11 Fuzzy Linear Regression I

11.1 Introduction

This is the first of four chapters on fuzzy regression. This chapter and Chapter 14 are about fuzzy linear regression and Chapters 12 and 13 consider fuzzy nonlinear regression. In this chapter the independent (predictor, explanatory) variables are crisp but the dependent (response) variable is fuzzy. In Chapter 14 both the independent variables and the dependent variable are fuzzy. This chapter is based on [1].

Fuzzy linear regression has become a very large area of research. Put "fuzzy regression" into your search engine and you can get too many web sites to visit. "Fuzzy linear regression" will eliminate a lot of web sites but the list is still quite long. We have selected a few recent and key references on fuzzy linear regression which are: (1) books (or articles in these books) ([4],[8],[14]); and (2) papers ([2],[3],[5],[9],[10],[12],[13],[15],[19]-[24],[27]-[32]). As far as the authors know our research is the only research on using Monte Carlo techniques in fuzzy linear regression. However, there have been other approaches employing random search (genetic algorithms) and others using neural nets. If we put "genetic algorithms" and "fuzzy linear regression" into the search engine there are less than 200 references. A recent reference is [11]. We feel that one problem with using a GA is that it can converge to a local minimum and to avoid this you need to start it with many different randomly generated initial populations. Also, we believe that our Monte Carlo method is easier to apply than a genetic algorithm, once you have a quasi-random number generator in your computer. Next we searched for "neural nets" and "fuzzy linear regression" getting less than 100 references. A key reference on this topic is [6].

Consider a fuzzy linear regression model

$$\overline{Y} = \overline{A}_0 + \overline{A}_1 x_1 + ... + \overline{A}_m x_m, \tag{11.1}$$

where the $x_1, ..., x_m$ are crisp real numbers and the $\overline{A}_0, ..., \overline{A}_m$ and $\overline{Y}$ are all triangular fuzzy numbers. In this model the independent (predictor, explanatory) variables are crisp but the dependent (response) variable is fuzzy. The data will

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be $((x_{1l},...,x_{ml}),\overline{Y}_l)$, $1 \leq l \leq n$, for the x_{il} given real numbers and $\overline{Y}_l$ are given triangular fuzzy numbers. The best way to fit the model to the data is to have the $\overline{A}_j$, $0 \leq j \leq m$, also triangular fuzzy numbers. Given the data the objective is to find the "best" $\overline{A}_j$, $0 \leq j \leq m$. We propose to employ Monte Carlo methods to approximate the "best" values for the $\overline{A}_j$, $j = 0,1,...,m$.

In Monte Carlo we randomly generate a possible solution, evaluate how "good" it is, discard inferior solutions, and continue N times. N is usually large like $10,000$ or $100,000$. In the next section we discuss how to randomly produce vectors $\overline{V}_k = (\overline{A}_{0k},...,\overline{A}_{mk})$, $k = 1,2,3,...,N$. Using the $\overline{V}_k$ we determine the predicted values

$$\overline{Y}^*_{lk} = \overline{A}_{0k} + \overline{A}_{1k}x_{1l} + ... + \overline{A}_{mk}x_{ml}, \tag{11.2}$$

for $k = 1,2,3,..,N$ and $l = 1,2,...,m$. To see how good this $\overline{V}_k$ is we find the error between the given values $\overline{Y}_l$ and the predicted values $\overline{Y}^*_{lk}$. We will have two error measures in this chapter. The first error measure is

$$E_{1k}(E_1) = \sum_{l=1}^{n}[\int_{-\infty}^{\infty} |\overline{Y}_l(x) - \overline{Y}^*_{lk}(x)|dx]/[\int_{-\infty}^{\infty} \overline{Y}_l(x)dx], \tag{11.3}$$

where the integrals are really only over interval(s) containing the support of the fuzzy numbers. Let $\overline{Y}_l = (y_{l1}/y_{l2}/y_{l3})$ and $\overline{Y}^*_{lk} = (y_{lk1}/y_{lk2}/y_{lk3})$ all triangular fuzzy numbers. Then our second error measure is

$$E_{2k}(E_2) = \sum_{l=1}^{n}[|y_{l1} - y_{lk1}| + |y_{l2} - y_{lk2}| + |y_{l3} - y_{lk3}|]. \tag{11.4}$$

So we calculate $\overline{V}_k$, E_{1k} and E_{2k} for $k = 1,2,...,N$. A "best" solution is a value of $\overline{V}_k$ that minimizes E_{1k} (E_{2k}) for all k. An approximate "best" solution is a $\overline{V} \in \{\overline{V}_1,...,\overline{V}_N\}$ that minimizes an error measure. So we can have two approximate "best" solutions one with respect to E_1 and an other for E_2. Next we see how we will produce sequences of random vectors $\overline{V}_k$, $k = 1,2,3,..,N$.

11.2 Random Fuzzy Vectors

To obtain random sequences $\overline{V}_k = (\overline{X}_{0k},...,\overline{X}_{mk})$, where the $\overline{X}_{ik}$ are all triangular fuzzy numbers, we first randomly generate crisp vectors $v_k = (x_{1k},...,x_{3m+3,k})$ using our Sobol quasi-random number generator (Chapter 3) with all the x_{ik} in $[0,1]$, $k = 1,2,..,N$. We choose the first three numbers in v_k and order them from smallest to largest. Assume that $x_{3k} < x_{1k} < x_{2k}$. Then the first triangular fuzzy number $\overline{X}_0 = (x_{3k}/x_{1k}/x_{2k})$ which becomes $\overline{A}_0$. Continue with the next three numbers in v_k, etc. making $\overline{X}_i = \overline{A}_i$, $i = 1,2,...,m$.

However the $\overline{A}_i$ will need to be in certain intervals. Suppose $\overline{A}_i$ is to be in interval $I_i = [a_i,b_i]$, $i = 0,1,2,...,m$. These intervals are very important to the Monte Carlo process because: (1) if they are wrong and/or too small we can miss a "good" solution; and (2) if they are too big the simulation can spend too much time looking at situations that will not produce a "good" solution. Since each

Table 11.1. Data for the Application

Fuzzy Output $\overline{Y}_l$	x_{1l}	x_{2l}	x_{3l}
(2.27/5.83/9.39)	2.00	0.00	15.25
(0.33/0.85/1.37)	0.00	5.00	14.13
(5.43/13.93/22.43)	1.13	1.50	14.13
(1.56/4.00/6.44)	2.00	1.25	13.63
(0.64/1.65/2.66)	2.19	3.75	14.75
(0.62/1.58/2.54)	0.25	3.50	13.75
(3.19/8.18/13.17)	0.75	5.25	15.25
(0.72/1.85/2.98)	4.25	2.00	13.50

$\overline{X}_i$ starts out in $[0,1]$ we may easily put it into $[a_i, b_i]$ by $\overline{A}_i = a_i + (b_i - a_i)\overline{X}_i$, $i = 0, 1, ..., m$.

11.3 Application

The data for this application was taken from [16] and is shown in Table 11.1. There are three ($m = 3$) independent variables x_1, x_2 and x_3. Also, there are only eight ($n = 8$) items in the data set. We will need to find intervals I_i, $i = 0, 1, 2, 3$, as explained above for the $\overline{A}_i$. We will solve for these intervals two ways: (1) first, in the next subsection, using the solutions for the $\overline{A}_i$, $i = 0, 1, 2, 3$, from [7]; and (2) secondly, in the second subsection, we use two optimization procedures to determine these intervals.

The authors in [16] compared their method, applied to the data in Table 11.1, to that in [25] and [26] applied to the same data set, in their Table 5. They give the predicted values for the dependent variable using the three methods. We have also studied this data set in [7] and obtained predicted values for the dependent variable using a least absolute values estimator. In this chapter we apply our Monte Carlo method to compute predicted values and compare our new results to the other four methods using error measures E_1 and E_2. All programs were written in MATLAB [18]. A copy of the MATLAB program may be obtained from the authors. For all our calculations, we used a Pentium III, Processor: 933 MHz.

11.3.1 First Choice of Intervals

After studying the solutions for the $\overline{A}_i$ using the methods in [7] we first decided on the following intervals for our fuzzy Monte Carlo method: (1) $[-1, 0]$ for $\overline{A}_0$; (2) $[-1, 0]$ for $\overline{A}_1$; (3) $[-1.5, -0.5]$ for $\overline{A}_2$; and (4) $[0, 1]$ for $\overline{A}_3$. The exact solutions in [7] for the $\overline{A}_i$ are given below.

Using our Sobol quasi-random number generator we produced $70,000$ vectors $v_k = (x_{1k}, ..., x_{12,k})$ which defined the $\overline{A}_i$, $i = 0, 1, 2, 3$, as described in Section 11.2. Results for the $\overline{A}_i$ are shown in Table 11.2 with minimum error values in Table 11.8. Since we will have four Monte Carlo studies on this data we call this one MCI.

Table 11.2. Results of the Monte Carlo (MCI) Method, First Choice of Intervals, to Minimize the Error

Coefficient	$MCI\ E_1$	$MCI\ E_2$
$\overline{A}_0$	$(-0.4953/-0.4306/-0.3393)$	$(-0.8902/-0.6815/-0.3355)$
$\overline{A}_1$	$(-0.5005/-0.4656/-0.0059)$	$(-0.6808/-0.5436/-0.5194)$
$\overline{A}_2$	$(-0.7965/-0.7864/-0.7165)$	$(-1.0640/-1.0476/-0.8578)$
$\overline{A}_3$	$(0.3335/0.3540/0.3920)$	$(0.4756/0.5379/0.6112)$

Table 11.3. Results of the Monte Carlo (MCII) Method, First Choice of Intervals, to Minimize the Error

Coefficient	$MCII\ E_1$	$MCII\ E_2$
$\overline{A}_0$	$(0.2464/0.4892/0.7266)$	$(0.0285/0.3569/0.8847)$
$\overline{A}_1$	$(-0.4815/-0.2852/-0.1398)$	$(-0.5654/-0.5329/-0.3708)$
$\overline{A}_2$	$(-0.8760/-0.8303/-0.7575)$	$(-1.0999/-1.0600/-0.9360)$
$\overline{A}_3$	$(0.3174/0.3361/0.3398)$	$(0.4052/0.4381/0.5280)$

Next we experimented with other intervals. We started with larger intervals, shifted them and shortened them, until we arrived at: (1) $[0,1]$ for $\overline{A}_0$; (2) $[-1,0]$ for $\overline{A}_1$; (3) $[-1.5,-0.5]$ for $\overline{A}_2$; and (4) $[0,1]$ for $\overline{A}_3$. The only difference is for $\overline{A}_0$. After another run of $70,000$ quasi-random vectors the results for the $\overline{A}_i$ are in Table 11.3 with minimum error values in Table 11.8. This Monte Carlo study is called MCII. For these choices of intervals, in MCI and MCII, the computational time was between 25 and 28 minutes.

11.3.2 Second Choice of Intervals

The first thing to do is to determine the intervals $I_i = [a_i, b_i]$ for the $\overline{A}_i$, $0 \le i \le 3$. We first describe an optimization method used to determine these intervals. This procedure will be called *MCIII*. A second optimization method will be used and it will be described below. Let

$$[L_l, R_l] = I_0 + I_1 x_{1l} + I_2 x_{2l} + I_3 x_{3l}, \tag{11.5}$$

evaluated using interval arithmetic, for $l = 1, 2, ..., 8$. Recall the data $\overline{Y}_l = (y_{l1}/y_{l2}/y_{l3})$. Define

$$W = \sum_{l=1}^{8} (L_l - y_{l1})^2 + \sum_{l=1}^{8} (R_l - y_{l3})^2. \tag{11.6}$$

The optimization problem is to minimize W subject to $a_i \le b_i$ all i. We want to find the intervals that make L_l and R_l closest, in the sense of minimizing W, to the end points of the bases of the dependent fuzzy numbers $\overline{Y}_l$ in the

Table 11.4. First Method of Determining the Intervals (MCIII) for Monte Carlo

Interval	Value
I_0	$[-18.174, -18.174]$
I_1	$[-1.083, -1.083]$
I_2	$[-1.150, -1.150]$
I_3	$[1.733, 2.149]$

Table 11.5. Results of the Monte Carlo Method (MCIII), Second Choice of Intervals, to Minimize the Error

Coefficient	$MCIII\ E_1$	$MCIII\ E_2$
A_0	-18.174	-18.174
A_1	-1.083	-1.083
A_2	-1.150	-1.150
$\overline{A}_3$	$(1.736/1.752/1.792)$	$(1.733/1.799/1.958)$

data. We solved this using Maple [17]. The results, rounded to three decimal places, are in Table 11.4. It is very interesting that the first three "intervals" are degenerate and are just real numbers. Using these intervals $\overline{A}_0 = A_0 = -18.174$, $\overline{A}_1 = A_1 = -1.083$, $\overline{A}_2 = A_2 = -1.150$ and the support of $\overline{A}_3$ is a subset of $[1.733, 2.149]$ with only one fuzzy number.

We now produce a sequence of random crisp vectors $v_k = (x_{1k}, ..., x_{3k})$, $k = 1, 2, ..., N$, using our Sobol quasi-random number generator as described in Section 11.2, to get a sequence of triangular fuzzy numbers $\overline{A}_{3k}$ and computed E_{1k} and E_{2k}. After a run of $N = 70,000$ the smallest E_1 value and the minimum E_2 value found are shown in Table 11.8 with corresponding $\overline{A}_3$ shown in Table 11.5.

We also investigated a second optimization procedure for finding the intervals. L_l and R_l are defined as above. This method is called MCIV. Let

$$W = \sum_{l=1}^{8}(y_{l1} - L_l) + \sum_{l=1}^{8}(R_l - y_{l3}). \qquad (11.7)$$

The linear programming problem is to minimize W subject to the constraints: (1) $L_l \leq y_{l1}$ all l; (2) $R_l \geq y_{l3}$ all l; and (3) $a_i \leq b_i$ all i. We solved this problem using Maple [17] and the results are in Table 11.6. It is again interesting that the last three "intervals" are degenerate and are just real numbers. Using these intervals with the support of $\overline{A}_0$ a subset of $[28.000, 47.916]$, $\overline{A}_1 = A_1 = -2.542$, $\overline{A}_2 = A_2 = -2.323$ and $\overline{A}_3 = A_3 = -1.354$ with only one fuzzy number.

We now produce a sequence of random crisp vectors $v_k = (x_{1k}, ..., x_{3k})$, $k = 1, 2, ..., N$, using a quasi-random number generator as described in Section 11.2, to get a sequence of triangular fuzzy numbers $\overline{A}_{0k}$ and computed E_{1k} and E_{2k}. After a run of $N = 70,000$ the smallest E_1 value and the minimum E_2 value

Table 11.6. Second Method of Determining the Intervals (MCIV) for Monte Carlo

Interval	Value
I_0	$[28.000, 47.916]$
I_1	$[-2.542, -2.542]$
I_2	$[-2.323, -2.323]$
I_3	$[-1.354, -1.354]$

Table 11.7. Results of the Monte Carlo Method (MCIV), Second Choice of Intervals, to Minimize the Error

Coefficient	$MCIV\ E_1$	$MCIV\ E_2$
$\overline{A}_0$	(35.842/36.030/36.030)	(31.062/33.336/36.228)
A_1	−2.542	−2.542
A_2	−2.323	−2.323
A_3	−1.354	−1.354

found are shown in Table 11.8 with corresponding $\overline{A}_0$ shown in Table 11.7. The total computational time for the second choice of intervals, MCIII and MCIV, was between 15 and 18 minutes.

11.3.3 Comparison of Solutions

Table 5 in [16] gives the predicted values for the dependent variable for the techniques used in [16],[25] and [26]. The predicted values were not given in [7] but we know the optimal solution for the $\overline{A}_i$: $\overline{A}_0 = (-0.71/-0.539/-0.524)$, $\overline{A}_1 = (-0.61/-0.473/-0.472)$, $\overline{A}_2 = (-1.09/-1.089/-1.088)$ and $\overline{A}_3 = (0.459/0.487/0.68)$. From this we may compute the predicted values. From the Monte Carlo methods discussed above we take the solutions for the $\overline{A}_i$ and determine the predicted values. From the predicted values we can find E_1 (equation (11.3)) and E_2 (equation (11.4)). The results are shown in Table 11.8.

The dependent variable represents “response time” and can not be negative. If a predictive value is $(-1.15/2.33/3.04)$ the authors in [16] round up the negative to zero and present $(0/2.33/3.04)$ as the predicted response time in their Table 5. Since we did not have access to the original predicted values we used those in their Table 5 with the zero value in our calculations for our Table 11.8. By rounding the negative left end point up to zero E_2 will decrease and E_1 may decrease or increase.

We see from Table 11.8 that our Monte Carlo method obtained the smallest values for error measure E_1. However, our Monte Carlo procedure did not get the smallest values for E_2. The smallest value for E_2 was gotten by [7]. Let us explain why we did not expect Monte Carlo to do better than [7] on E_2.

Table 11.8. Error Measures in the Application

Error	[25]	[26]	[16]	[7]	MCI	$MCII$	$MCIII$	$MCIV$
E_1	53.82	48.79	16.98	9.26	6.169	5.812	7.125	8.201
E_2	143.45	131.83	70.99	61.86	64.878	63.590	66.463	94.092

Let $\overline{A}_i = (a_{i1}/a_{i2}/a_{i3})$, $i = 0,1,2,3$. Also let $LL_l = a_{01} + \sum_{i=1}^{3} a_{i1}x_{il}$,$C_l = a_{02} + \sum_{i=1}^{3} a_{i2}x_{il}$ and $RR_l = a_{03} + \sum_{i=1}^{3} a_{i3}x_{il}$. In [7] they first solve

$$min \sum_{l=1}^{8} |C_l - y_{l2}|, \tag{11.8}$$

for the a_{i2}, $i = 0,1,2,3$. Let the solution be a_{i2}^*, $i = 0,1,2,3$. Then they solve for the a_{i1} from

$$min \sum_{l=1}^{8} |LL_l - y_{l1}|, \tag{11.9}$$

subject to $a_{i1} \leq a_{i2}^*$ all i, and solve for the a_{i3} from

$$min \sum_{l=1}^{8} |RR_l - y_{l3}|, \tag{11.10}$$

subject to $a_{i3} \geq a_{i2}^*$ all i. This is like finding the $\overline{A}_i$ to minimize E_2. Hence, we expected [7] to have a minimum value for E_2.

11.4 Summary and Conclusions

In this chapter we studied the fuzzy linear regression problem given in equation (11.1). We employed our fuzzy Monte Carlo method to approximate the "best" solutions for the coefficients $\overline{A}_i$, $0 \leq i \leq m$. Best will be measured by two error measures E_1 (equation (11.3)) and E_2 (equation (11.4)). We showed in an example problem that our Monte Carlo method was best according to E_1 with respect to the results on the same data set in four other publications. Monte Carlo did not get the smallest E_2 value, but MCI and MCII came close, and we explained above why [7] was expected to show the smallest E_2 value. Given any error measure E^* we conjecture that our Monte Carlo method, allowing the number of iterations N to be sufficiently large, will be best (minimizing E^*), or approximately best. If this conjecture is true, then the estimation technique in fuzzy linear regression may become Monte Carlo. We can easily extend out method to trapezoidal fuzzy numbers, quadratic fuzzy numbers and other more general fuzzy numbers.

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