18 Fuzzy Shortest Path Problem

18.1 Introduction

In this chapter we consider the shortest route problem where distances/costs are not known precisely and are modeled using fuzzy numbers. The fuzzy shortest route problem is outlined in the next section. We have previously used an evolutionary algorithm to solve an example problem (Section 6.5.1 of [2] and [3]). In Section 18.3 we plan to apply our fuzzy Monte Carlo method to obtain a solution to this example problem and then compare both solution methods.

18.2 Fuzzy Shortest Path Problem

Consider a network having n nodes linked by arcs. We allow all nodes to be connected by an arc. All arcs can be two-way streets (can travel in both directions). Define $\overline{a}_{ij}$ to be the fuzzy distance (or cost) of traveling from node i to node j, $1 \leq i, j \leq n$. Set $\overline{a}_{ii} = 0$ for $1 \leq i \leq n$. If there is no arc/street connecting node i to node j we set $\overline{a}_{ij} = M$, M is a large positive number. Since this is a minimization problem the optimal solution will not select a path from i to j if $\overline{a}_{ij} = M$. We wish to find paths from node 1 to node n with smallest total fuzzy distance/cost.

For a survey of the literature on the fuzzy shortest path problem see the papers/books ([1],[4]-[14]) and the references in these papers/books. Our solution concept using an evolutionary algorithm, or fuzzy Monte Carlo, is different from those used by other researchers.

Define a path of length $K+1$, $1 \leq K \leq n-1$, to be $(1, i_1, \cdots, i_K, n)$ where i_1, i_2, $\cdots$, i_K are distinct numbers in the set $\{2, \cdots, n-1\}$. A path of length 1 is $(1, n)$ and a path of length 2 is $(1, i_1, n)$ for $2 \leq i_1 \leq n-1$. Let Ω be all paths from 1 to n. If $\Gamma \in \Omega$, then define $\overline{D}(\Gamma) = \overline{a}_{1,i_1} + \overline{a}_{i_1,i_2} + \cdots + \overline{a}_{i_K,n}$. If all the $\overline{a}_{ij}$ are triangular fuzzy numbers, or real numbers in $[0, M]$, then $\overline{D}(\Gamma)$ is also a triangular fuzzy number. We will be using Buckley's Method (Section 2.6.1 of Chapter 2), with $\eta = 0.8$, to evaluate $\leq$, $<$ and $\approx$ between fuzzy numbers since that is what we used in our previous research on this topic. We wish to

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find paths Γ with minimum $\overline{D}(\Gamma)$. We will employ the "tie" breaking strategy in Section 2.6.4 so we expect a unique solution.

18.3 Monte Carlo Method

Now we want to apply our fuzzy Monte Carlo method to obtain solutions to this problem. First randomly choose $K \in \{1, 2, ..., n-2\}$ and using this K randomly choose, without replacement, K numbers in $\{2, 3, ..., n-1\}$. Let the first number chosen be i_1, the second i_2,..., and the last i_K. Then the path Γ is $1 \rightarrow i_1 \rightarrow i_2 \rightarrow ... \rightarrow i_K \rightarrow n$. In this application we used a pseudo-random number generator. Compute $\overline{D}(\Gamma)$. At some point in the simulation let $\overline{D}^*(\Gamma)$ be the smallest fuzzy distance from node 1 to node n. Let the next random path produce $\overline{D}_0(\Gamma)$. If $\overline{D}_0(\Gamma) < \overline{D}^*(\Gamma)$, then replace $\overline{D}^*(\Gamma)$ with $\overline{D}_0(\Gamma)$, otherwise discard $\overline{D}_0(\Gamma)$. Since we are using a method of breaking "ties", we do not expect to get $\overline{D}_0(\Gamma) \approx \overline{D}^*(\Gamma)$.

Apply this algorithm to a network with $n = 6$ nodes and fuzzy distances/costs given in Table 18.1. All the fuzzy distances/costs are triangular fuzzy numbers and we will use $M = 10,000$. So $\overline{D}(\Gamma)$ will also be a triangular fuzzy number. If we use all the vertices of these triangular fuzzy numbers we have a crisp network and its solution for the shortest distance from node 1 to node 6 is 18 using path $1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$.

In this application we do not need to generate random fuzzy numbers, only random paths. The results of our Monte Carlo study, after N iterations, would

Table 18.1. Fuzzy Distances Between the Nodes in Example Problem

	1	2	3	4	5	6
1	0	(1/3/4)	(4/6/8)	(11/13/16)	(13/15/18)	M
2	(2/4/5)	0	(3/5/7)	(6/8/9)	(10/12/15)	(14/19/23)
3	(1/3/4)	(3/4/6)	0	(3/4/6)	M	(11/14/18)
4	(9/11/14)	(5/8/10)	(5/7/10)	0	(1/2/3)	(7/9/12)
5	(11/15/18)	(8/10/13)	M	(5/6/7)	(0	(5/6/8)
6	M	(16/20/26)	(8/10/13)	(7/9/11)	(2/3/4)	0

Table 18.2. Fuzzy Shortest Routes from Our Evolutionary Algorithm

Path	Fuzzy Distance
1–3–4–5–6	(13/18/25)
1–2–4–5–6	(13/19/24)
1–3–4–6	(14/19/26)
1–2–4–6	(14/20/25)
1–3–6	(15/20/26)
1–2–3–4–5–6	(13/20/28)
1–5–6	(18/21/26)

be: (1) shortest fuzzy distance from node 1 to node 6 is $\overline{D}(\Gamma) = TBC$; and (2) the path is $\Gamma = 1 \rightarrow TBC \rightarrow n$ where TBC means "to be completed".

The results of the evolutionary algorithm applied to the same problem are given in Table 18.2. We obtained seven "solutions" because we did not use any "tie breakers".

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