11 Evolutionary Optimization of Interval Type-2 Membership Functions Using the Human Evolutionary Model

Uncertainty is an inherent part in controllers used for real-world applications. The use of new methods for handling incomplete information is of fundamental importance in engineering applications. We simulated the effects of uncertainty produced by the instrumentation elements in type-1 and type-2 fuzzy logic controllers to perform a comparative analysis of the systems' response, in the presence of uncertainty. We are presenting an innovative idea to optimize interval type-2 membership functions using an average of two type-1 systems with the Human Evolutionary Model, we are showing comparative results of the optimized proposed method. We found that the optimized membership functions for the inputs of a type-2 system increases the performance of the system for high noise levels.

11.1 Introduction

Uncertainty affects decision-making and appears in a number of different forms. The concept of information is fully connected with the concept of uncertainty. The most fundamental aspect of this connection is that the uncertainty involved in any problem-solving situation is a result of some information deficiency, which may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way (Klir and Yuan, 1995). The general framework of fuzzy reasoning allows handling much of this uncertainty, fuzzy systems employ type-1 fuzzy sets, which represents uncertainty by numbers in the range [0, 1]. When something is uncertain, like a measurement, it is difficult to determine its exact value, and of course, type-1 fuzzy sets makes more sense than using crisp sets (Zadeh, 1975). However, it is not reasonable to use an accurate membership function for something uncertain, so in this case what we need is another type of fuzzy sets, those which are able to handle these uncertainties, the so called type-2 fuzzy sets (Mendel, 2000). So, the amount of uncertainty in a system can be reduced by using type-2 fuzzy logic because it offers better capabilities to handle linguistic uncertainties by modeling vagueness and

O. Castillo and P. Melin: Type-2 Fuzzy Log.: Theory and Appl., STUDFUZZ 223, pp. 133–144, 2008.
springerlink.com

unreliability of information (Karnik and Mendel, 2001). In industry, type-2 fuzzy logic and neural networks was used in the control of non-linear dynamic plants (Hagras, 2004) (Melin and Castillo, 2004).

This chapter deals with the optimization of interval type-2 membership functions in a fuzzy logic controller (FLC). Since, uncertainty is inherent in controllers for real world applications, as a first step, we are presenting how to deal with it using type-2 FLC to diminish the effects of imprecise information. We are supporting this statement with experimental results, qualitative observations, and quantitative measures of errors. For quantifying the errors, we utilized three widely used performance criteria, these are: Integral of Square Error (ISE), Integral of the Absolute value of the Error (IAE), and Integral of the Time multiplied by the Absolute value of the Error (ITAE) (Deshpande and Ash, 1988). Then as a second step, we optimized the parameters of the Gaussian membership functions (MFs) using the Human Evolutionary Model (HEM) which will be explained in section 11.3, and ISE as the fitness function. In this case, we used as an output, the average of two type-1 system.

11.2 Fuzzy Controllers

A FLS, described completely in terms of type-1 fuzzy sets is called a type-1 fuzzy logic system (type-1 FLS). It is composed by a knowledge base that comprises the information given by the process operator in form of linguistic control rules; a fuzzification interface, who has the effect of transforming crisp data into fuzzy sets; an inference system, that uses them in conjunction with the knowledge base to make inference by means of a reasoning method; and a defuzzification interface, which translates the fuzzy control action so obtained to a real control action using a defuzzification method.

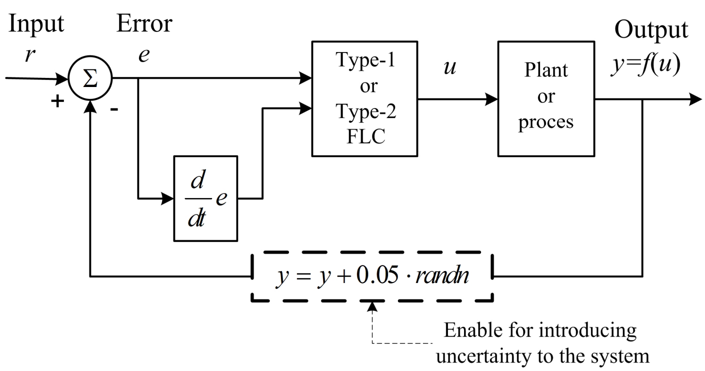

Fig. 11.1. System used for obtaining the experimental results

In this chapter, the implementation of the fuzzy controller in terms of type-1 fuzzy sets, has two input variables such as the error $e(t)$, the difference between the reference signal and the output of the process, as well as the error variation $\Delta e(t)$,

$$e(t) = r(t) - y(t) \tag{11.1}$$

$$\Delta e(t) = e(t) - e(t-1) \tag{11.2}$$

so the control system can be represented as in Fig. 11.1.

A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs are unable to directly handle rule uncertainties, because they use type-1 fuzzy sets that are certain. On the other hand, type-2 FLSs, are very useful in circumstances where it is difficult to determine an exact certainty, and measurement uncertainties (Mendel, 2000).

Similar to a type-1 FLS, a type-2 FLS includes fuzzifier, rule base, fuzzy inference engine, and output processor. The output processor includes type-reducer and defuzzifier; it generates a type-1 fuzzy set output (from the type-reducer) or a crisp number (from the defuzzifier) (Mendel, 2000). A type-2 FLS is again characterized by IF-THEN rules, but its antecedent or consequent sets are now type-2. In the case of the implementation of the type-2 FLC, we have the same characteristics as in type-1 FLC, but we used type-2 fuzzy sets as membership functions for the inputs and for the output.

For evaluating the transient closed-loop response of a computer control system we can use the same criteria that normally are used for adjusting constants in PID (Proportional Integral Derivative) controllers. These are:

1. Integral of Square Error (ISE).

$$\text{ISE} = \int_0^\infty [e(t)]^2\, dt \tag{11.3}$$

2. Integral of the Absolute value of the Error (IAE).

$$\text{IAE} = \int_0^\infty |e(t)|\, dt \tag{11.4}$$

3. Integral of the Time multiplied by the Absolute value of the Error (ITAE).

$$\text{ITAE} = \int_0^\infty t\,|e(t)|\, dt \tag{11.5}$$

The selection of the criteria depends on the type of response desired, the errors will contribute different for each criterion, so we have that large errors will increase the value of ISE more heavily than to IAE. ISE will favor responses with smaller overshoot for load changes, but ISE will give longer settling time. In ITAE, time appears as a factor, and therefore, ITAE will penalize heavily errors that occur late in time, but virtually ignores errors that occurs early in time.

11.3 The Human Evolutionary Model

The main idea of this computational model (Montiel et al., 2007),is to combine synergetically diferent techniques for performing search and optimization tasks. HEM was defined as follows (Montiel et al., 2007):

$$HEM = (H, AIIS, P, O, S, E, L, TL/PS, VRL, POS)$$

where

H	Human
AIIS	Adaptive Intelligent Intuitive System
P	Population of size *N* individuals
O	Single or a multiple objective optimization goals
S	Evolutionary strategy used for reaching the objectives expressed in *O*
E	Environment, here we can have predators, etc.
L	Landscape, i.e., the scenario where the evolution must be performed
TL/S	Tabu List formed by the bests solutions found/Pareto Set
VRL	Visited Regions List
POS	Pareto Optimal Set

Fig. 11.2 is a schematic representation of one individual which is comprised of three parts: a genetic representation *gr*, which can be codified using binary or floating-point representation; a set of genetic effects *ge*, that are attributes of each individual such as "physical structure", "gender", "actual age", "maximum age allowed", pheromone level", etc; these attributes give to the algorithm some of the human like characteristics that will define in great part, the individual behavior.

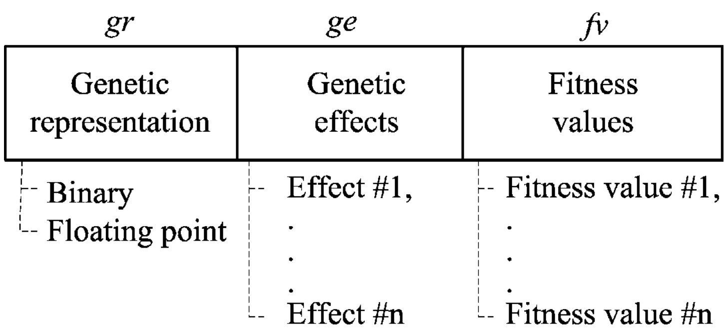

Fig. 11.2. Representing one individual in *HEM*

The third part in the individual representation is devoted to individual's fitness values. An individual p_i is defined as $p_i=(gr_i,ge_i,fv_i)$ where $gr_i=(gr_{i1},\ldots,gr_{iM})$ is a vector (a row) of the matrix *GR* of dimension $M \times N$. The genetic effects (ge_i) are rows in a matrix *GE*. In this method we can have one or several fitness values (fv), so we can handle single objective optimization problems (SOOP), and multi-objective optimization problems (MOOP). Fitness values are defined as vectors fv_i in the matrix $FV_{J\times N}$, in this way we have $fv=(fv_1,\ldots,fv_J)$. In this context, a population P_i is defined as $P_i=(GR_i+GE_i+FV_i)$. In the attribute $ge_{igender}$, we have the valid values set $\{M,F,0\}$, in this set *M* alludes a subpopulation of male individuals, *F* is used for the female subpopulation, and *0* means that this attribute will not be considered. The genetic at

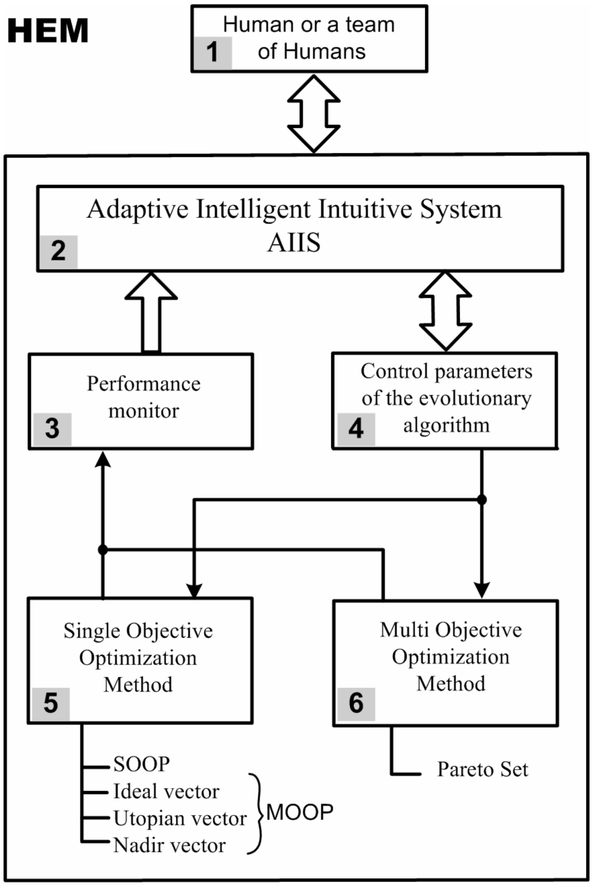

Fig. 11.3. General structure of HEM

tribute $ge_{iactAge}$ contains the actual age of an individual; its value corresponds to the number of generation that the individual has survived. We can set the maximum life expectance for each individual in the attribute $ge_{imaxAge}$. The task of the attribute $ge_{iphLevel}$ is to leave trace about which individuals have been involved in previous generations producing good offsprings. Fig. 11.3 shows the general structure of *HEM*.

In Figure 11.3, we have a general description of HEM containing six main blocks. In the first block, we show that the human or group of humans is part of the system. HEM is an intelligent evolutionary algorithm that learns from experts their rational and intuitive procedures that they use to solve optimization problems. In this model, we consider that we have two kinds of humans: real human beings and artificial humans. In the first block of Figure 11.3 we show that real human beings form one class. In the second block, the artificial human implemented in the AIIS of HEM is shown. Humans as part of the system are in charge of teaching the artificial human all the knowledge needed for realizing the searching task. The AIIS should learn the rational and intuitive knowledge from the experts; the final purpose is that the artificial human eventually can substitute the human beings most of the times. HEM has a feedback control system formed by blocks three and four; they work coordinately for monitoring and evaluating the evolution of the problem to be solved. In the fifth block, we have a single objective optimization (SOO) method for solving single objective optimization problems (SOOP). In addition, using the SOO method we can to find the ideal, utopian and nadir vectors for multiple objective optimization problems (MOOP). In the sixth block, we have a multiple objective optimization (MOO) method, which is dedicated to find the Pareto optimal set (POS) in MOOP.

11.4 Experimental Results

Figure 11.1 shows, the feedback control system that was used for achieving the results of this paper. It was implemented in Matlab where the controller was designed to follow the input as closely as possible.

The plant was modeled using equation (11.6)

$$y(i)=0.2\cdot y(i-3)\cdot 0.07y(i-2)+0.9\cdot y(i-1)+ \\ 0.05\cdot u(i-1)+0.5\cdot u(i-2) \tag{11.6}$$

The controller's output was applied directly to the plant's input. Since we are interested in comparing the performance between normal type-1 and type-2 FLC system versus optimized type-2 FLC system, we tested the controller in three ways:

1. Considering the system as ideal, that is, we did not introduce in the modules of the control system any source of uncertainty. See experiments 1, and 2.
2. Simulating the effects of uncertain modules (subsystems) response introducing some uncertainty, and diverse noise levels. See experiments 3, 4 and 5.
3. After optimization of the interval type-2 MFs, we repeated case two above. See experiment 6.

For case one, as is shown in Fig. 11.1, the system's output is directly connected to the summing junction, but in the second case, the uncertainty was simulated introducing random noise with normal distribution (the dashed square in Fig. 1). We added noise to the system's output $y(i)$ using the Matlab's function "randn" which generates random numbers with Gaussian distribution. The signal and the added noise in turn, were obtained with the programmer's expression (11.7), the result $y(i)$ was introduced to the summing junction of the controller system. Note that in (11.7) we are using the value 0.05, for experiments 3 and 4, but in the set of tests for experiment 5 we varied this value to obtain different SNR values.

$$y(i) = y(i) + 0.05 \cdot randn \tag{11.7}$$

We tested the system using as input, a unit step sequence free of noise, $r(i)$. For evaluating the system's response and compare between type 1 and type 2 fuzzy controllers, we used the performance criteria ISE, IAE, and ITAE. In Table 11.3, we summarized the values obtained for each criterion considering 200 units of time. For calculating ITAE we considered a sampling time $T_s = 0.1$ sec.

For experiments 1, 2, 3, and 4 the reference input r is stable and noisy free. In experiments 3 and 4, although the reference appears clean, the feedback at the summing junction is noisy since we introduced deliberately noise for simulating the overall existing uncertainty in the system, in consequence, the controller's inputs $e(t)$ (error), and $\Delta e(t)$ contains uncertainty data.

In experiment 5, we tested the systems, type-1 and type-2 FLCs, introducing diverse values of noise η, this is modifying the signal to noise ratio SNR (Ingle and Proakis, 2000),

$$SNR = \frac{\sum |s|^2}{\sum |\eta|^2} = \frac{P_{signal}}{P_{noise}} \tag{11.8}$$

Because many signals have a very wide dynamic range, SNRs are usually expressed in terms of the logarithmic decibel scale, SNR(db),

$$SNR(db) = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \tag{11.9}$$

In Table 11.4, we show, for different values of SNR(db), the behavior of ISE, IAE, ITAE for type-1 and type-2 FLCs. In almost all the cases the results for type-2 FLC are better than type-1 FLC.

In type-1 FLC, we selected Gaussian MFs for the inputs and for the output. A Gaussian MF is specified by two parameters {c,σ}:

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \tag{11.10}$$

c represents the MFs center and σ determines the MFs standard deviation.

For each input of the type-1 FLC, $e(t)$ and $\Delta e(t)$, we defined three type-1 fuzzy Gaussian MFs: negative, zero, positive. The universe of discourse for these membership functions is in the range [-10 10]; their centers are -10, 0 and 10 respectively, and their standard deviations are 9, 2 and 9 respectively.

For the output of the type-1 FLC, we have five type-1 fuzzy Gaussian MFs: NG, N, Z, P and PG. They are in the interval [-10 10], their centers are -10, -4.5, 0, 4, and 10 respectively; and their standard deviations are 4.5, 4, 4.5, 4 and 4.5. Table 11.1 illustrates the characteristics of the inputs and output of the FMs of the type-1 FLC.

For the type-2 FLC, as in type-1 FLC we also selected Gaussian MFs for the inputs and for the output, but in this case we have an interval type-2 Gaussian MFs with a fixed standard deviation, σ, and an uncertain center, ie.,

$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \qquad c \in [c_1, c_2] \tag{11.11}$$

Table 11.1. Characteristics of the MFs of the inputs and output of the type-1 FLC

Variable	Term	Center c	Standard Deviation σ
Input e	negative	-10	9
	zero	0	2
	positive	10	9
Input Δe	negative	-10	9
	zero	0	2
	positive	10	9
Output cde	NG	-10	4.5
	N	-4.5	4
	Z	0	4.5
	P	4	4
	PG	10	4.5

In terms of the upper and lower membership functions, we have for $\overline{\mu}_{\tilde{A}}(x)$,

$$\overline{\mu}_{\tilde{A}}(x) = \begin{cases} N(c_1, \sigma; x) & x < c_1 \\ 1 & c_1 \le x \le c_2 \\ N(c_2, \sigma; x) & x > c_2 \end{cases} \tag{11.12}$$

and for the lower membership function $\underline{\mu}_{\tilde{A}}(x)$,

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} N(c_2, \sigma; x) & x \le \dfrac{c_1 + c_2}{2} \\ N(c_1, \sigma; x) & x > \dfrac{c_1 + c_2}{2} \end{cases} \tag{11.13}$$

where $N(c_1, \sigma, x) \equiv e^{-\frac{1}{2}\left(\frac{x - c_1}{\sigma}\right)^2}$,

and $N(c_2, \sigma, x) \equiv e^{-\frac{1}{2}\left(\frac{x - c_2}{\sigma}\right)^2}$.

Hence, in type-2 FLC, for each input we defined three interval type-2 fuzzy GaussianMFs: negative, zero, positive in the interval [-10 10], as illustrates Fig. 11.4 for input *e*. For computing the output we have five interval type-2 fuzzy Gaussian MFs NG, N, Z, P and PG, with uncertain center and fixed standard deviations in the interval [-10 10], as can be seen in Fig. 11.5. Table 11.2 shows the characteristics of the MFs of the inputs and output of the type-2 FLC.

In experiment 6, to simulate the interval type-2 MFs of the FLC, we used two type-1 FLCs. Using HEM as the optimization method, ISE as a fitness function, we found the best values, see table V, for the MFs of the inputs of these controllers. Through

Table 11.2. Characteristics of the MFs of the inputs and output of the type-2 FLC

Variable	Term	Center c_1	Center c_2	Standard Deviation σ
Input e	negative	-10.25	-9.75	9.2
	zero	-0.25	0.25	2.2
	positive	9.75	10.25	9.2
Input Δe	negative	-10.25	-9.75	9.2
	zero	-0.25	0.25	2.2
	positive	9.75	10.25	9.2
Output cde	NG	-10.25	-9.75	4.5
	N	-4.75	-4.5	4
	Z	-0.25	0.25	4.5
	P	3.75	4.25	4
	PG	9.75	10.25	4.5

an average of the two type-1 optimized FLCs, we repeated experiment 5, and calculated again the values of ISE, IAE and ITAE, as can be seen in table 11.6.

For the experiments with interval type-2 MFs not optimized, we used, basically, the type-2 toolbox that we developed.

Experiment 1. Ideal system using a type-1 FLC.
In this experiment, we did not add uncertainty data to the system. The system trends to stabilize with time and the output will follow accurately the input. In Table 11.3, we listed the obtained values of ISE, IAE, and ITAE for this experiment.

Experiment 2. Ideal system using a type-2 FLC.
Here, we used the same test conditions of Experiment 1, but in this case, we implemented the controller's algorithm with type-2 fuzzy logic. The corresponding performance criteria are listed in Table 11.3.

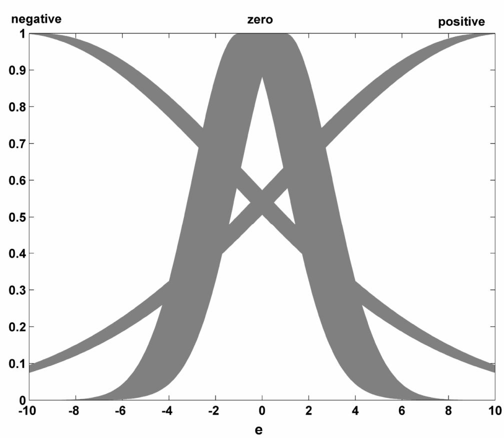

Fig. 11.4. Input *e* membership functions for the type-2 FLC

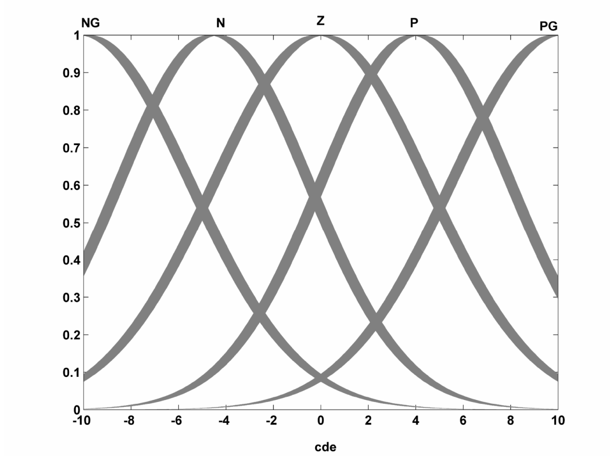

Fig. 11.5. Output *cde* membership functions fot the type-2 FLC

Experiment 3. System with uncertainty using a type-1 FLC.
In this case, we simulated using equation (7), the effects of uncertainty introduced to the system by transducers, amplifiers, and any other element that in real world applications affects expected values. In Table 11.3, we can see the obtained values of ISE, IAE, and ITAE for a simulated 10 db signal noise ratio.

Experiment 4. System with uncertainty using a type-2 FLC. In this experiment, we introduced uncertainty in the system, in the same way as in Experiment 3. In this case, we used a type-2 FLC and we improved those results obtained with a type-1 FLC in Experiment 3, see table 11.3.

Table 11.3. Comparison of performance criteria for type-1 and type-2 fuzzy logic controllers for 10 db signal noise ratio. values obtained after 200 samples.

Performance Criteria	**Type-1 FLC**		**Type-2 FLC**	
	Ideal System	Syst. with uncertainty	Ideal System	Syst. with uncertainty
ISE	5.2569	205.019 1	5.2572	149.309 7
IAE	13.8055	155.941 2	13.7959	131.77
ITAE	46.0651	1583.4	45.8123	1262.2

Experiment 5. Varying the signal to noise ratio in type-1 and type-2 FLCs.
To test the robustness of the type-1 and type-2 FLCs, we repeated experiments 3 and 4 giving different noise levels, going from 30 db to 6 db of SNR ratio in each experiment. In Table 11.4, we summarized the values for ISE, IAE, ITAE considering 200

Table 11.4. Behavior of type -1 and type-2 fuzzy logic controllers after variation of signal noise ratio. Values obtained for 200 samples.

SNR	Type-1 FLC			Type-2 FLC		
db	ISE	IAE	ITAE	ISE	IAE	ITAE
6	1208	392.3	4903	1113	368.76	4388
8	1004	352.4	4526	903	330.38	4104
10	205.0	155.9	1583.4	149.3	131.77	1262.2
12	89.77	102.1	974.97	89.8	102.21	974.93
14	56.47	80.88	769.51	56.78	80.85	770.25
16	36.28	64.36	610.86	36.39	64.21	610.65
18	23.76	51.54	485.19	23.81	51.32	485.16
20	16.14	41.75	386.45	16.04	41.59	386.6
22	11.36	34.65	310.04	11.25	34.28	308.87
24	8.54	29.25	249.67	8.39	28.78	247.89
26	6.87	25.15	202.46	6.72	24.78	201.07
28	5.9	22.2	166.38	5.78	21.92	165.21
30	5.38	20.12	139.27	5.27	19.77	137.47

Table 11.5. Characteristics of the optimized MFs of the inputs and output of the type-2 FLC

Variable	**Term**	**Center c_1**	**Standard Deviation σ_1**	**Center c_2**	**Standard Deviation σ_2**
Input e	negative	-10	9	-10	8.0298
	zero	0	2	0	1.0987
	positive	10	9	10	8.1167
Input Δe	negative	-10	9.2	-10	8.7767
	zero	0	2.2	0	1.0987
	positive	10	9.2	10	8.5129
Output cde	NG	-10	4.5	-10	4.5
	N	-4.5	4	-4.5	4
	Z	0	4.5	0	4.5
	P	4	4	4	4
	PG	10	4.5	10	4.5

units of time with a P_{signal} of 22.98 db in all cases. As it can be seen in Table 11.4, in presence of diverse noise levels, the behavior of type-2 FLC is better than type-1 FLC above 10 db.

From Table 11.4, taking two examples, the extreme cases; we have for an SNR ratio of 8 db, in type-1 FLC the next performance values ISE=1004, IAE=352.45, ITAE=4526; for the same case, in type-2 FLC, we have ISE=903, IAE=330.38, ITAE=4104.

For 10 db of SNR ratio, we have for type-1 FLC, ISE=205.01, IAE=155.94, ITAE=1583.4, and for type-2 FLC, ISE=149.3, IAE=131.77, ITAE=1262.2.

These values indicate a better performance of type-2 FLC than type-1 FLC above certain noise values, because they are a representation of the errors and as bigger they are the performance of the system is worst

To optimize the interval type-2 MFs of the FLC, we simulated the system using two type-1 FLCs . We maintain constant the centers of the Gaussian MFs of the inputs and varied its standard deviations.

After using HEM as the optimization method, and taking ISE as the fitness function, we found the best values of the MFs, as can be seen in table 11.5. With the new values of the MFs of both type-1 FLCs, we repeated experiment 5, but in this case, we used the average of the two type-1 FLCs as the output of the type-2 system. Table 11.6, shows the results for this experiment, as can be seen, all the values of ISE were improved, and in general we can see that the performance of the system is better.

Experiment 6. Optimizing the interval type-2 MFs of the FLC.

Table 11.6. Comparison of performance criteria for type-1 FLC, and type-2 fuzzy logic controller with optimized MFs, for 10 db signal noise ratio. Values obtained after 200 samples.

SNR	**Type-1 FLC**			**Type-2 FLC**		
db	ISE	IAE	ITAE	ISE	IAE	ITAE
6	1208	392.3	4903	616.4	274.7	3005
8	1004	352.4	4526	437.3	226.7	2509
10	205.0	155.9	1583.4	115	116.6	1119.6
12	89.77	102.1	974.97	72.8	90.9	866.8
14	56.47	80.88	769.51	45.6	71.3	674.1
16	36.28	64.36	610.86	28.9	56.3	528.4
18	23.76	51.54	485.19	18.6	45.2	419.4
20	16.14	41.75	386.45	12.6	37	337
22	11.36	34.65	310.04	8.9	30.8	273.8
24	8.54	29.25	249.67	6.8	26.3	227.7
26	6.87	25.15	202.46	5.6	23.1	195.6
28	5.9	22.2	166.38	4.9	21	172.8
30	5.38	20.12	139.27	4.5	19.6	157.8

11.5 Summary

We observed and quantified using performance criteria such as ISE, IAE, and ITAE that in systems without uncertainties (ideal systems) is a better choice to select a type-1 FLC since it works a little better than a type-2 FLC, and it is easier to implement it. It is known that type-1 FLC can handle nonlinearities, and uncertainties up to some extent.

Unfortunately, real systems are inherently noisy and nonlinear, since any element in the system contributes with deviations of the expected measures because of thermal noise, electromagnetic interference, etc., moreover, they add nonlinearities from element to element in the system.

In the simulation of real systems, systems with uncertainty, we observed that the results presented in Table 11.4 demonstrated that the performance of this kind of controllers is better under high noise levels. After optimizing the interval type- 2 MFs the performance of the system is improved as we can see in table 11.6.

We can say that using a type-2 FLC in real world applications can be a better choice since this type of system is a more suitable system to manage uncertainty, as we can see in the results shown in tables 11.5 and 11.6.