-b$.
4. *x-dimension* decrease and *y-dimension* decrease, the distance is $max(|a|, k-b)$.

We know that $dist((0,0),(a,b))=min(|a|+b, max(l+a, b), l+a+k-b, max(|a|, k-b))$, then we discuss the routing problem in two cases:

(1) *k-b>l+a*

If $b\geq l+a$, $dist((0,0),(a,b))=min(|a|+b, b)=b$. So the routing is *x-dimension* increase and *y-dimension* increase:

$$(0,0) \rightarrow (1,1) \rightarrow ... \rightarrow (\lceil l/2 \rceil - 1, \lceil l/2 \rceil - 1) \rightarrow (-\lfloor l/2 \rfloor, \lceil l/2 \rceil) \rightarrow ... \rightarrow (a, a+l) \rightarrow (a, a+l+1) \rightarrow ... \rightarrow (a,b).$$

Example 7. The routing from (0,0) to (-2,4) in Fig.3 is as follows:

(0,0) →(1,1) →(2,2)→(-2,3) →(-2,4).

If $b<l+a$, $dist((0,0),(a,b))=min(|a|+b, l+a)$.

When $|a|+b\geq l+a$, that is $b\geq l+2a$, we have $dist((0,0),(a,b))=l+a$. So the routing is *x-dimension* increase and *y-dimension* increase:

$$(0,0)\to(1,1)\to\ldots\to(b,b)\to(b+1,b)\to\ldots\to(\lceil l/2\rceil-1,b)\to$$
$$(-\lfloor l/2\rfloor,b)\to(-\lfloor l/2\rfloor+1,b)\to\ldots\to(a,b).$$

Example 8. The routing from (0,0) to (-2,2) in Fig.3 is as follows:

(0,0) →(1,1) →(2,2)→(-2,2) .

When $|a|+b<l+a$, that is $b<l+2a$, we have $dist((0,0),(a,b))=b-a$. So the routing is *x-dimension* decrease and *y-dimension* increase:

$(0,0)\to(-1,0)\to\ldots\to(a,0)\to(a,1)\to\ldots\to(a,b)$.

Example 9. The routing from (0,0) to (-1,2) in Fig.3 is as follows:
(0,0) →(-1,0) →(-1,1)→(-1,2).

(2) *k-b≤l+a*

Because $max(l+a,b)=l+a$, $l+a+k-b\geq l+a$, and $l+a\geq max(|a|,k-b)$, we get $dist((0,0),(a,b))=min(|a|+b, max(|a|, k-b))$.

If $k-b\geq -a$, $dist((0,0),(a,b))=min(a+|b|, k-b)$.

When $b-a\geq k-b$, that is $a\leq 2b-k$, we have $dist((0,0),(a,b))=k-b$. So the routing is *x-dimension* decrease and *y-dimension* decrease:

$$(0,0)\to(-1,-1)\to\ldots\to(a,a)\to(a,a-1)\to\ldots\to$$
$$(a,-\lfloor k/2\rfloor)\to(a,\lceil k/2\rceil-1)\to(a,\lceil k/2\rceil-2)\to\ldots\to(a,b).$$

Example 10. The routing from (0,0) to (-2,2) in Fig.2 is as follows:

(0,0) →(-1,-1) →(-2,-2)→(-2,2).

When $b-a<k-b$, that is $a>2b-k$, we have $dist((0,0),(a,b))=b-a$. So the routing is *x-dimension* decrease and *y-dimension* increase:
$(0,0)\to(-1,1)\to\ldots\to(a,0)\to(a,1)\to\ldots\to(a,b)$.

Example 11. The routing from (0,0) to (-2,1) in Fig.2 is as follows:

(0,0) →(-1,0) →(-2,0) →(-2,1).

If $k-b<-a$, $dist((0,0),(a,b))=min(a+|b|, |a|)=|a|$. So the routing is *x-dimension* decrease and *y-dimension* decrease:

$$(0,0)\to(-1,-1)\to\ldots\to(-\lfloor k/2\rfloor,-\lfloor k/2\rfloor)\to(-\lfloor k/2\rfloor-1,\lceil k/2\rceil-1)$$
$$\to(b-k,b)\to(b-k-1,b)\to\ldots\to(a,b).$$

Example 12. The routing from (0,0) to (-4,2) in Fig.2 is as follows:

(0,0) →(-1,-1) →(-2,-2)→(-3,2) →(-4,2).

The routing algorithm can get the optimal routing according to the Proposition 3.

5 Diameter of Hexagonal Torus

For any digraph Γ, $D(\Gamma)$ denotes the diameter of Γ, defined as the longest distance between any pair of vertices in Γ. In **[17]** ,we have the following result about the diameter.

Theorem 1. For $g \in S, S \subseteq G$, ,the mapping $\phi: g \rightarrow Kg$ is a homomorphism from $Cay(G,S)$ to $Cos(G,K,S)$.

Theorem 2. Assume that G is a finite group, $K \leq G, \Gamma = Cay(G,S)$, $\triangle = Cos(G,K,S)$ for some generating set S of G, and $D(\Gamma_K)$ denote the longest distance between vertices of K in Γ. Then we have $D(\Gamma) \leqslant D(\triangle) + D(\Gamma_K)$.
Proof. See [17].

Proposition 4. Assume the hexagonal torus $\triangle = Cay(Z_l \times Z_k, S)$,where $S=\{(\pm1,0),(0,\pm1),(1,1),(-1,-1)\}$, we have $\lceil (\max(l,k))/2 \rceil \leq D(\Delta) \leq \lfloor l/2 \rfloor + \lfloor k/2 \rfloor$.

Proof. Let $K=\{ (-\lfloor k/2 \rfloor,0)$, $(-\lfloor k/2 \rfloor+1,0)$,…,(0,0),(1,0),…, $(\lceil k/2 \rceil-1,0)$ }, then $Cos(G,K,S)$ is an 1-D torus according to Theorem1and, $D(Cos(G,K,S)) = \lfloor l/2 \rfloor$. The K in $\triangle$ is an 1-D torus too, and $D(\Gamma_K) = \lfloor k/2 \rfloor$. From the Theorem 2, we get $D(\triangle) \leqslant \lfloor l/2 \rfloor + \lfloor k/2 \rfloor$.For any $l \geq 3, k \geq 3$, either $dis((0,0),(1,\lceil k/2 \rceil)$or $dis((0,0),(\lceil l/2 \rceil,1))$ is $\lceil (\max(l,k))/2 \rceil$. Assume $k \geq l$, we can get $D(\triangle) \geq \lceil (\max(l,k))/2 \rceil$ according to the Proposion 2. □

6 Conclusion

The routing algorithm for the Hexagonal Torus is an open problem. In this paper, we use Cayley-formulations for the hexagonal torus to develop an optimal routing algorithm for the Hexagonal Torus. Then we discuss the diameter of the Hexagonal Torus, and give an upper bound and a lower bound to the diameter. We are currently investigating the Hexagonal Torus diameter in order to give an accurate value.

References

1. Biggs, N.: Algebraic Graph Theory. Cambridge University Press, Cambridge (1993)
2. Chen, M.S., Shin, K.G., Kandlur, D.D.: Addressing, Routing and Broadcasting in Hexagonal Mesh Multiprocessors. IEEE Trans. Computers 39(1), 10–18 (1990)
3. Dolter, J.W., Ramanathan, P., Shin, K.G.: Performance Analysis of Virtual Cut-Through Switching in HARTS: A Hexagonal Mesh Multicomputer. IEEE Trans. Computers 40(6), 669–680 (1991)
4. Decayeux, C., Seme, D.: 3D hexagonal network: modeling, topological properties, addressing scheme, and optimal routing algorithm. IEEE Trans. on Parallel and Distrib. Sys. 16(9), 875–884 (2005)
5. García, F., Solano, J., Stojmenovic, I., Stojmenovic, M.: Higher dimensional hexagonal networks. Journal of Parallel and Distributed Computing 63(11), 1164–1172 (2003)

6. Heydemann, M.: Cayley Graphs and Interconnection Networks. In: Graph Symmetry: Algebraic Methods and Applications, pp. 167–224 (1997)
7. He, M.X., Xiao, W.J.: A Unified Addressing Schema for Hexagonal and Honeycomb Networks with Isomorphic Cayley Graphs. In: IMSCCS 2006. Proc. of 1st Int. Multi-Symp. of Computer and Computational Sciences, Hangzhou, China, vol. 1, pp. 363–368 (2006)
8. Leighton, F.T.: Introduction to Parallel Algorithms and Architectures: Arrays, Trees, Hypecubes. Morgan Kaufmann, San Francisco (1992)
9. Nocetti, F.G., Stojmenovic, I., Zhang, J.Y.: Addressing and Routing in Hexagonal Networks with Applications for Tracking Mobile Users and Connection Rerouting in Cellular Networks. IEEE Trans. Parallel and Distributed Systems 13(9), 963–971 (2002)
10. Parhami, B.: Introduction to Parallel Processing: Algorithms and Architectures, Plenum (1999)
11. Robic, B., Silc, J.: High performance Computing on a Honeycomb Arcbhitecture. In: Proc. Int. ACPC Parallel Computation Conf. (1993)
12. Stojmenovic, I.: Honeycomb Networks: Topological Properties and Communication Algorithms. IEEE Trans. Parallel and Distributed systems 8(10), 1036–1042 (1997)
13. Shin, K.G.: HARTS: A Distributed Real-Time Architectrue. Computer 24(5), 25–35 (1991)
14. Tosic, R., Masulovic, D., Stojmenovic, I., et al.: Enumeration of Polyhex Hydrocarbons up to h=17. J. Chemical Infromation and Computer Sciences 35, 181–187 (1995)
15. Xiao, W.J., Parhami, B.: Some Mathematical Properties of Cayley Digraphs with Applications to Interconnection Network Design. Int. J. Computer Mathematics 82, 521–528 (2005)
16. Xiao, W.J., Parhami, B.: Further Mathematical Properties of Cayley Digraphs Applied to Hexagonal and Honeycomb Meshes. Discrete Applied Mathematics (to appear, 2007)
17. Xiao, W.J., Parhami, B.: Hexagonal and Pruned Torus Networks as Cayley Graphs. In: Proc. International Conf. on Communications in Computing, Las Vegas, June 21-24, 2004, pp. 107–112 (2004)
18. Xiao, W.J., Parhami, B.: Structural Properties of Cayley Digraphs with Applications to Mesh and Pruned Torus Interconnection Networks. Int. J. of Computer and System Sciences, Special Issue on Network-Based Computing (to appear, 2007)
19. Xiao, W.J., Parhami, B.: A Group Consruction Method with Applications to Deriving Pruned Interconnection Networks. IEEE Trans. on Parallel and Distrib. Sys. 18(5), 637–643 (2007)