9 Systematic Design of a Stable Type-2 Fuzzy Logic Controller

Stability is one of the more important aspects in the traditional knowledge of Automatic Control. Type-2 Fuzzy Logic is an emerging and promising area for achieving Intelligent Control (in this case, Fuzzy Control). In this chapter we use the Fuzzy Lyapunov Synthesis as proposed by Margaliot to build a Lyapunov Stable Type-1 Fuzzy Logic Control System, and then we make an extension from a Type-1 to a Type-2 Fuzzy Logic Control System, ensuring the stability on the control system and proving the robustness of the corresponding fuzzy controller.

9.1 Introduction

Stability has been one of the central issues concerning fuzzy control since Mamdani's pioneer work (Mamdani and Assilian, 1975). Most of the critical comments to fuzzy control are due to the lack of a general method for its stability analysis.

But as Zadeh often points out, fuzzy control has been accepted by the fact that it is task-oriented control, while conventional control is characterized as setpoint-oriented control, and hence do not need a mathematical analysis of stability. Also, as Sugeno has mentioned, in general, in most industrial applications, the stability of control is not fully guaranteed and the reliability of a control hardware system is considered to be more important than the stability (Sugeno, 1999).

The success of fuzzy control, however, does not imply that we do not need a stability theory for it. Perhaps the main drawback of the lack of stability analysis would be that we cannot take a model-based approach to fuzzy control design. In conventional control theory, a feedback controller can be primarily designed so that a close-loop system becomes stable (Paul and Yang, 1999). This approach of course restricts us to setpoint-oriented control, but stability theory will certainly give us a wider view on the future development of fuzzy control.

Therefore, many researchers have worked to improve the performance of the FLC's and ensure their stability. Li and Gatland in 1995 proposed a more systematic design method for PD and PI-type FLC's. Choi, Kwak and Kim (Choi et al., 2000)

O. Castillo and P. Melin: Type-2 Fuzzy Log.: Theory and Appl., STUDFUZZ 223, pp. 109–120, 2008.
springerlink.com
Springer-Verlag Berlin Heidelberg 2008

present a single-input FLC ensuring stability. Ying in 1994 presented a practical design method for nonlinear fuzzy controllers, and many other researchers have results on the matter of the stability of FLC's, in (Castillo et al., 2005) and (Cázarez et al., 2005) presents an extension of the Margaliot work (Margaliot and G. Langholz, 2000) to built stable type-2 fuzzy logic controllers in Lyapunov sense.

This work is based on Margaliot´s work (Margaliot and Langholtz, 2000), we use the Fuzzy Lyapunov Synthesis to build an Stable Type-2 Fuzzy Logic Controller for a 1 Degree of Freedom (DOF) manipulator robot, first without gravity effect to prove stability, and then with gravity effect to prove the robustness of the controller. The same criteria can be used for any number of DOF manipulator robots, linear or nonlinear, and any kind of plants.

This chapter is organized as follows: In Section 9.2 we present an introductory explanation of type-1 and type-2 FLC's. In Section 9.3 we extend Margaliot's result to build a general rule base for any type (1 or 2) of FLC's. Experimental results are presented in Section 9.4 and the summary is given in Section 9.5.

9.2 Fuzzy Logic Controllers

9.2.1 Type-1 Fuzzy Logic Control

Type-1 FLCs are both intuitive and numerical systems that map crisp inputs to a crisp output. Every FLC is associated with a set of rules with meaningful linguistic interpretations, such as

$$R^l : \text{If } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ and } \ldots \text{ and } x_n \text{ is } F_n^l \text{ Then } w \text{ is } G^l$$

which can be obtained either from numerical data, or experts familiar with the problem at hand. Based on this kind of statement, actions are combined with rules in an antecedent/consequent format, and then aggregated according to approximate reasoning theory, to produce a nonlinear mapping from input space $U = U_1 x U_2 x \ldots U_n$ to the output space W, where $F_k^l \subset U_k, k = 1,2,\ldots,n$, are the antecedent type-1 membership functions, and $G^l \subset W$ is the consequent type-1 membership function. The input linguistic variables are denoted by $u_k, k = 1,2,\ldots,n$, and the output linguistic variable is denoted by w.

A Fuzzy Logic System (FLS), as the kernel of a FLC, consist of four basic elements (Fig. 9.1): the type-1 fuzzyfier, the fuzzy rule-base, the inference engine, and the type-1 defuzzyfier. The fuzzy rule-base is a collection of rules in the form of R^l, which are combined in the inference engine, to produce a fuzzy output. The type-1 fuzzyfier maps the crisp input into type-1 fuzzy sets, which are subsequently used as inputs to the inference engine, whereas the type-1 defuzzyfier maps the type-1 fuzzy sets produced by the inference engine into crisp numbers.

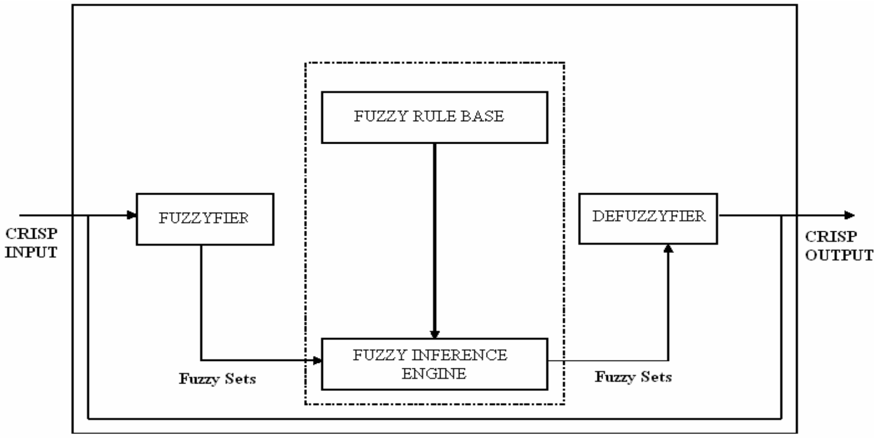

Fig. 9.1. Structure of type-1 fuzzy logic system

Fuzzy sets can be interpreted as membership functions u_X that associate with each element x of the universe of discourse, U, a number $u_X(x)$ in the interval [0,1]:

$$u_X : U \to [0,1] \tag{9.1}$$

For more detail of Type-1 FLS see (Chen and Pham, 2000).

9.2.2 Type-2 Fuzzy Logic Control

As with the type-1 fuzzy set, the concept of type-2 fuzzy set was introduced by Zadeh as an extension of the concept of an ordinary fuzzy set (Zadeh, 1975).

A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain. On the other hand, type-2 FLSs, are very useful in circumstances where it is difficult to determine an exact, and measurement uncertainties (Mendel, 2000).

It is known that type-2 fuzzy set let us to model and to minimize the effects of uncertainties in rule-based FLS. Unfortunately, type-2 fuzzy sets are more difficult to use and understand that type-1 fuzzy sets; hence, their use is not widespread yet.

Similar to a type-1 FLS, a type-2 FLS includes type-2 fuzzyfier, rule-base, inference engine and substitutes the defuzzifier by the output processor. The output processor includes a type-reducer and a type-2 defuzzyfier; it generates a type-1 fuzzy set output (from the type reducer) or a crisp number (from the defuzzyfier). A type-2 FLS is again characterized by IF-THEN rules, but its antecedent of consequent sets are now type-2. Type-2 FLSs, can be used when the circumstances are too uncertain to determine exact membership grades. A model of a type-2 FLS is shown in Fig. 9.2.

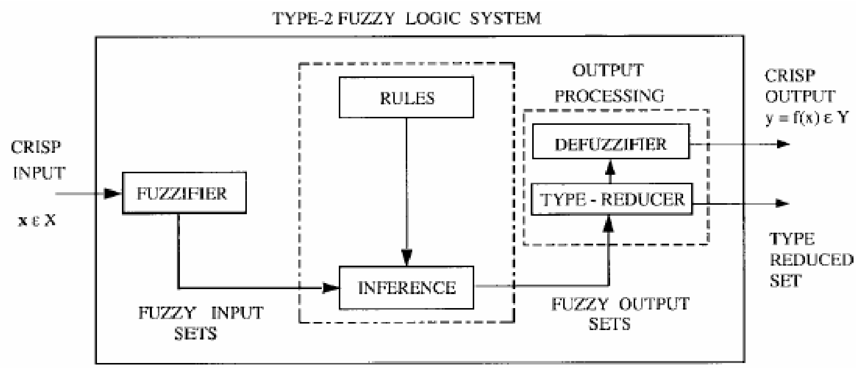

Fig. 9.2. Structure of type-2 fuzzy logic system

In the case of the implementation of type-2 FLCs, we have the same characteristics as in type-1 FLC, but we now use type-2 fuzzy sets as membership functions for the inputs and for the outputs. Fig. 9.3 shows the structure of a control loop with a FLC.

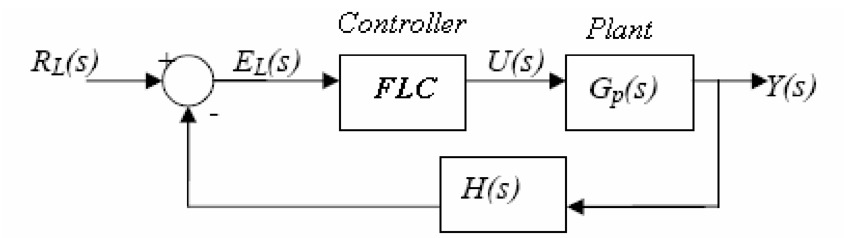

Fig. 9.3. Fuzzy control loop

9.3 Systematic and Design of Stable Fuzzy Controllers

For our description we consider the problem of designing a stabilizing controller for a 1DOF manipulator robot system depicted in Fig.9.4. The state-variables are $x_1 = \theta$ - the robot arm angle, and $x_2 = \dot{\theta}$ - its angular velocity. The system's actual dynamical equation, which we will assume unknown, is as shown in equation (9.2) (Paul and Yang, 1999):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{9.2}$$

To apply the fuzzy Lyapunov synthesis method, we assume that the exact equations are unknown and that we have only the following partial knowledge about the plant (see Fig. 9.4):

1. The system may have really two degrees of freedom θ and $\dot{\theta}$, referred to as x_1 and x_2, respectively. Hence, $\dot{x}_1 = x_2$.

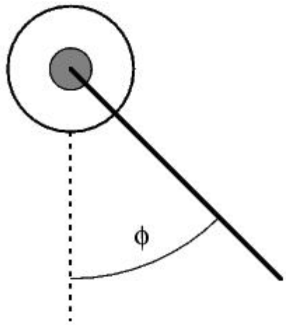

Fig. 9.4. 1DOF Manipulator robot

2. $\dot{x}_2$ is proportional to u, that is, when u increases (decreases) $\dot{x}_2$ increases (decreases).

To facilitate our control design we are going to suppose no gravity effect in our model, see (equation 9.3).

$$ml^2\ddot{q} = \tau \tag{9.3}$$

Our objective is to design the rule-base of a fuzzy controller that will carry the robot arm to a desired position $x_1 = \theta d$. We choose (9.4) as our Lyapunov function candidate. Clearly, V is positive-definite.

$$V(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2) \tag{9.4}$$

Differentiating V, we have (9.5),

$$\dot{V} = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1x_2 + x_2\dot{x}_2 \tag{9.5}$$

Hence, we require:

$$x_1x_2 + x_2\dot{x}_2 < 0 \tag{9.6}$$

We can now derive sufficient conditions so that condition (9.6) holds: If x_1 and x_2 have opposite signs, then $x_1x_2 < 0$ and (9.6) will hold if $\dot{x}_2 = 0$; if x_1 and x_2 are both positive, then (9.6) will hold if $\dot{x}_2 < -x_1$; and if x_1 and x_2 are both negative, then (9.6) will hold if $\dot{x}_2 > -x_1$.

We can translate these conditions into the following fuzzy rules:

- If x_1 is *positive* and x_2 is *positive* then $\dot{x}_2$ must be *negative big*
- If x_1 is *negative* and x_2 is *negative* then $\dot{x}_2$ must be *positive big*
- If x_1 is *positive* and x_2 is *negative* then $\dot{x}_2$ must be *zero*
- If x_1 is *negative* and x_2 is *positive* then $\dot{x}_2$ must be *zero*

However, using our knowledge that $\dot{x}_2$ is proportional to u, we can replace each $\dot{x}_2$ with u to obtain the fuzzy rule-base for the stabilizing controller:

- If x_1 is *positive* and x_2 is *positive* Then u must be *negative big*
- If x_1 is *negative* and x_2 is *negative* Then u must be *positive big*
- If x_1 is *positive* and x_2 is *negative* Then u must be *zero*
- If x_1 is *negative* and x_2 is *positive* Then u must be *zero*

It is interesting to note that the fuzzy partitions for x_1, x_2, and u follow elegantly from expression (9.5). Because $\dot{V} = x_2(x_1 + \dot{x}_2)$, and since we require that $\dot{V}$ be negative, it is natural to examine the signs of x_1 and x_2; hence, the obvious fuzzy partition is *positive*, *negative*. The partition for $\dot{x}_2$, namely *negative big*, *zero*, *positive big* is obtained similarly when we plug the linguistic values *positive*, *negative* for x_1 and x_2 in (9.5). To ensure that $\dot{x}_2 < -x_1$ $(\dot{x}_2 > -x_1)$ is satisfied even though we do not know x_1's exact magnitude, only that it is *positive* (*negative*), we must set $\dot{x}_2$ to *negative big* (*positive big*). Obviously, it is also possible to start with a given, pre-defined, partition for the variables and then plug each value in the expression for $\dot{V}$ to find the rules. Nevertheless, regardless of what comes first, we see that fuzzy Lyapunov synthesis transforms classical Lyapunov synthesis from the world of exact mathematical quantities to the world of computing with words (Zadeh, 1996).

To complete the controllers design, we must model the linguistic terms in the rule-base using fuzzy membership functions and determine an inference method. Following (Wang, 1997), we characterize the linguistic terms *positive*, *negative*, *negative big*, *zero* and *positive big* by the type-1 membership functions shown in Fig. 9.5 for a Type-1 Fuzzy Logic Controller, and by the type-2 membership functions shown in Fig. 9.6 for a Type-2 Fuzzy Logic Controller. Note that the type-2 membership functions are extended type-1 membership functions.

To this end, we had systematically developed a FLC rule-base that follows the Lyapunov Stability criterion. In Section 9.4 we present some experimental results using our fuzzy rule-base to build a Type-2 Fuzzy Logic Controller.

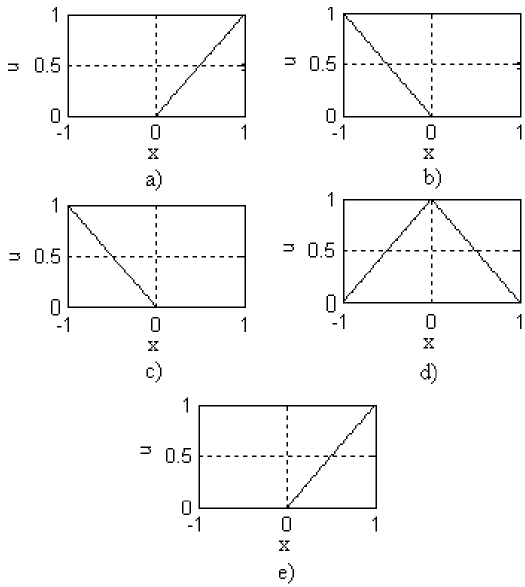

Fig. 9.5. Set of type-1 membership functions: a) positive, b)negative, c) negative big, d) zero and e) positive big

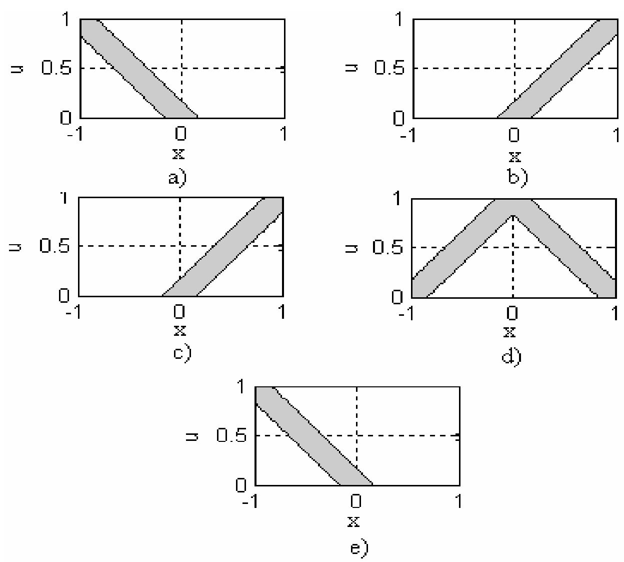

Fig. 9.6. Set of type-2 membership functions: a)negative, b) positive, c) positive big, d) zero and e) negative big

9.4 Experimental Results

In Section 9.3 we had systematically developed a stable FLC rule-base, and now we are going to show some experimental results using our stable rule-base to build a Type-2 FLC. The plant description used in the experiments is the same shown in Section 9.3.

Our experiments were done with Type-1 Fuzzy Sets and Interval Type-2 Fuzzy Sets. In the Type-2 Fuzzy Sets the membership grade of every domain point is a crisp set whose domain is some interval contained in [0,1] (Mendel, 2000). On Fig. 9.6 we show some Interval Type-2 Fuzzy Sets, and for each fuzzy set, the grey area is known as the Footprint of Uncertainty (FOU) (Mendel, 2000), and this is bounded by an upper and a lower membership function as shown in Fig. 9.7.

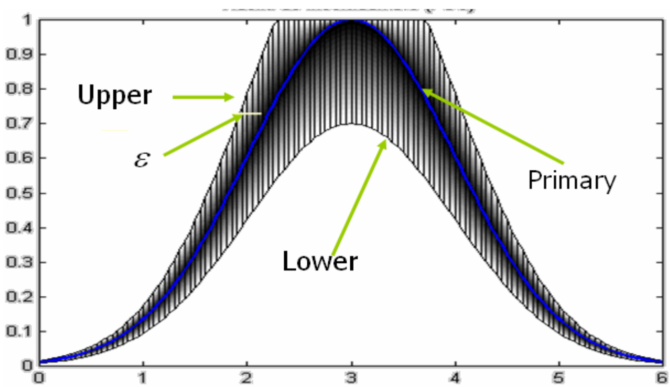

Fig. 9.7. Interval Type-2 Fuzzy Set

In our experiments we increase and decrease the value of ε to the left and to the right side having a εL and a εR values respectively to determine how much the FOU can be extended or perturbed without losing stability in the FLC.

We did make simulations with initial conditions of θ having values in the whole circumference [0, 2π], and the desired angle θd having values in the same range. The initial conditions considered in the experiments shown in this paper are an angle $\theta = 0 rad$ and $\theta_d = 0.1 rad$.

In Fig. 9.8 we show a simulation of the plant made with a Type-1 FLC, as can be seen, the plant has been regulated in around 8 seconds, and in Fig. 9.9 we show the graph of equation (9.5) which is always negative defined and consequently the system is stable.

Figure 9.10 shows the simulation results of the plant made with the Type-2 FLC increasing and decreasing ε in the range of [0,1], and as can be seen the plant has been regulated in around 10 seconds, and the graph of (9.5), which is depicted in Fig. 9.11 is always negative defined and consequently the system is stable. As we can seen, the time response is increasing when the value of ε is increasing.

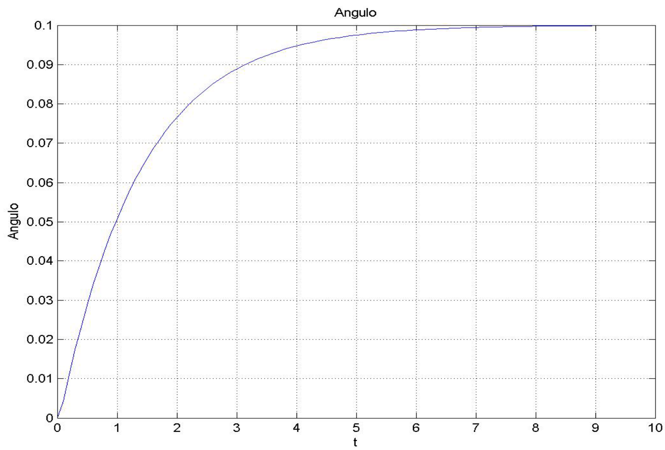

Fig. 9.8. Response for the Type-1 FLC

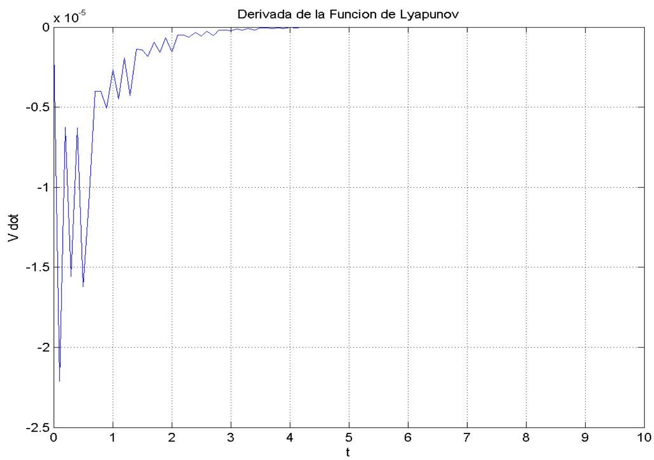

Fig. 9.9. $\dot{V}$ for the Type-1 FLC

With the variation of ε in the definition of the FOU, the control surface changes proportional to the change of ε, for this reason, the value of *u* for $\varepsilon \geq 1$ is practically zero, and the plant does not have physical response. To test the robustness of the built Fuzzy Controller, now we are going to use the same controller designed in Section 9.3, but at this time, we are going to use it to control equation (9.2) considering the gravity effect as shown in equation (9.7).

$$ml^2\ddot{q} + gml\cos q = \tau \tag{9.7}$$

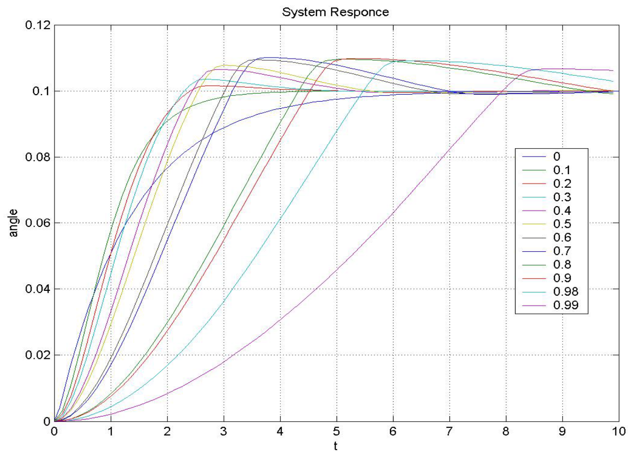

Fig. 9.10. Response for the Type-2 FLC ($\varepsilon \rightarrow [0,1)$)

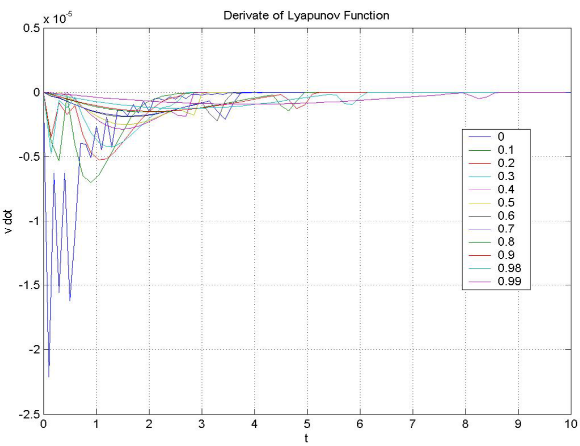

Fig. 9.11. $\dot{V}$ for the Type-2 FLC ($\varepsilon \rightarrow [0,1]$)

In Figure 9.12 we can see a simulation of the plant obtained with a Type-1 FLC, and as can be seen, the plant has been regulated in approximately 8 seconds and Figure 9.13 shows the graph of (9.5) which is always negative defined and consequently the system is stable.

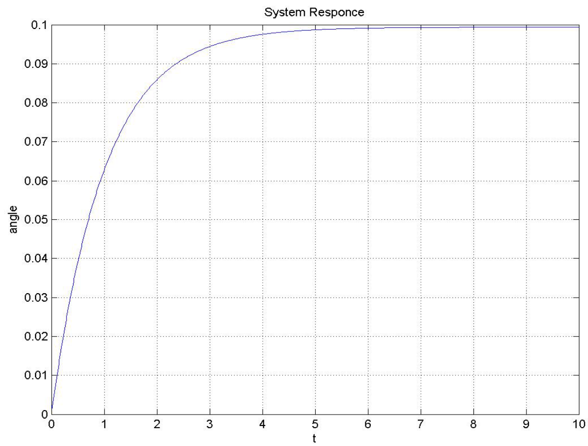

Fig. 9.12. Response for the Type-1 FLC

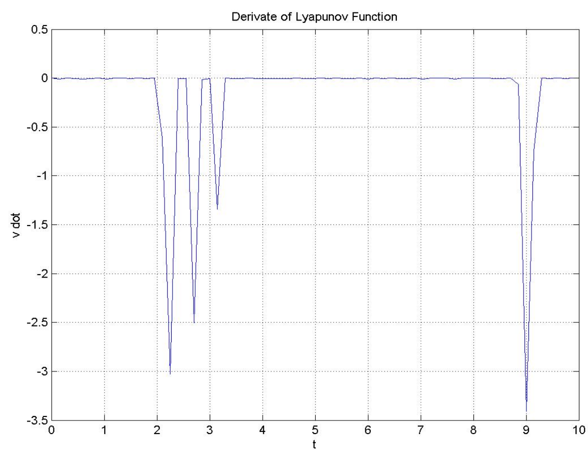

Fig. 9.13. $\dot{V}$ for the Type-1 FLC

Figure 9.14 shows the simulation results of the plant obtained with the Type-2 FLC with increasing and decreasing ε values in the range of [0,1], and the graph of (9.5) depicted at Fig. 9.15 is always negative defined and consequently the system is stable. As we can seen, if we use an adaptive gain like in (Castillo et al., 2005) all the cases of ε can be regulated around 8 seconds.

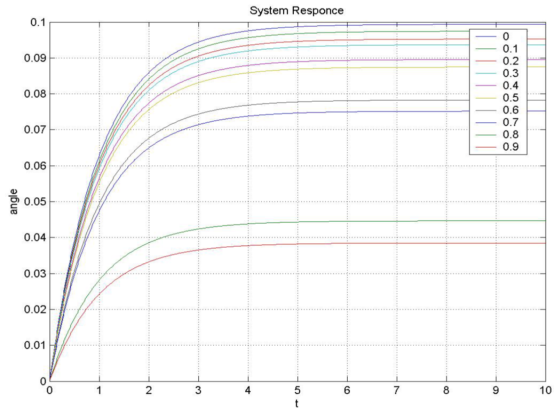

Fig. 9.14. Response for the Type-2 FLC ($\varepsilon \rightarrow [0,1]$)

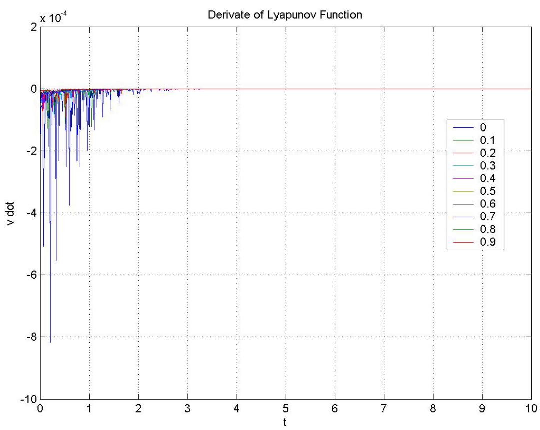

Fig. 9.15. $\dot{V}$ for the Type-2 FLC ($\varepsilon \rightarrow [0,1]$)

9.5 Summary

Margaliot's approach for the design of FLC's is now proved to be valid for both, Type-1 and Type-2 Fuzzy Logic Controllers. In the case of Type-2 FLC's membership functions, we can perturb or change the definition domain of the FOU without losing of stability of the controller; in the case described in this chapter, like in (Castillo et al.,2005) we have to use an adaptive gain to regulate the plant in a desired time. For our example of the 1DOF manipulator robot, stability holds when extending the FOU on the domain [0,1), and we also have shown that a FLC designed following the Fuzzy Lyapunov Synthesis is stable and robust.