

Property-Preserving Composition of Distributed System Components

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Abstract. Augmented marked graphs possess a special structure for modelling common resources as well as some desirable properties pertaining to liveness, boundedness, reversibility and conservativeness. This paper investigates the property-preserving composition of augmented marked graphs for the synthesis of distributed systems. It is proposed that distributed system components are specified as augmented marked graphs. An integrated system is then obtained by composing these augmented marked graphs via their common resource places. Based on the preservation of properties, the liveness, boundedness, reversibility and conservativeness of the integrated system can be readily derived. This effectively solves the difficult problem of ensuring design correctness in the composition of distributed system components.

1 Introduction

In the past decade, component-based system design has emerged as a promising paradigm to meet the ever increasing needs for managing system complexity and maximising re-use as well as for deriving software engineering into standards. When applied to distributed systems which usually involve concurrent (parallel) and asynchronous processes, one need to be aware that errors such as deadlock and capacity overflow may occur. Even though the system components are correct in the sense that they are live (implying freedom of deadlock), bounded (implying absence of capacity overflow) and reversible (implying the capability of being reinitialised from any reachable states), the integrated system may not be correct, especially as competition of common resources exists.

This paper investigates the component-based approach to synthesising a given set of distributed system components into an integrated system. Our focus is placed on the preservation of four essential properties which include liveness, boundedness, reversibility and conservativeness. Based on the property-preserving composition of augmented marked graphs, we propose a formal method for synthesising the given distributed system components into an integrated system whose design correctness (in terms of liveness, boundedness, reversibility and conservativeness) can be readily derived and verified.

A subclass of Petri nets, augmented marked graphs possess a special structure for modelling common resources. They exhibit some desirable properties pertaining to liveness, boundedness, reversibility and conservativeness. Chu and Xie first studied their liveness and reversibility using siphons and mathematical programming [1]. We proposed siphon-based and cycle-based characterisations for live and reversible augmented marked graphs, and transform-based characterisations for bounded and conservative augmented marked graphs [2, 3, 4]. Besides, the composition of augmented marked graphs via common resource places was preliminarily studied [5, 6].

In this paper, after a brief review of augmented marked graphs, we investigate the composition of augmented marked graphs via common resource places and show that this composition preserves boundedness and conservativeness whereas liveness and reversibility can be preserved under a pretty simple condition. The results are then applied to the composition of distributed system components, where liveness, boundedness, reversibility and conservativeness of the integrated system can be readily derived. These will be illustrated using examples.

The rest of this paper is organised as follows. Section 2 introduces augmented marked graphs. Section 3 presents the composition of augmented marked graphs with a special focus on the preservation of properties. Section 4 shows its application to the composition of distributed system components. Section 5 briefly concludes this paper. Readers of this paper are expected to have knowledge of Petri nets [7, 8].

2 Augmented Marked Graphs

This section introduces augmented marked graphs and summarises their known properties and characterisations.

Definition 2.1 [1]. An augmented marked graph $(N, M_0; R)$ is a PT-net (N, M_0) with a specific subset of places R called resource places, satisfying the following conditions : (a) Every place in R is marked by M_0 . (b) The net (N', M'_0) obtained from $(N, M_0; R)$ by removing the places in R and their associated arcs is a marked graph. (c) For each $r \in R$, there exist $k_r \geq 1$ pairs of transitions $D_r = \{ \langle t_{s1}, t_{h1} \rangle, \langle t_{s2}, t_{h2} \rangle, \dots, \langle t_{skr}, t_{hkr} \rangle \}$ such that $r^\bullet = \{ t_{s1}, t_{s2}, \dots, t_{skr} \} \subseteq T$ and ${}^\bullet r = \{ t_{h1}, t_{h2}, \dots, t_{hkr} \} \subseteq T$ and that, for each $\langle t_{si}, t_{hi} \rangle \in D_r$, there exists in N' an elementary path ρ_{ri} connecting t_{si} to t_{hi} . (d) In (N', M'_0) , every cycle is marked and no ρ_{ri} is marked.

Definition 2.2. For a PT-net (N, M_0) , a set of places S is called a siphon if and only if ${}^\bullet S \subseteq S^\bullet$. S is said to be minimal if and only if there does not exist a siphon S' in N such that $S' \subset S$. S is said to be empty at a marking $M \in [M_0]$ if and only if S contains no places marked by M .

Definition 2.3. For a PT-net (N, M_0) , a set of places Q is called a trap if and only if $Q^\bullet \subseteq {}^\bullet Q$. Q is said to be maximal if and only if there does not exist a trap Q' in N such that $Q \subset Q'$. Q is said to be marked at a marking $M \in [M_0]$ if and only if Q contains a place marked by M .

Property 2.1 [1]. An augmented marked graph is live and reversible if and only if it does not contain any potential deadlock. (Note : A potential deadlock is a siphon which would eventually become empty.)

Definition 2.4. For an augmented marked graph $(N, M_0; R)$, a minimal siphon is called a R -siphon if and only if it contains at least one place in R .

Property 2.2 [1, 2, 3]. An augmented marked graph $(N, M_0; R)$ is live and reversible if every R -siphon contains a marked trap.

Property 2.3 [2, 3]. An augmented marked graph $(N, M_0; R)$ is live and reversible if and only if no R -siphons eventually become empty.

Definition 2.5 [4]. Suppose an augmented marked graph $(N, M_0; R)$ is transformed into a PT-net (N', M_0') : For each $r \in R$, where $D_r = \{ \langle t_{s1}, t_{h1} \rangle, \langle t_{s2}, t_{h2} \rangle, \dots, \langle t_{skr}, t_{hkr} \rangle \}$, replace r with a set of places $\{ q_1, q_2, \dots, q_{kr} \}$ such that $M_0'[q_i] = M_0[r]$ and $q_i^\bullet = \{ t_{si} \}$ and ${}^\bullet q_i = \{ t_{hi} \}$ for $i = 1, 2, \dots, k_r$. (N', M_0') is called the R -transform of $(N, M_0; R)$.

Property 2.4 [4]. Augmented marked graph $(N, M_0; R)$ is bounded and conservative if and only if every place in its R -transform (N', M_0') belongs to a cycle.

Fig. 1 shows an augmented marked graph $(N, M_0; R)$, where $R = \{ r_1, r_2 \}$. Every R -siphon contains a marked trap and would never become empty. It follows from Properties 2.2 and 2.3 that $(N, M_0; R)$ is live and reversible. As every place in the R -transform of $(N, M_0; R)$ belongs to a cycle, according to Property 2.4, $(N, M_0; R)$ is bounded and conservative.

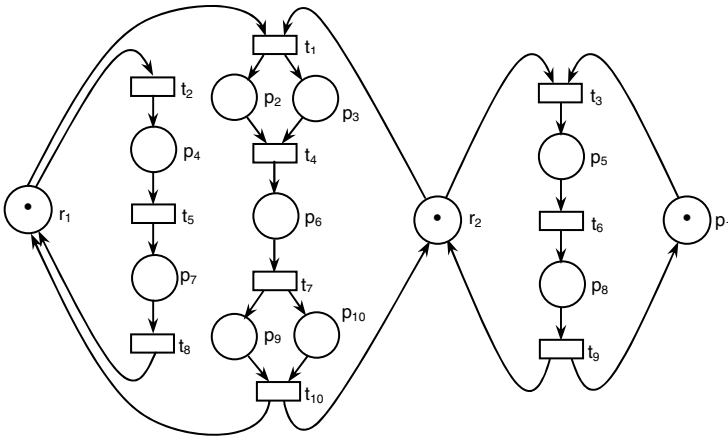


Fig. 1. An augmented marked graph

3 Composition of Augmented Marked Graphs

This section first describes the composition of augmented marked graphs via common resource places. Preservation of properties is then studied.

Property 3.1. Let $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ be two augmented marked graphs, where $R_1' = \{ r_{11}, r_{12}, \dots, r_{1k} \} \in R_1$ and $R_2' = \{ r_{21}, r_{22}, \dots, r_{2k} \} \in R_2$ are the common places that r_{11} and r_{21} are to be fused as one single place r_1 , r_{12} and r_{22} into r_2, \dots, r_{1k}

and r_{2k} into r_k . Then, the resulting net is also an augmented marked graph $(N, M_0; R)$, where $R = (R_1 \setminus R_1') \cup (R_2 \setminus R_2') \cup \{ r_1, r_2, \dots, r_k \}$. (obvious)

Definition 3.1. With reference to Property 3.1, $(N, M_0; R)$ is called the composite augmented marked graph of $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ via a set of common resource places $\{ (r_{11}, r_{21}), (r_{12}, r_{22}), \dots, (r_{1k}, r_{2k}) \}$, where $r_{11}, r_{12}, \dots, r_{1k} \in R_1$ and $r_{21}, r_{22}, \dots, r_{2k} \in R_2$. $R_F = \{ r_1, r_2, \dots, r_k \}$ is called the set of fused resource places that are obtained after fusing $(r_{11}, r_{21}), (r_{12}, r_{22}), \dots, (r_{1k}, r_{2k})$.

Fig. 2 shows two augmented marked graphs $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$. Fig. 3 shows the composite augmented marked graph $(N, M_0; R)$ of $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ via $\{ (r_{11}, r_{21}) \}$, where $R_F = \{ r_1, r_2 \}$.

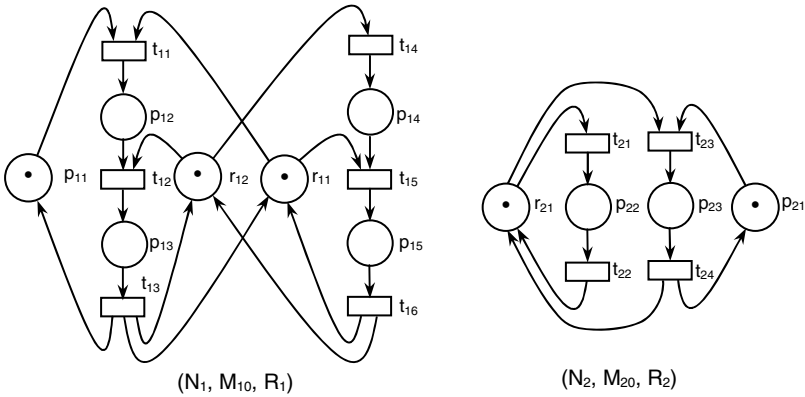


Fig. 2. Two augmented marked graphs (N_1, M_{10}, R_1) and (N_2, M_{20}, R_2)

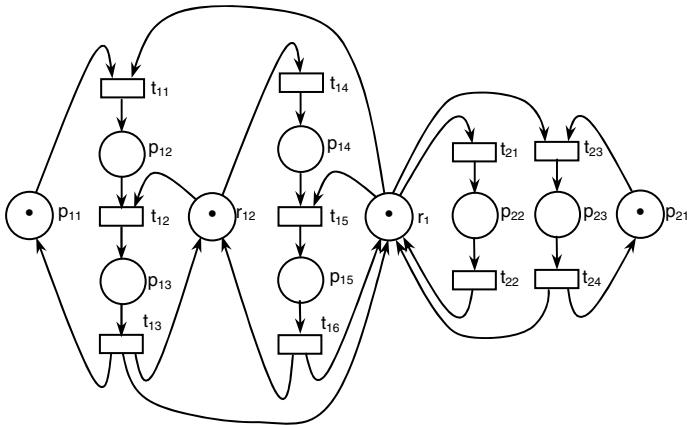


Fig. 3. An augmented marked graph obtained by composing the two augmented marked graphs in Fig. 2 via $\{ (r_{11}, r_{21}) \}$

Property 3.2 [5, 6]. Let $(N, M_0; R)$ be the composite augmented marked graph of two augmented marked graphs $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ via a set of common resource places. $(N, M_0; R)$ is bounded if and only if $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ are bounded.

Property 3.3 [5]. Let $(N, M_0; R)$ be the composite augmented marked graph of two augmented marked graphs $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ via a set of common resource places. $(N, M_0; R)$ is conservative if and only if $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ are conservative.

Definition 3.2. Let $(N, M_0; R)$ be the composite augmented marked graph of two augmented marked graphs via a set of common resource places, and $R_F \subseteq R$ be the set of fused resource places. For $(N, M_0; R)$, a minimal siphon is called a R_F -siphon if and only if it contains at least one place in R_F .

Property 3.4 [5]. Let $(N, M_0; R)$ be the composite marked graph of two augmented marked graphs $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ via a set of common resource places. $(N, M_0; R)$ is live and reversible if and only if $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ are live and no R_F -siphons eventually become empty.

Consider the augmented marked graphs $(N_1, M_{10}; R_1)$ and $(N_2, M_{20}; R_2)$ in Fig. 2. $(N_1, M_{10}; R_1)$ is neither live nor reversible but is bounded and conservative. $(N_2, M_{20}; R_2)$ is live, bounded, reversible and conservative. According to Properties 3.2 and 3.3, the composite augmented marked graph $(N, M_0; R)$ as shown in Fig. 3 is bounded and conservative. According to Property 3.4, $(N, M_0; R)$ is neither live nor reversible.

4 Application to Distributed Systems

In component-based system design, a system is synthesised from a set of components [9, 10]. It may not be live, bounded and reversible even all its components are live, bounded and reversible. For distributed systems which usually involve concurrent (parallel) and asynchronous processes, because of competition of common resources, errors such as deadlock and capacity overflow are easily induced. This section shows the application of composition of augmented marked graphs to the synthesis of a distributed system whose design correctness can be readily derived.

Fig. 4 shows a distributed system consisting of four system components, C_1 , C_2 , C_3 and C_4 . Owing to the "distributed processing" nature, the components exhibit

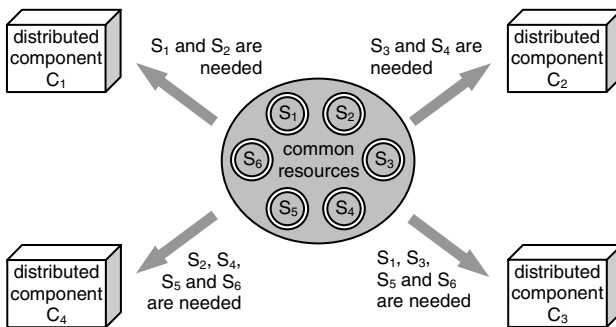


Fig. 4. Example of a distributed system with shared resources

concurrent (parallel) and asynchronous processes. There are six pieces of common resources, S_1, S_2, S_3, S_4, S_5 and S_6 , used to be shared among the components.

The functions of the distributed system components C_1, C_2, C_3 and C_4 are briefly described as follows.

C_1 : At its initial idle state, C_1 invokes operation o_{11} only if S_1 is available. While o_{11} is being processed, S_1 is occupied. Once o_{11} finishes processing, operation o_{12} is invoked only if S_2 is available. S_1 is then released. While o_{12} is being processed, S_2 is occupied. Once o_{12} finishes processing, S_2 is released and C_1 returns to idle state. At any moment, S_1 is withheld on receipt of signal m_{11} and released on receipt of signal m_{12} . S_2 is withheld on receipt of signal m_{13} and released on receipt of signal m_{14} .

C_2 : At its initial idle state, C_2 invokes operation o_{21} only if S_3 is available. While o_{21} is being processed, S_3 is occupied. Once o_{21} finishes processing, operation o_{22} is invoked only if S_4 is available. S_3 is then released. While o_{22} is being processed, S_4 is occupied. Once o_{22} finishes processing, S_4 is released and C_2 returns to idle state. At any moment, S_3 is withheld on receipt of signal m_{21} and released on receipt of signal m_{22} . S_4 is withheld on receipt of signal m_{23} and released on receipt of signal m_{24} .

C_3 : At its initial idle state, C_3 invokes operation o_{31} only if S_1, S_3, S_5 and S_6 are all available. While o_{31} is being processed, S_1, S_3, S_5 and S_6 are occupied. Once o_{31} finishes processing, S_1, S_3, S_5 and S_6 are released and C_3 returns to idle state.

C_4 : At its initial idle state, C_4 invokes operation o_{41} only if S_2, S_4, S_5 and S_6 are all available. While o_{41} is being processed, S_2, S_4, S_5 and S_6 are occupied. Once o_{41} finishes processing, S_2, S_4, S_5 and S_6 are released and C_4 returns to idle state.

Our method begins with specifying each component as an augmented marked graph. We identify the event occurrences and their pre-conditions and post-conditions in the component. For each event occurrence, a transition is created for denoting the location of occurrence. Input and output places are created to denote the locations of its pre-conditions and post-conditions. An initial marking is created to denote the system initial state. Execution for the component begins at this initial marking which semantically means its initial idle state, and ends at the same marking.

Component C_1 is specified as augmented marked graph $(N_1, M_{10}; R_1)$, where $R_1 = \{ r_{11}, r_{12} \}$. C_2 is specified as $(N_2, M_{20}; R_2)$, where $R_2 = \{ r_{21}, r_{22} \}$. C_3 is specified as $(N_3, M_{30}; R_3)$, where $R_3 = \{ r_{31}, r_{32}, r_{33}, r_{34} \}$. C_4 is specified as $(N_4, M_{40}; R_4)$, where $R_4 = \{ r_{41}, r_{42}, r_{43}, r_{44} \}$. They are shown in Fig. 5.

According to Properties 2.1, 2.2, 2.3 and 2.4, $(N_1, M_{10}; R_1)$, $(N_2, M_{20}; R_2)$, $(N_3, M_{30}; R_3)$ and $(N_4, M_{40}; R_4)$ are live, bounded, reversible and conservative.

Resource places r_{11} in $(N_1, M_{10}; R_1)$ and r_{31} in $(N_3, M_{30}; R_3)$ refer to the same resource S_1 . r_{12} in $(N_1, M_{10}; R_1)$ and r_{42} in $(N_4, M_{40}; R_4)$ refer to the same resource S_2 . r_{21} in $(N_2, M_{20}; R_2)$ and r_{33} in $(N_3, M_{30}; R_3)$ refer to the same resource S_3 . r_{22} in $(N_2, M_{20}; R_2)$ and r_{44} in $(N_4, M_{40}; R_4)$ refer to the same resource S_4 . r_{32} in $(N_3, M_{30}; R_3)$ and r_{41} in $(N_4, M_{40}; R_4)$ refer to the same resource S_5 . r_{34} in $(N_3, M_{30}; R_3)$ and r_{43} in $(N_4, M_{40}; R_4)$ refer to the same resource S_6 . $(N_1, M_{10}; R_1)$, $(N_2, M_{20}; R_2)$, $(N_3, M_{30}; R_3)$ and $(N_4, M_{40}; R_4)$ are to be composed via these common resource places.

We first obtain the composite augmented marked graphs $(N', M_0'; R')$ of $(N_1, M_{10}; R_1)$ and $(N_3, M_{30}; R_3)$ via $\{ (r_{11}, r_{31}) \}$, and the composite augmented marked graph $(N'', M_0''; R'')$ of $(N_2, M_{20}; R_2)$ and $(N_4, M_{40}; R_4)$ via $\{ (r_{22}, r_{44}) \}$. Fig. 6 shows $(N', M_0'; R')$, where r_1 is the place after fusing r_{11} and r_{31} . Fig. 7 shows $(N'', M_0''; R'')$, where r_4 is the place after fusing r_{22} and r_{44} .

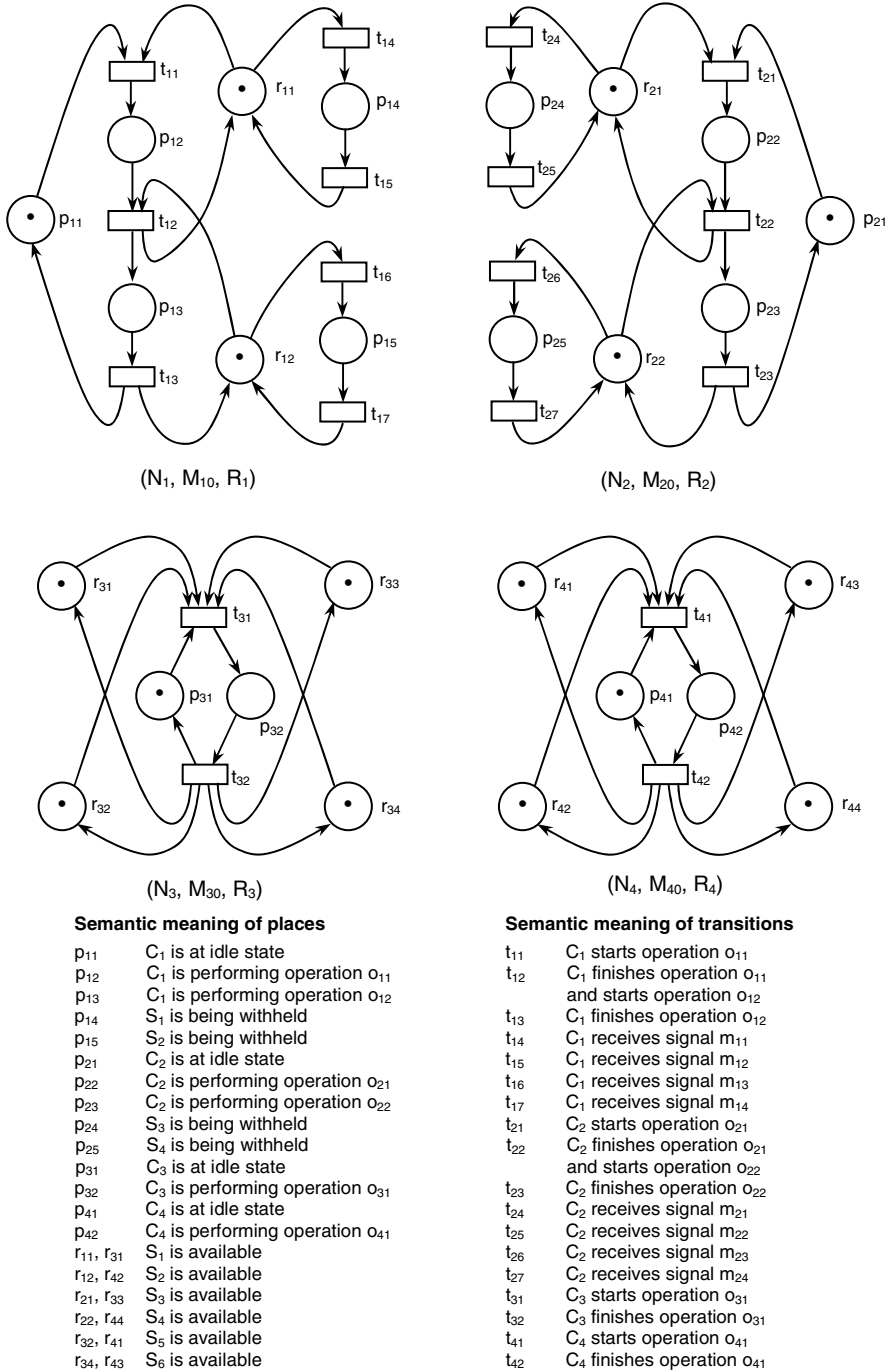


Fig. 5. Specification of distributed system components as augmented marked graphs

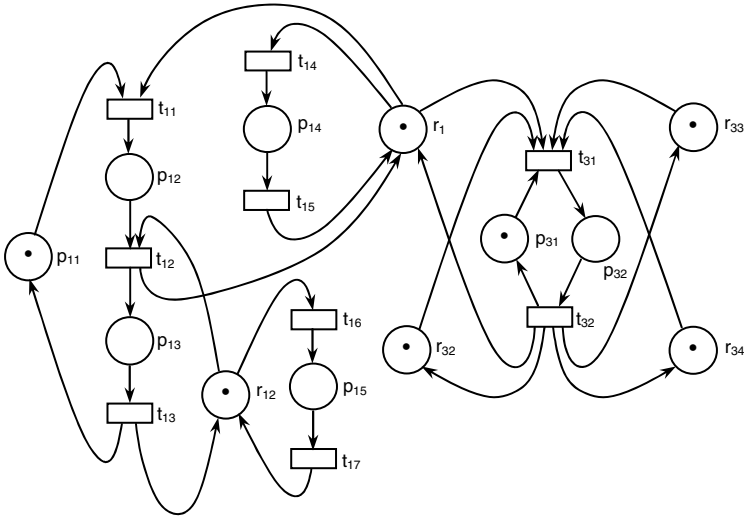


Fig. 6. Composite augmented marked graph $(N', M_0'; R')$

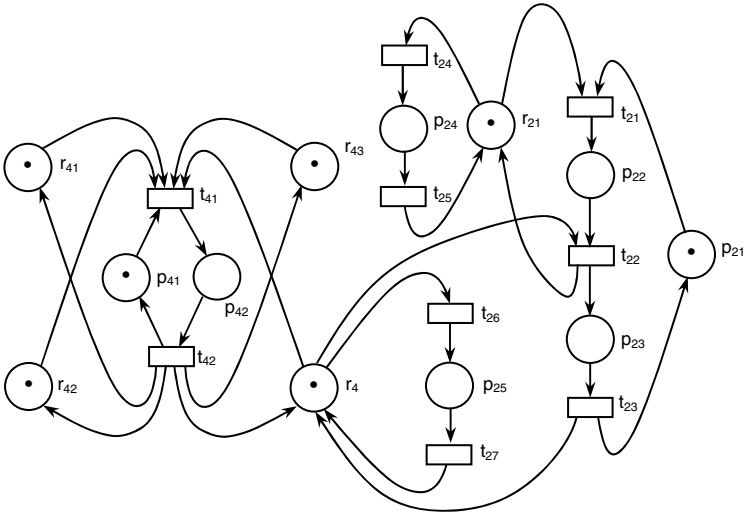
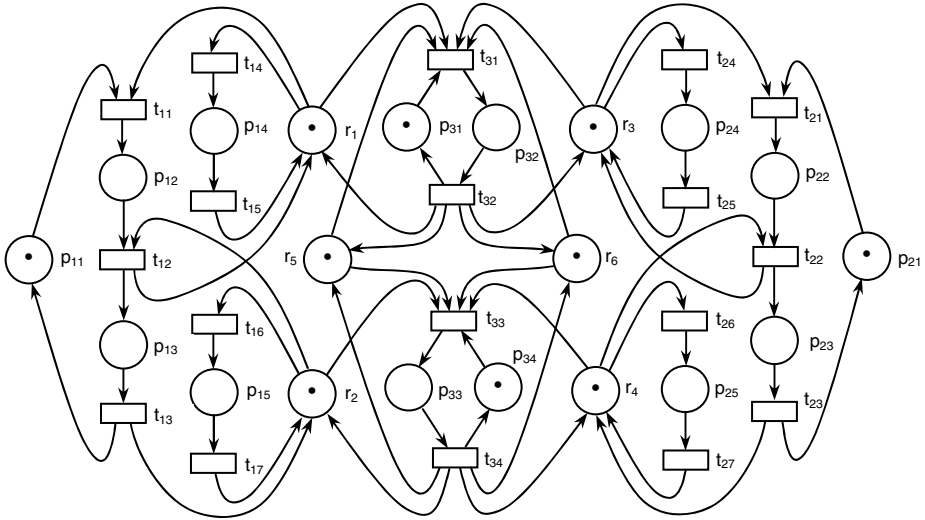


Fig. 7. Composite augmented marked graph $(N'', M_0''; R'')$

Since $(N_1, M_{10}; R_1)$, $(N_2, M_{20}; R_2)$, $(N_3, M_{30}; R_3)$ and $(N_4, M_{40}; R_4)$ are all bounded and conservative, according to Properties 3.2 and 3.3, the composite augmented marked graphs $(N', M_0'; R')$ and $(N'', M_0''; R'')$ are also bounded and conservative. On the other hand, $(N_1, M_{10}; R_1)$, $(N_2, M_{20}; R_2)$, $(N_3, M_{30}; R_3)$ and $(N_4, M_{40}; R_4)$ are all live and reversible. For $(N', M_0'; R')$, where $R_{F'} = \{ r_1 \}$, no $R_{F'}$ -siphons would eventually become empty. According to Property 3.4, $(N', M_0'; R')$ is also live and reversible. For $(N'', M_0''; R'')$, where $R_{F''} = \{ r_4 \}$, no $R_{F''}$ -siphons would eventually become empty. According to Property 3.4, $(N'', M_0''; R'')$ is also live and reversible.

We obtain the final composite augmented marked graph $(N, M_0; R)$ of $(N', M_0'; R')$ and $(N'', M_0''; R'')$ via $\{ (r_{12}, r_{42}), (r_{33}, r_{21}), (r_{32}, r_{41}), (r_{34}, r_{43}) \}$. Fig. 8 shows $(N, M_0; R)$, where r_2 is the place after fusing r_{12} and r_{42} , r_3 is the place after fusing r_{21} and r_{33} , r_5 is the place after fusing r_{32} and r_{41} , and r_6 is the place after fusing r_{34} and r_{43} .

Since $(N', M_0'; R')$ and $(N'', M_0''; R'')$ are bounded and conservative, according to Properties 3.2 and 3.3, the composite augmented marked graph $(N, M_0; R)$ is also bounded and conservative. On the other hand, $(N', M_0'; R')$ and $(N'', M_0''; R'')$ are live and reversible. For $(N, M_0; R)$, where $R_F = \{ r_2, r_3, r_5, r_6 \}$, no R_F -siphons would eventually become empty. According to Property 3.4, $(N, M_0; R)$ is also live and reversible. Hence, it may be concluded that the integrated system is live, bounded, reversible and conservative. In other words, the integrated system is well-behaved.



Semantic meaning of places

- p₁₁ C₁ is at idle state
- p₁₂ C₁ is performing operation o₁₁
- p₁₃ C₁ is performing operation o₁₂
- p₁₄ S₁ is being withheld
- p₁₅ S₂ is being withheld
- p₂₁ C₂ is at idle state
- p₂₂ C₂ is performing operation o₂₁
- p₂₃ C₂ is performing operation o₂₂
- p₂₄ S₃ is being withheld
- p₂₅ S₄ is being withheld
- p₃₁ C₃ is at idle state
- p₃₂ C₃ is performing operation o₃₁
- p₃₃ C₄ is at idle state
- p₃₄ C₄ is performing operation o₄₁
- s₁ S₁ is available
- s₂ S₂ is available
- s₃ S₃ is available
- s₄ S₄ is available
- s₅ S₅ is available
- s₆ S₆ is available

Semantic meaning of transitions

- t₁₁ C₁ starts operation o₁₁
- t₁₂ C₁ finishes operation o₁₁ and starts operation o₁₂
- t₁₃ C₁ finishes operation o₁₂
- t₁₄ C₁ receives signal m₁₁
- t₁₅ C₁ receives signal m₁₂
- t₁₆ C₁ receives signal m₁₃
- t₁₇ C₁ receives signal m₁₄
- t₂₁ C₂ starts operation o₂₁
- t₂₂ C₂ finishes operation o₂₁ and starts operation o₂₂
- t₂₃ C₂ finishes operation o₂₂
- t₂₄ C₂ receives signal m₂₁
- t₂₅ C₂ receives signal m₂₂
- t₂₆ C₂ receives signal m₂₃
- t₂₇ C₂ receives signal m₂₄
- t₃₁ C₃ starts operation o₃₁
- t₃₂ C₃ finishes operation o₃₁
- t₄₁ C₄ starts operation o₄₁
- t₄₂ C₄ finishes operation o₄₁

Fig. 8. The final composite augmented marked graphs $(N, M_0; R)$

5 Conclusion

We investigate the property-preserving composition of augmented marked graphs and its application to the synthesis of distributed systems. It is shown that, in composing two augmented marked graphs via their common resource places, boundedness and conservativeness are preserved while liveness and reversibility are preserved under a pretty simple condition. By modelling the distributed system components as augmented marked graphs with common resources denoted by resource places, an integrated system can be obtained by composing these augmented marked graphs via the common resource places. Based on preservation of properties, liveness, boundedness, reversibility and conservativeness of the integrated system can be readily derived.

Liveness, boundedness, reversibility and conservativeness are essential properties that collectively characterise a well-behaved system. For distributed systems which usually involve concurrent (parallel) and asynchronous processes, as competition of common resources exists, it is important for one to assure design correctness in the sense that these essential properties are maintained. By making good use of the special structure and properties of augmented marked graphs as well as the property-preserving composition of augmented marked graphs, our method effectively solves the problem of ensuring design correctness in the composition of distributed system components, which has perplexed designers of distributed systems for a long time.

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