5 Design of Intelligent Systems with Interval Type-2 Fuzzy Logic

Uncertainty is an inherent part of intelligent systems used in real-world applications. The use of new methods for handling incomplete information is of fundamental importance. Type-1 fuzzy sets used in conventional fuzzy systems cannot fully handle the uncertainties present in intelligent systems. Type-2 fuzzy sets that are used in type-2 fuzzy systems can handle such uncertainties in a better way because they provide us with more parameters. This chapter deals with the design of intelligent systems using interval type-2 fuzzy logic for minimizing the effects of uncertainty produced by the instrumentation elements, environmental noise, etc. Experimental results include simulations of feedback control systems for non-linear plants using type-1 and type-2 fuzzy logic controllers; a comparative analysis of the systems' response is performed, with and without the presence of uncertainty.

5.1 Introduction

Uncertainty affects decision-making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem-solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way (Klir and Yuan, 1995). Uncertainty is an attribute of information (Zadeh, 2005). The general framework of fuzzy reasoning allows handling much of this uncertainty, fuzzy systems employ type-1 fuzzy sets, which represent uncertainty by numbers in the range [0, 1]. When something is uncertain, like a measurement, it is difficult to determine its exact value, and of course type-1 fuzzy sets make more sense than using sets (Zadeh, 1975). However, it is not reasonable to use an accurate membership function for something uncertain, so in this case what we need is another type of fuzzy sets, those which are able to handle these uncertainties, the so called type-2 fuzzy sets (Mizumoto and Tanaka, 1976) (Mendel, 2001). So, the amount of uncertainty in a system can be reduced by using type-2 fuzzy logic because it offers better capabilities to handle linguistic uncertainties by modeling vagueness and unreliability of information (Liang and Mendel, 2000).

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Recently, we have seen the use of type-2 fuzzy sets in Fuzzy Logic Systems (FLS) in different areas of application (Lee et al., 2003). A novel approach for realizing the vision of ambient intelligence in ubiquitous computing environments (UCEs), is based on embedding intelligent agents that use type-2 fuzzy systems which are able to handle the different sources of uncertainty and imprecision in UCEs to give a good response (Doctor et al., 2005). There are also papers with emphasis on the implementation of type-2 FLS (Karnik and Mendel, 1999) and in others, it is explained how type-2 fuzzy sets let us model and minimize the effects of uncertainties in rule-base FLS (Wu and Mendel, 2001). There is also a paper that provides mathematical formulas and computational flowcharts for computing the derivatives that are needed to implement steepest-descent parameter tuning algorithms for type-2 fuzzy logic systems (Mendel, 2004). Some research works are devoted to solve real world applications in different areas, for example in signal processing, type-2 fuzzy logic is applied in prediction of the Mackey-Glass chaotic time-series with uniform noise presence (Mendel, 2000). In medicine, an expert system was developed for solving the problem of Umbilical Acid-Base (UAB) assessment (Ozen and Garibaldi, 2003). In industry, type-2 fuzzy logic and neural networks was used in the control of non-linear dynamic plants (Melin and Castillo, 2004); also we can find interesting studies in the field of mobile robots (Hagras, 2004).

In this chapter we deal with the application of interval type-2 fuzzy control to non-linear dynamic systems. It is a well known fact, that in the control of real systems, the instrumentation elements (instrumentation amplifier, sensors, digital to analog, analog to digital converters, etc.) introduce some sort of unpredictable values in the information that has been collected (Castillo and Melin, 2004). So, the controllers designed under idealized conditions tend to behave in an inappropriate manner (Castillo and Melin, 2001). Since, uncertainty is inherent in the design of controllers for real world applications, we are presenting how to deal with this problem using type-2 Fuzzy Logic Controller (FLC), to reduce the effects of imprecise information. We are supporting this statement with experimental results, qualitative observations, and quantitative measures of errors. For quantifying the errors, we utilized three widely used performance criteria, these are: Integral of Square Error (ISE), Integral of the Absolute value of the Error (IAE), and Integral of the Time multiplied by the Absolute value of the Error (ITAE) (Deshpande and Ash, 1988). We also consider the application of interval type-2 fuzzy logic to the problem of forecasting chaotic time series.

5.2 Fuzzy Logic Systems

In this section, a brief overview of type-1 and type-2 fuzzy systems is presented. This overview is considered as necessary to understand the basic concepts needed to understand the methods and algorithms presented later in the chapter.

5.2.1 Type-1 Fuzzy Logic Systems

In the 40's and 50's, many researchers proved that dynamic systems could be mathematically modeled using differential equations. In these works we have the foundations of the Control Theory, which in addition with the Transform Theory (Laplace's Theory), provided an extremely powerful means of analyzing and designing control systems (Mamdani, 1993). These theories were developed until the 70's, when the area was called Systems Theory to indicate its definitiveness.

Soft computing techniques have become an important research topic, which can be applied in the design of intelligent controllers (Jang et al., 1997). These techniques have tried to avoid the above-mentioned drawbacks, and they allow us to obtain efficient controllers, which utilize the human experience in a more natural form than the conventional mathematical approach (Zadeh, 1973). In the cases in which a mathematical representation of the controlled system is difficult to obtain, the process operator has the knowledge, the experience to express the relationships existing in the process behavior.

A FLS, described completely in terms of type-1 fuzzy sets is called a type-1 fuzzy logic system (type-1 FLS). It is composed by a knowledge base, which comprises the information given by the process operator in form of linguistic control rules. A fuzzification interface, that has the effect of transforming crisp data into fuzzy sets. An inference system, that uses the fuzzy sets in conjunction with the knowledge base to make inferences by means of a reasoning method. Finally, a defuzzification interface, which translates the fuzzy control action so obtained to a real control action using a defuzzification method (Mendel, 2001).

In this chapter, the implementation of the fuzzy controller in terms of type-1 fuzzy sets, has two input variables, which are the error $e(t)$, the difference between the reference signal and the output of the process, as well as the error variation $\Delta e(t)$,

$$e(t) = r(t) - y(t) \tag{5.1}$$

$$\Delta e(t) = e(t) - e(t-1) \tag{5.2}$$

so the control system can be represented as in Figure 5.1.

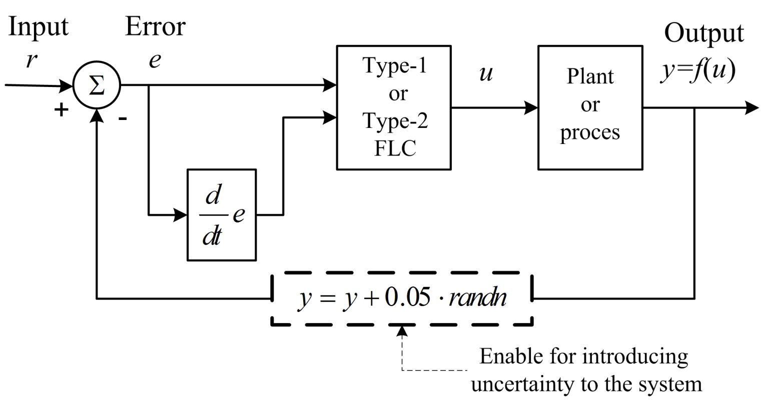

Fig. 5.1. System used for obtaining the experimental results for control

5.2.2 Type-2 Fuzzy Logic Systems

If for a type-1 membership function, as in Figure 5.2, we blur it to the left and to the right, as illustrated in Figure 5.3, then a type-2 membership function is obtained. In this case, for a specific value x', the membership function (u'), takes on different values, which are not all weighted the same, so we can assign an amplitude distribution to all of those points.

Doing this for all $x \in X$, we create a three-dimensional membership function –a type-2 membership function– that characterizes a type-2 fuzzy set (Mendel, 2001)

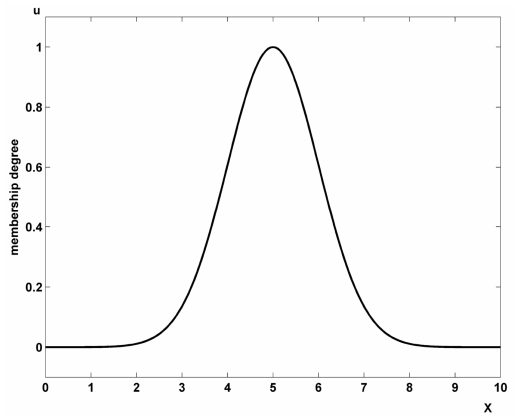

Fig. 5.2. Type-1 membership function

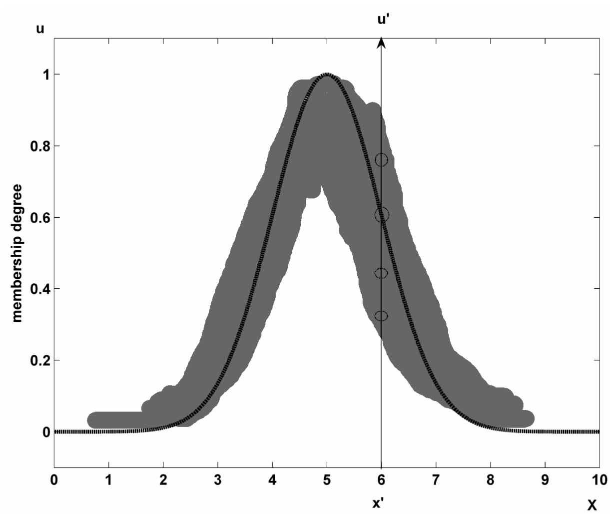

Fig. 5.3. Blurred type-1 membership function

(Mendel and Mouzouris, 1999). A type-2 fuzzy set $\tilde{A}$, is characterized by the membership function:

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{5.3}$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. Another expression for $\tilde{A}$ is,

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \quad J_x \subseteq [0,1] \tag{5.4}$$

Where $\int\int$ denotes the union over all admissible input variables *x* and *u*. For discrete universes of discourse $\int$ is replaced by $\sum$ (Mendel and John, 2002). In fact $J_x \subseteq [0,1]$ represents the primary membership of *x*, and $\mu_{\tilde{A}}(x,u)$ is a type-1 fuzzy set known as the secondary set. Hence, a type-2 membership grade can be any subset in [0,1], the primary membership, and corresponding to each primary membership, there is a secondary membership (which can also be in [0,1]) that defines the possibilities for the primary membership (Liang and Mendel, 2000). Uncertainty is represented by a region, which is called the footprint of uncertainty (FOU). When $\mu_{\tilde{A}}(x,u) = 1, \forall u \in J_x \subseteq [0,1]$ we have an interval type-2 membership function, as shown in Figure 5.4. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described in terms of an upper membership function $\overline{\mu}_{\tilde{A}}(x)$ and a lower membership function $\underline{\mu}_{\tilde{A}}(x)$.

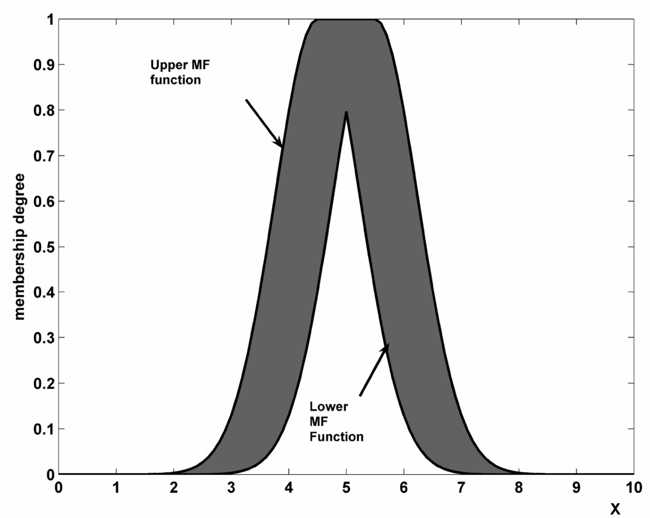

Fig. 5.4. Interval type-2 membership function

A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain. On the other hand, type-2 FLSs, are very useful in circumstances where it is difficult to determine an exact membership function, and there are measurement uncertainties (Mendel, 2001).

It is known that type-2 fuzzy sets enable modeling and minimizing the effects of uncertainties in rule-based FLS. Unfortunately, type-2 fuzzy sets are more difficult to use and understand than type-1 fuzzy sets; hence, their use is not widespread yet. As a justification for the use of type-2 fuzzy sets, in (Sepulveda et al., 2007) are mentioned at least four sources of uncertainties not considered in type-1 FLSs:

1. The meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people).
2. Consequents may have histogram of values associated with them, especially when knowledge is extracted from a group of experts who do not all agree.
3. Measurements that activate a type-1 FLS may be noisy and therefore uncertain.
4. The data used to tune the parameters of a type-1 FLS may also be noisy.

All of these uncertainties translate into uncertainties about fuzzy set membership functions. Type-1 fuzzy sets are not able to directly model such uncertainties because their membership functions are totally crisp. On the other hand, type-2 fuzzy sets are able to model such uncertainties because their membership functions are themselves fuzzy. A type-1 fuzzy set is a special case of a type-2 fuzzy set; its secondary membership function is a subset with only one element, unity.

A type-2 FLS is again characterized by IF-THEN rules, but its antecedent or consequent sets are now of type-2. Type-2 FLSs, can be used when the circumstances are too uncertain to determine exact membership grades such as when the training data is corrupted by noise. Similar to a type-1 FLS, a type-2 FLS includes a fuzzifier, a rule

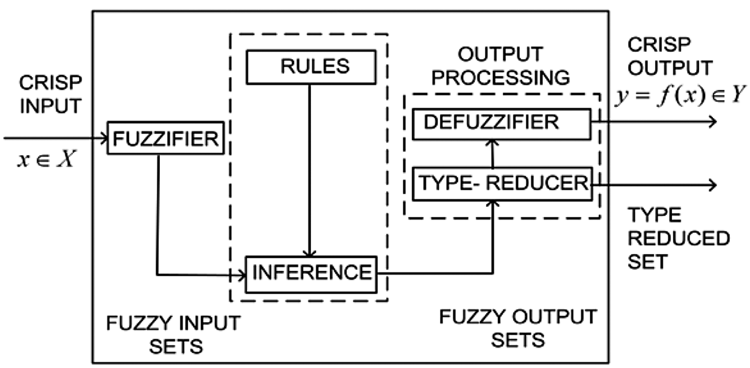

Fig. 5.5. Type-2 Fuzzy Logic System

base, fuzzy inference engine, and an output processor, as we can see in Fig. 5.5. The output processor includes type-reducer and defuzzifier; it generates a type-1 fuzzy set output (from the type-reducer) or a crisp number (from the defuzzifier) (Mendel, 2005). Now we will explain each of the blocks of Figure 5.5.

5.2.2.1 Fuzzifier

The fuzzifier maps a crisp point $\mathbf{x}=(x_1,\ldots,x_p)^T \in X_1 x X_2 x \ldots x X_p \equiv \mathbf{X}$ into a type-2 fuzzy set $\tilde{A}_x$ in **X** (Mendel, 2001), interval type-2 fuzzy sets in this case. We will use type-2 singleton fuzzifier, in a singleton fuzzification, the input fuzzy set has only a single point on nonzero membership. $\tilde{A}_x$ is a type-2 fuzzy singleton if $\mu_{\tilde{A}_x}(x) = 1/1$ for $\mathbf{x}=\mathbf{x}'$ and $\mu_{\tilde{A}_x}(x) = 1/0$ for all other $\mathbf{x}\neq\mathbf{x}'$[7].

5.2.2.2 Rules

The structure of rules in a type-1 FLS and a type-2 FLS is the same, but in the latter the antecedents and the consequents will be represented by type-2 fuzzy sets. So for a type-2 FLS with p inputs $x_1 \in X_1,\ldots,x_p \in X_p$ and one output $y \in Y$, Multiple Input Single Output (MISO), if we assume there are M rules, the lth rule in the type-2 FLS can be written as follows:

$$R^l\text{: IF } x_1 \text{ is } \tilde{F}_1^{\,l} \text{ and } \cdots \text{and } x_p \text{ is } \tilde{F}_p^{\,l}\text{ , THEN } y \text{ is } \tilde{G}^l \qquad l=1,\ldots,M \tag{5.5}$$

5.2.2.3 Inference

In the type-2 FLS, the inference engine combines rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. It is necessary to compute the join $\sqcup$, (unions) and the meet $\sqcap$ (intersections), as well as extended sup-star compositions (sup star compositions) of type-2 relations. If $\tilde{F}^l{}_1 \times\cdots\times \tilde{F}^l{}_p = \tilde{A}^l$, equation (5.5) can be re-written as

$$R^l : \tilde{F}^l{}_1 \times\cdots\times \tilde{F}^l{}_p \rightarrow \tilde{G}^l = \tilde{A}^l \rightarrow \tilde{G}^l \quad l=1,\ldots,M \tag{5.6}$$

R^l is described by the membership function $\mu_{R^l}(\mathbf{x}, y) = \mu_{R^l}(x_1,\ldots,x_p, y)$, where

$$\mu_{R^l}(\mathbf{x}, y) = \mu_{\tilde{A}^l\rightarrow\tilde{G}^l}(\mathbf{x}, y) \tag{5.7}$$

can be written as (Mendel, 2001):

$$\begin{aligned}\mu_{R^l}(\mathbf{x}, y) &= \mu_{\tilde{A}^l\rightarrow\tilde{G}^l}(\mathbf{x}, y) = \mu_{\tilde{F}_1^l}(x_1) \sqcap\cdots\sqcap \mu_{\tilde{F}_p{}^l}(x_p) \sqcap \mu_{\tilde{G}^l}(y) \\ &= [\sqcap_{i=1}^{p} \mu_{\tilde{F}^l{}_i}(x_i)] \sqcap \mu_{\tilde{G}^l}(y)\end{aligned} \tag{5.8}$$

In general, the p-dimensional input to R^l is given by the type-2 fuzzy set $\tilde{A}_x$ whose membership function is

$$\mu_{\tilde{A}_x}(\mathbf{x}) = \mu_{\tilde{x}_1}(x_1) \sqcap \cdots \sqcap \mu_{\tilde{x}p}(x_p) = \sqcap_{i=1}^{p} \mu_{\tilde{x}i}(x_i) \tag{5.9}$$

where $\tilde{X}_i (i = 1,..., p)$ are the labels of the fuzzy sets describing the inputs. Each rule R^l determines a type-2 fuzzy set $\tilde{B}^l = \tilde{A}_x \circ R^l$ such that:

$$\mu_{\tilde{B}^l}(y) = \mu_{\tilde{A}_x \circ R^l} = \sqcup_{x \in \mathbf{X}} \left[\mu_{\tilde{A}_x}(\mathbf{x}) \sqcap \mu_{R^l}(\mathbf{x}, y) \right] \qquad y \in Y \;\; l=1,\ldots,M \tag{5.10}$$

This equation is the input/output relation in Figure 5.5 between the type-2 fuzzy set that activates one rule in the inference engine and the type-2 fuzzy set at the output of that engine (Mendel, 2001).

In the FLS we used interval type-2 fuzzy sets and meet under product t-norm, so the result of the input and antecedent operations, which are contained in the firing set $\sqcap_{i=1}^{p} \mu_{\tilde{F}_{i_i}}(x'_i \equiv F^l(\mathbf{x}')$, is an interval type-1 set,

$$F^l(\mathbf{x}') = \left[\underline{f}^l(\mathbf{x}'), \bar{f}^l(\mathbf{x}') \right] \equiv \left[\underline{f}^l, \bar{f}^l \right] \tag{5.11}$$

Where

$$\underline{f}^l(\mathbf{x}') = \underline{\mu}_{\tilde{F}_1^l}(x_1') * \cdots * \underline{\mu}_{\tilde{F}_p^l}(x_p') \tag{5.12}$$

And

$$\bar{f}^l(\mathbf{x}') = \bar{\mu}_{\tilde{F}_1^l}(x_1') * \cdots * \bar{\mu}_{\tilde{F}_p^l}(x_p') \tag{5.13}$$

where * is the product operation.

5.2.2.4 Type Reducer

The type-reducer generates a type-1 fuzzy set output, which is then converted in a crisp output through the defuzzifier. This type-1 fuzzy set is also an interval set, for the case of our FLS we used center of sets (cos) type reduction, $Y_{\cos}$ which is expressed as (Mendel, 2001)

$$Y_{\cos}(\mathbf{x}) = [y_l, y_r] = \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \bar{f}^M]} 1 \Big/ \frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i} \tag{5.14}$$

this interval set is determined by its two end points, y_l and y_r, which corresponds to the centroid of the type-2 interval consequent set $\tilde{G}^i$,

$$C_{\tilde{G}^i} = \int_{\theta_1 \in J_{y1}} \cdots \int_{\theta_N \in J_{yN}} 1 / \frac{\sum_{i=1}^{N} y_i \theta_i}{\sum_{i=1}^{N} \theta_i} = [y_l^{\,i}, y_r^{\,i}] \tag{5.15}$$

before the computation of $Y_{\cos}(\mathbf{x})$, we must evaluate equation (5.15), and its two end points, y_l and y_r. If the values of f_i and y_i that are associated with y_l are denoted f_l^i and y_l^i, respectively, and the values of f_i and y_i that are associated with y_r are denoted f_r^i and y_r^i, respectively, from equation (15.14), we have (Mendel, 2001)

$$y_l = \frac{\sum_{i=1}^{M} f_l^{\,i} y_l^{\,i}}{\sum_{i=1}^{M} f_l^{\,i}} \tag{5.16}$$

$$y_r = \frac{\sum_{i=1}^{M} f_r^{\,i} y_r^{\,i}}{\sum_{i=1}^{M} f_r^{\,i}} \tag{5.17}$$

5.2.2.5 Defuzzifier

From the type-reducer we obtain an interval set $Y_{\cos}$, to defuzzify it we use the average of y_l and y_r, so the defuzzified output of an interval singleton type-2 FLS is (Mendel, 2001)

$$y(\mathbf{x}) = \frac{y_l + y_r}{2} \tag{5.18}$$

In this chapter, we are simulating the fact that the instrumentation elements (instrumentation amplifier, sensors, digital to analog, analog to digital converters, etc.) are introducing some sort of unpredictable values in the collected information. In the case of the implementation of the type-2 FLC, we have the same characteristics as in type-1 FLC, but we used type-2 fuzzy sets as membership functions for the inputs and for the output.

5.2.3 Performance Criteria

For evaluating the transient closed-loop response of a computer control system we can use the same criteria that normally are used for adjusting constants in PID (Proportional Integral Derivative) controllers. These are (Sepulveda et al., 2007):

1. Integral of Square Error (ISE).

$$\text{ISE} = \int_0^{\infty} [e(t)]^2 \, dt \tag{5.19}$$

2 Integral of the Absolute value of the Error (IAE).

$$\mathrm{IAE} = \int_0^\infty | e(t) | \, dt \qquad (5.20)$$

3. Integral of the Time multiplied by the Absolute value of the Error (ITAE).

$$\mathrm{ITAE} = \int_0^\infty t \, | e(t) | \, dt \qquad (5.21)$$

The selection of the criteria depends on the type of response desired, the errors will contribute different for each criterion, so we have that large errors will increase the value of ISE more heavily than to IAE. ISE will favor responses with smaller overshoot for load changes, but ISE will give longer settling time. In ITAE, time appears as a factor, and therefore, ITAE will penalize heavily errors that occur late in time, but virtually ignores errors that occur early in time. Designing using ITAE will give us the shortest settling time, but it will produce the largest overshoot among the three criteria considered. Designing considering IAE will give us an intermediate result, in this case, the settling time will not be so large than using ISE nor so small than using ITAE, and the same applies for the overshoot response. The selection of a particular criterion is depending on the type of desired response.

5.3 Experimental Results for Intelligent Control

The experimental results are devoted to show comparisons in the system's response in a feedback controller when using a type-1 FLC or a type-2 FLC. A set of five experiments is described in this section. The first two experiments were performed in ideal conditions, i.e., without any kind of disturbance. In the last three experiments, Gaussian noise was added to the feedback loop with the purpose of simulating, in a global way, the effects of uncertainty from several sources.

Figure 5.1 shows the feedback control system that was used for obtaining the simulation results. The complete system was simulated in the Matlab programming language, and the controller was designed to follow the input as closely as possible. The plant is a non-linear system that is modeled using equation (5.22)

$$y(i) = 0.2 \cdot y(i-3) \cdot 0.07 y(i-2) + 0.9 \cdot y(i-1) + 0.05 u(i-1) + 0.5 \cdot u(i-2) \qquad (5.22)$$

To illustrate the dynamics of this non-linear system, two different inputs are applied, first the input indicated by equation (5.23), which is shown in Figure 5.6, and whose system's response is in Figure 5.7.

$$u(i) = \begin{cases} 0 & 1 \le i < 5 \\ .1 & 5 \le i < 10 \\ .5 & 10 \le i < 15 \\ 1 & 15 \le i < 20 \\ .5 & 20 \le i < 25 \\ 1 & 25 \le i < 30 \\ 0 & 30 \le i < 35 \\ 1.47 & 35 \le i < 40 \end{cases} \tag{5.23}$$

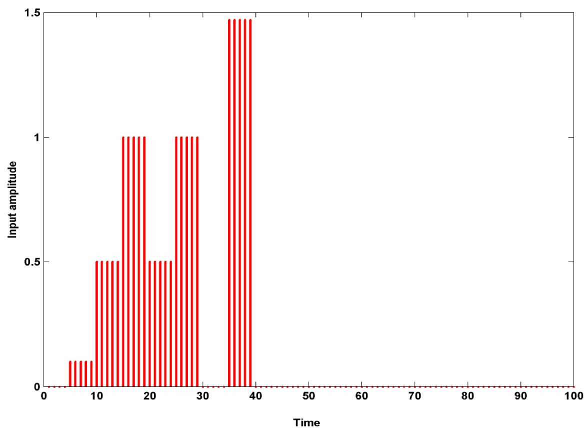

Fig. 5.6. Test sequence applied to the model of the plant given in equation (5.23)

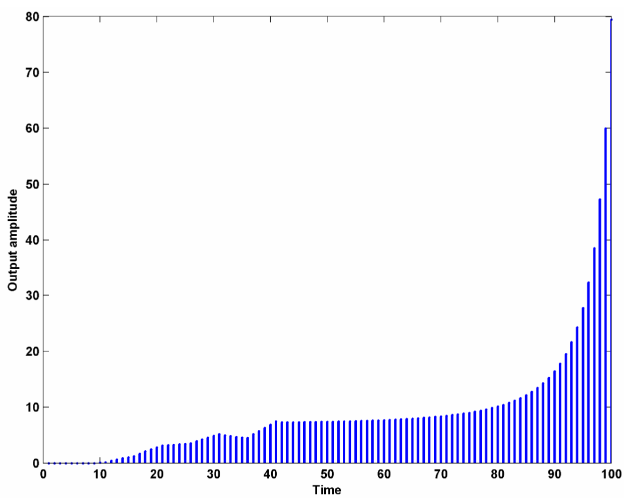

Fig. 5.7. System´s response for the inputs given in equation (5.23) which is illustrated in Fig. 5.6

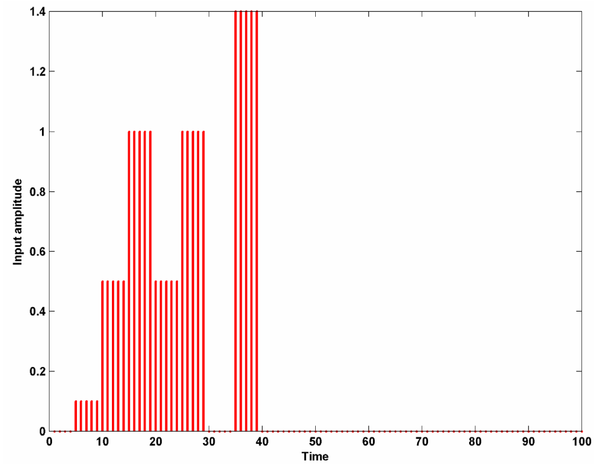

Fig. 5.8. A second input to the model for testing the plant response

Now, for a slightly different input given by equation (5.24), see Figure 5.8, we have the corresponding system´s response in Figure 5.9.

$$u(i)=\begin{cases} 0 & 1\le i<5 \\ .1 & 5\le i<10 \\ .5 & 10\le i<15 \\ 1 & 15\le i<20 \\ .5 & 20\le i<25 \\ 1 & 25\le i<30 \\ 0 & 30\le i<35 \\ 1.4 & 35\le i<40 \end{cases} \tag{5.24}$$

Going back to the control problem, this system given by equation (5.22) was used in Figure 5.1, under the name of plant or process, in this figure we can see that the controller's output is applied directly to the plant's input. Since we are interested in comparing the performance between type-1 and type-2 FLC systems, the controller was tested in two ways:

1. One is considering the system as ideal, that is, not introducing in the modules of the control system any source of uncertainty (experiments 1 and 2).
2. The other one is simulating the effects of uncertain modules (subsystems) response introducing some uncertainty (experiments 3, 4 and 5).

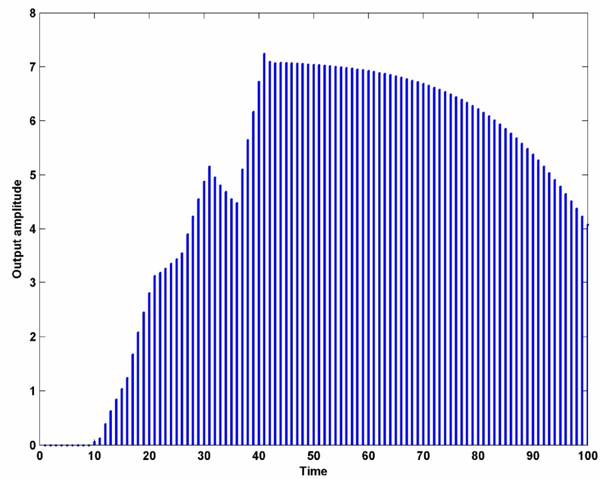

Fig. 5.9. Output of the plant when we applied the input given by equation (5.24) illustrated in Fig. 5.8

For both cases, as it is shown in Figure 5.1, the system's output is directly connected to the summing junction, but in the second case, the uncertainty was simulated introducing random noise with normal distribution (the dashed square in Figure 5.1). We added noise to the system's output $y(i)$ using the Matlab's function "randn", which generates random numbers with Gaussian distribution. The signal and the added noise in turn, were obtained with the programmer's expression (5.25), the result $y(i)$ was introduced to the summing junction of the controller system. Note that in expression (5.25) we are using the value 0.05, for experiments 3 and 4, but in the set of tests for experiment 5, we varied this value to obtain different SNR values.

$$y(i) = y(i) + 0.05 \cdot randn \tag{5.25}$$

The system was tested using as input, a unit step sequence free of noise, $r(i)$. For evaluating the system's response and comparing between type 1 and type 2 fuzzy controllers, the performance criteria ISE, IAE, and ITAE were used. In Table 5.3, we summarized the values obtained in an ideal system for each criterion considering 400 units of time. For calculating ITAE a sampling time of $T_s = 0.1$ sec. was considered.

For all experiments the reference input r is stable and noisy free. In experiments 3 and 4, although the reference appears clean, the feedback at the summing junction is noisy since noise for simulating the overall existing uncertainty in the system was introduced deliberately, in consequence, the controller's inputs $e(t)$ (error), and $\Delta e(t)$ contain uncertainty in the data.

In Experiment 5, we tested the systems, type-1 and type-2 FLCs, introducing different values of noise η, this was done by modifying the signal to noise ratio SNR (Proakis and Manolakis, 1996),

$$SNR = \frac{\sum |s|^2}{\sum |\eta|^2} = \frac{P_{signal}}{P_{noise}} \tag{5.26}$$

Because many signals have a very wide dynamic range (Ingle and Proakis, 2000), SNRs are usually expressed in terms of the logarithmic decibel scale, SNR(db),

$$SNR(db) = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \tag{5.27}$$

In Table 5.4, we show, for different values of SNR(db), the behavior of ISE, IAE, ITAE for type-1 and type-2 FLCs. In all the cases the results for type-2 FLC are better than type-1 FLC.

In the type-1 FLC, Gaussian membership functions (Gaussian MFs) for the inputs and for the output were used. A Gaussian MF is specified by two parameters {c,σ}:

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{5.28}$$

c represents the MFs center and σ determines the MFs standard deviation.

For each of the inputs of the type-1 FLC, $e(t)$ and $\Delta e(t)$, three type-1 fuzzy Gaussian MFs were defined as: negative, zero, positive. The universe of discourse for these membership functions is in the range [-10 10]; their centers are -10, 0 and 10 respectively, and their standard deviations is 4.2466 as is illustrated in Figures 5.10 and 5.11.

For the output of the type-1 FLC, we have five type-1 fuzzy Gaussian MFs: NG, N, Z, P and PG. They are in the interval [-10 10], their centers are -10, -.5, 0, 5, and 10 respectively; and their standard deviation is 2.1233 as can be seen in Figure 5.12. Table 1 illustrates the characteristics of the MFs of the inputs and output of the type-1 FLC.

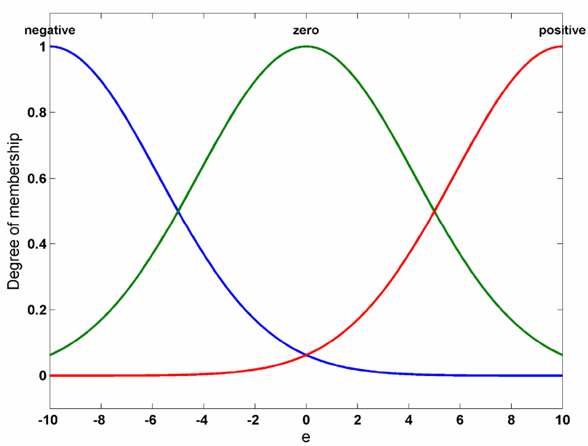

Fig. 5.10. Input e membership functions for the type-1 FLC

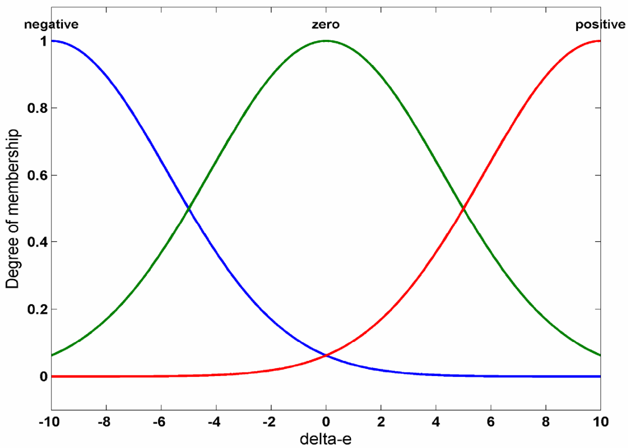

Fig. 5.11. Input Δe membership functions for the type-1 FLC

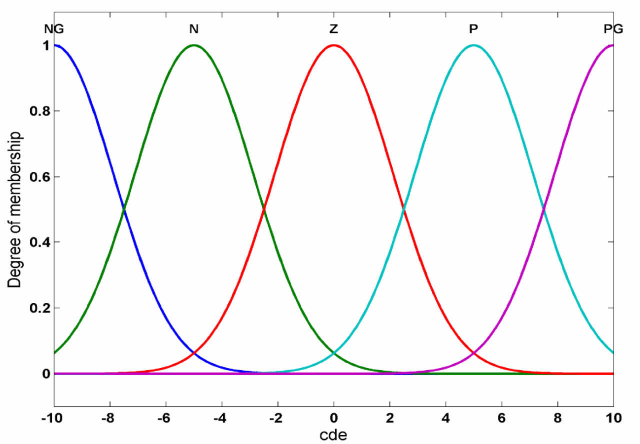

Fig. 5.12. Output cde membership functions for the type-1 FLC

In experiments 2, 4, and 5, for the type-2 FLC, as in type-1 FLC, we also selected Gaussian MFs for the inputs and for the output, but in this case we have interval type-2 Gaussian MFs with a fixed center, *c*, and an uncertain standard deviation, σ , i.e.,

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{5.29}$$

In terms of the upper and lower membership functions, we have for $\mu_{\tilde{A}}(x)$,

$$\overline{\mu}_{\tilde{A}}(x) = \ N(c,\ \sigma_2; x) \tag{5.30}$$

Table 5.1. Characteristics of the inputs and output of type-1 FLC

Variable	Term	Center c	Standard deviation σ
Input e	negative	-10	4.2466
	zero	0	4.2466
	positive	10	4.2466
Input Δe	Negative	-10	4.2466
	Zero	0	4.2466
	positive	10	4.2466
Output cde	NG	-10	2.1233
	N	-5	2.1233
	Z	0	2.1233
	P	5	2.1233
	PG	10	2.1233

and for the lower membership function $\underline{\mu}_{\tilde{A}}(x)$,

$$\underline{\mu}_{\tilde{A}}(x) = N(c, \sigma_1; x) \tag{5.31}$$

where $N(c,\sigma_2,x) \equiv e^{-\frac{1}{2}\left(\frac{x-c}{\sigma_2}\right)^2}$, and $N(c,\sigma_1,x) \equiv e^{-\frac{1}{2}\left(\frac{x-c}{\sigma_1}\right)^2}$, (Mendel, 2001). Hence, in the type-2 FLC, for each input we defined three interval type-2 fuzzy Gaussian MFs: negative, zero, positive in the interval [-10 10], as illustrated in Figures 5.13 and 5.14. For computing the output we have five interval type-2 fuzzy Gaussian

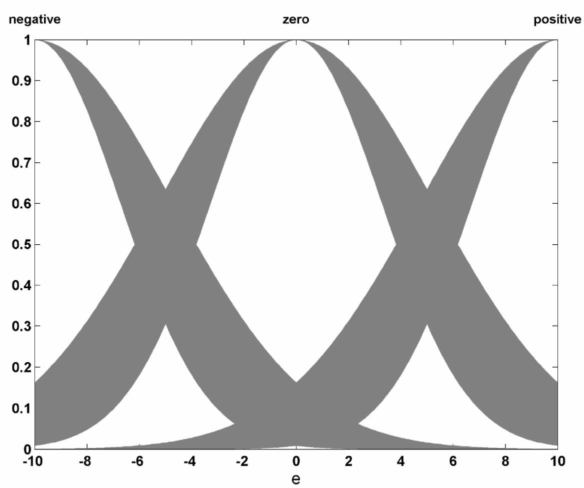

Fig. 5.13. Input e membership functions for the type-2 FLC

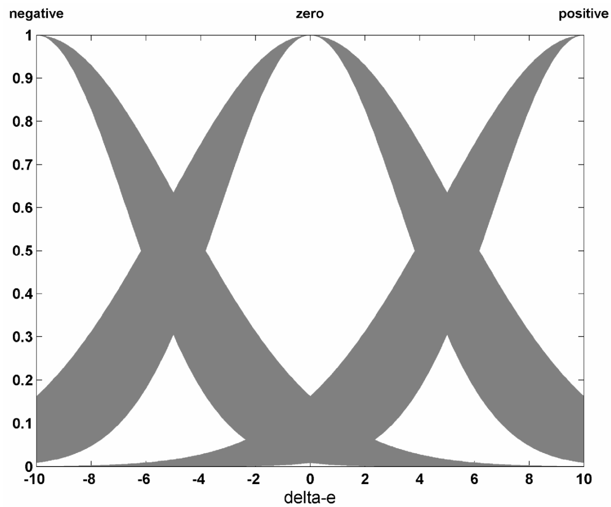

Fig. 5.14. Input Δe membership functions for the type-2 FLC

MFs, which are NG, N, Z, P and PG, in the interval [-10 10], as can be seen in Figures 5.15. Table 5.2 shows the characteristics of the inputs and output of the type-2 FLC.

For type-2 FLC we used, basically, the software for type-2 fuzzy logic developed by our research group. In all experiments, we have a dash-dot line for illustrating the system's response and behavior of type-1 FLC, in the same sense, a continuous line for type-2 FLC. The reference input *r* is shown with a dot line.

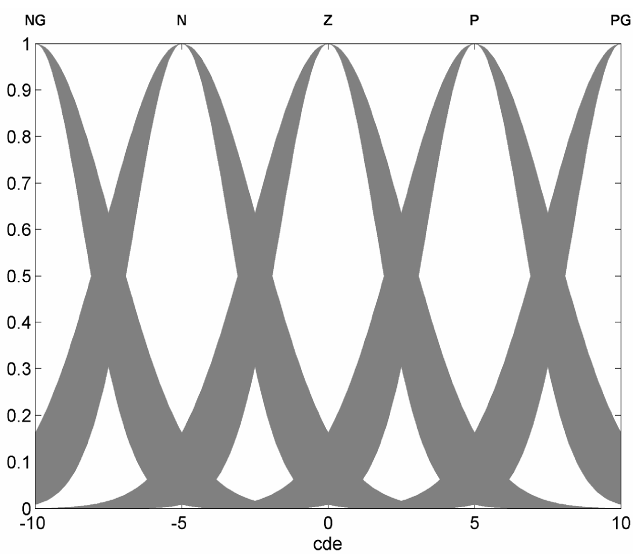

Fig. 5.15. Output cde membership functions for the type-2 FLC

Table 5.2. Characteristics of the inputs and output of type-2 FLC

Variable	Term	Center c	Standard deviation σ_1	Standard deviation σ_2
Input e	negative	-10	5.2466	3.2466
	zero	0	5.2466	3.2466
	positive	10	5.2466	3.2466
Input Δe	Negative	-10	5.2466	3.2466
	Zero	0	5.2466	3.2466
	positive	10	5.2466	3.2466
Output *cde*	NG	-10	2.6233	1.6233
	N	-5	2.6233	1.6233
	Z	0	2.6233	1.6233
	P	5	2.6233	1.6233
	PG	10	2.6233	1.6233

Experiment 1: Simulation of an ideal system with a type-1 FLC.
In this experiment, uncertainty data was not added to the system, and the system response is illustrated in Figure 5.16. Note that the settling time is of about 140 units of time; i.e., the system tends to stabilize with time and the output will follow accurately the input. In Table 5.3, we listed the obtained values of ISE, IAE, and ITAE for this

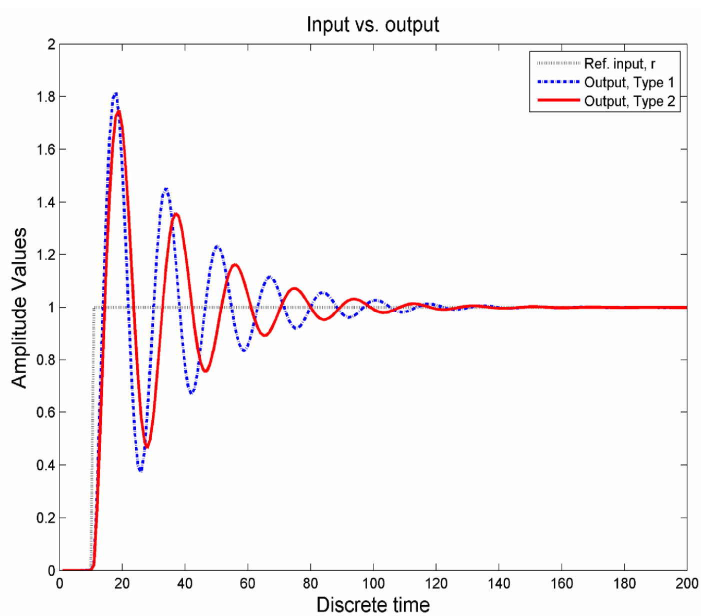

Fig. 5.16. This graphic shows the system's response to a unit step sequence. The input reference r is shown with pointed line, for the type-1 the systems' output $y(i)$ is shown with dash dot line; and for type-2, the system's output $y(i)$ with continuous line.

Table 5.3. Comparison of performance criteria for type-1 and type-2 fuzzy logic controllers for 20 db signal noise ratio. values obtained after 200 samples

Perform-ance Criteria	**Type-1 FLC**		**Type-2 FLC**	
	Ideal System	Syst. with uncertainty	Ideal System	Syst. with uncertainty
ISE	7.65	19.4	6.8	18.3
IAE	17.68	49.5	16.4	44.8
ITAE	62.46	444.2	56.39	402.9

experiment. In Figures 5.17, 5.18 and 5.19, the ISE, IAE, and ITAE behaviors of this experiment are shown.

Experiment 2: Simulation of an ideal system using the type-2 FLC.
Here, the same test conditions of Experiment 1 were used, but in this case, we implemented the controller's algorithm with type-2 fuzzy logic. The output sequence is illustrated in Figure 5.16, and the corresponding performance criteria are listed in Table 5.3, and we can observe that using a type-2 FLC we obtained the lower errors. By visual inspection, we can observe that the output system' response of the Experiment 1, and this one, are similar as it is shown in Figures 5.17, 5.18, and 5.19.

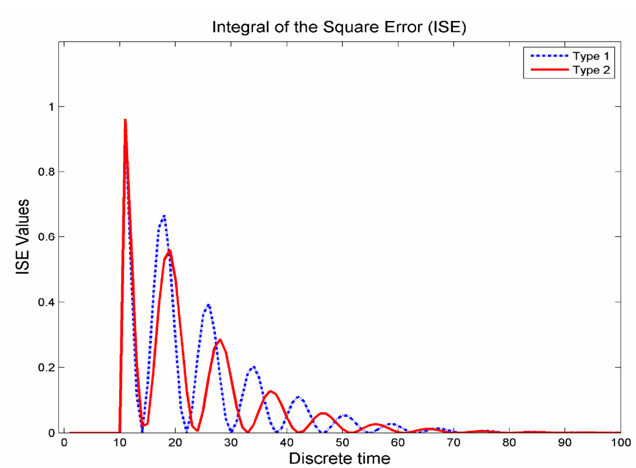

Fig. 5.17. In uncertainty absence, the ISE values are very similar for type-1 and type-2 FLCs

Experiment 3: System with uncertainty using a type-1 FLC.
In this case, equation (5.25) was used to simulate the effects of uncertainty introduced to the system by transducers, amplifiers, and any other element that in real world applications affects expected values. In this experiment the noise level was simulated in the range of 20 db of SNR ratio. Figure 5.20 shows the system's response output. In Figures 5.21, 5.22, and 5.23, the performance criteria ISE, IAE, ITAE are represented graphically.

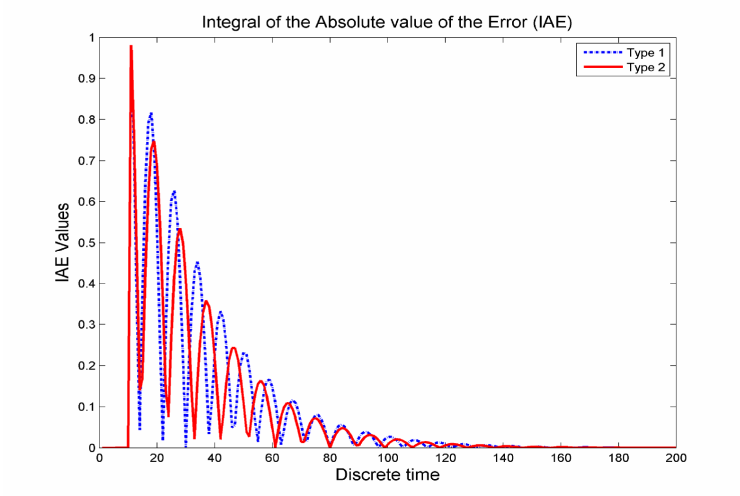

Fig. 5.18. In uncertainty absence, the IAE values obtained at the plant's output are very similar for type-1 and type-2 FLCs, here is more evident that a type-1 FLC works a little better than in Fig. 5.17

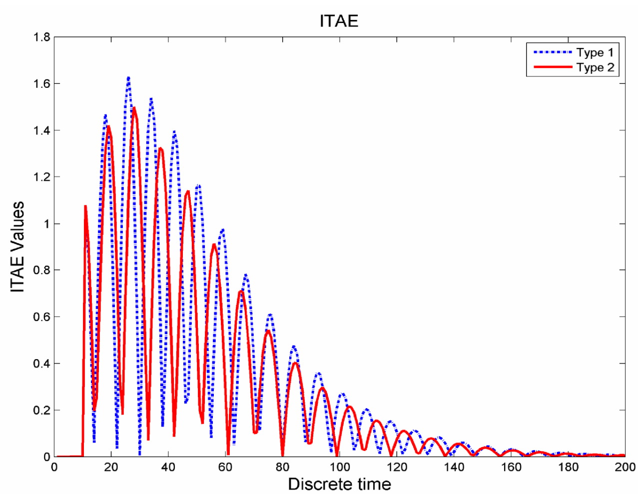

Fig. 5.19. In uncertainty absence, the ITAE values obtained at the plant's output are similar for type-1 and type-2 FLCs, in accordance with Figure 5.18, it is evident that a type-1 FLC works a little better

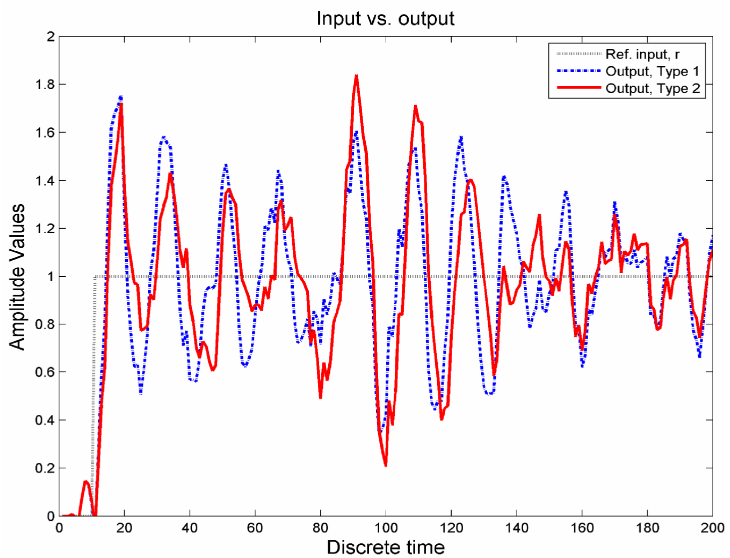

Fig. 5.20. This graphic was obtained with uncertainty presence; compare the system's outputs produced by type-1 and type-2 FLCs. Note that quite the opposite to Figure 5.16, a type-2 FLC works much better than a type-1 FLC when the system has uncertainty. The overshoot error is lower for a type-2 FLC.

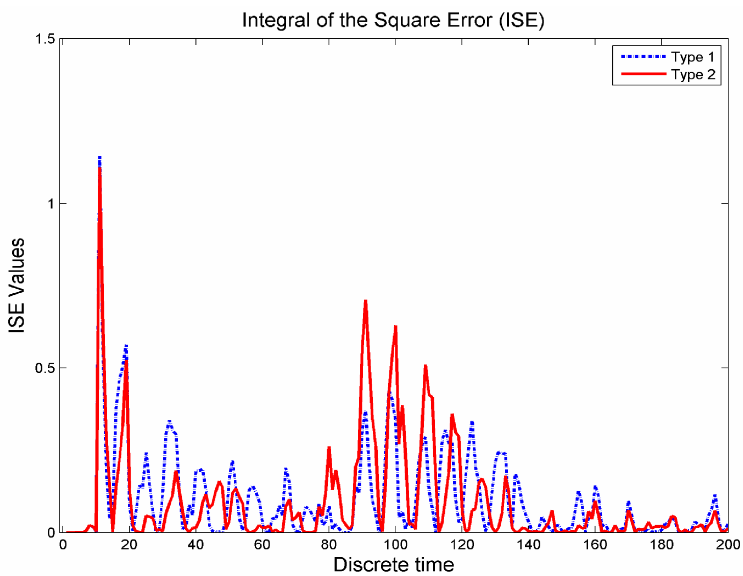

Fig. 5.21. We can see that a type-2 FLC produces lower overshoot errors, quantitatively the ISE overall error of using type-2 is 18.3 against 19.4 of the overall error produced by the type-1 FLC

Experiment 4: System with uncertainty using a type-2 FLC.
In this experiment, uncertainty was introduced in the system, in the same way as in Experiment 3. In this case, a type-2 FLC was used and the results obtained with a type-1 FLC (Experiment 3) were improved. We can appreciate from Figure 5.20, that the lower overshoot and the best settling times are reached using a type-2 FLC. In

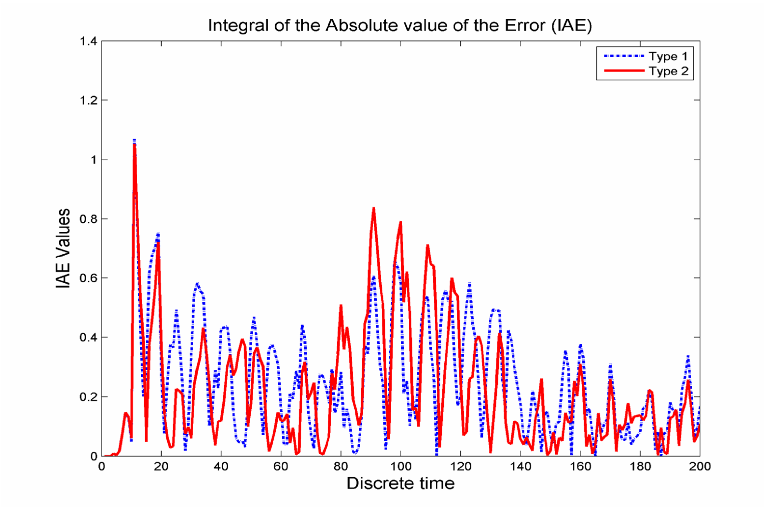

Fig. 5.22. In accordance with Fig. 5.20, IAE confirms that we obtained the best system response using a type-2 FLC with uncertainty presence. Moreover, the error of the settling time and steady state is lower using a type-2 FLC.

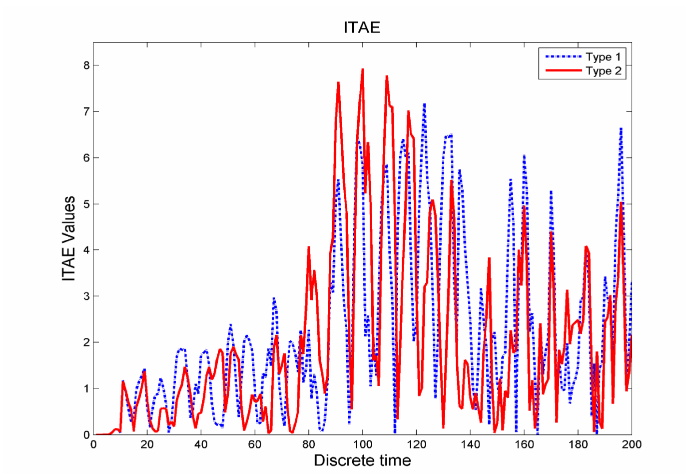

Fig. 5.23. Here we can see that the steady state error of the system produced by a type-2 FLC is lower than the error produced by a type-1 FLC with uncertainty present. ITAE will punish heavily all those errors produced with time.

Figures 5.21 and 5.22, we can see that with a type-2 FLC the overshoot error decreases very quickly and it remains lower than using a type-1 FLC. In Fig. 5.23, we can observe that through time the lower errors are obtained using a type-2 FLC.

Experiment 5: Varying the Signal to Noise Ratio (SNR) in type-1 and type-2 FLCs. To test the robustness of the type-1 and type-2 FLCs, we repeated experiments 3 and 4 giving different noise levels, going from 30 db to 8 db of SNR ratio in each experiment. In Table 5.4, we summarized the values for ISE, IAE, and ITAE considering 200 units of time with a P_{signal} of 22.98 db in all cases. As it can be seen in Table 5.4, in presence of different noise levels, the behavior of type-2 FLC is in general better than type-1 FLC.

Table 5.4. Behavior of type-1 and type-2 fuzzy logic controllers after variation of signal noise ratio. Values obtained for 200 samples.

Noise variation		**Type-1 FLC**			**Type-2 FLC**		
SNR (db)	Sum Noise (db)	ISE	IAE	ITAE	ISE	IAE	ITAE
8	22.72	321.1	198.1	2234.1	299.4	194.1	2023.1
10	20.762	178.1	148.4	1599.4	168.7	142.2	1413.5
12	18.783	104.7	114.5	1193.8	102.1	108.8	1057.7
14	16.785	64.1	90.5	915.5	63.7	84.8	814.6
16	14.78	40.9	72.8	710.9	40.6	67.3	637.8
18	12.78	27.4	59.6	559.1	26.6	54.2	504.4
20	10.78	19.4	49.5	444.2	18.3	44.8	402.9
22	8.78	14.7	42	356.9	13.2	37.8	324.6
24	6.78	11.9	36.2	289	10.3	32.5	264.2
26	4.78	10.1	31.9	236.7	8.5	28.6	217.3
28	2.78	9.1	28.5	196.3	7.5	25.5	180.7
30	0.78	8.5	25.9	164.9	7	23.3	152.6

From Table 5.4, considering two examples, the extreme cases; we have for an SNR ratio of 8 db, in type-1 FLC the following performance values ISE=321.1, IAE=198.1, ITAE=2234.1; and for the same case, in type-2 FLC, we have ISE=299.4, IAE=194.1, ITAE=2023.1.

For 30 db of SNR ratio, we have for the type-1 FLC, ISE=8.5, IAE=25.9, ITAE=164.9, and for the type-2 FLC, ISE=7, IAE=23.3, ITAE=152.6.

These values indicate a better performance of the type-2 FLC than type-1 FLC, because they are a representation of the errors, and as the error increases the performance of the system goes down.

5.4 Summary

We have presented the study of the controllers' design for nonlinear control systems using type-1 and type-2 fuzzy logic. We presented five experiments where we simulated the systems' responses with and without uncertainty presence. In the experiments, a quantification of errors was achieved and documented in tables for different criteria such as ISE, IAE, and ITAE, it was shown that the lower overshoot errors and the best settling times were obtained using a type-2 FLC. Based on the experimental results, we can say that the best results are obtained using type-2 fuzzy systems. In our opinion, this is because type-2 fuzzy sets that are used in type-2 fuzzy systems can handle uncertainties in a better way because they provide us with more parameters and more design degrees of freedom.