

Construction of a Hybrid HIBE Protocol Secure Against Adaptive Attacks (Without Random Oracle)

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Abstract. We describe a hybrid hierarchical identity based encryption (HIBE) protocol which is secure in the full model without using the random oracle heuristic and whose security is based on the computational hardness of the decisional bilinear Diffie-Hellman (DBDH) problem. The new protocol is obtained by augmenting a previous construction of a HIBE protocol which is secure against chosen plaintext attacks (CPA-secure). The technique for answering decryption queries in the proof is based on earlier work by Boyen-Mei-Waters. Ciphertext validity testing is done indirectly through a symmetric authentication algorithm in a manner similar to the Kurosawa-Desmedt public key encryption protocol. Additionally, we perform symmetric encryption and authentication by a single authenticated encryption algorithm. A net result of all these is that our construction improves upon previously known constructions in the same setting.

1 Introduction

Identity based encryption [29,8] is a kind of public key encryption where the public key can be the identity of the receiver. The secret key corresponding to the identity is generated by a private key generator (PKG) and is securely provided to the relevant user. The notion of IBE simplifies the issues of certificate management in public key infrastructure. The PKG issues the private key associated with an identity. The notion of hierarchical IBE (HIBE) [21,19] was introduced to reduce the workload of the PKG. The identity of any entity in a HIBE structure is a tuple (v_1, \dots, v_j) . The private key corresponding to such an identity can be generated by the entity whose identity is (v_1, \dots, v_{j-1}) and which possesses the private key corresponding to this identity. The security model for IBE was extended to that of HIBE in [21,19].

The first construction of an IBE which can be proved to be secure in the full model without the random oracle heuristic was given by Boneh and Boyen in [5]. Later, Waters [31] presented an efficient construction of an IBE which is secure in

the same setting. An extension of Waters' construction has been independently described in [13] and [26]. This leads to a controllable trade-off between the size of the public parameters and the efficiency of the protocol (see [13] for details).

A construction of a HIBE secure in the full model without using the random oracle heuristic was suggested in [31]. A recent work [14], describes a HIBE which builds on [31] by reducing the number of public parameters. The constructed HIBE is secure against chosen plaintext attacks (CPA-secure).

The Problem. We consider the problem of constructing a HIBE under the following conditions.

- Security is in the full model [8], i.e., the adversary can mount an adaptive chosen ciphertext attack and can choose the challenge identity adaptively.
- The reduction is from the decisional bilinear Diffie-Hellman problem.
- The security proof does not use the random oracle heuristic.

1.1 Our Contributions

We describe a hybrid HIBE protocol for the above setting. The new construction is obtained by augmenting the construction in [14]. The idea for this augmentation is based on the technique of [9] and algebraic ideas from the construction of IBE given in [4]. In addition, we make use of two new things. First, we incorporate information about the length of the identity into the ciphertext. Second, we use symmetric key authentication to verify ciphertext well formedness. We also show that the two tasks of symmetric key encryption and authentication can be combined by using an authenticated encryption (AE) protocol.

The idea of using symmetric authentication technique to verify the well formedness of the ciphertext is based on the PKE protocol due to Kurosawa-Desmedt (KD) [25]. To the best of our knowledge, this technique has not been earlier applied to the (H)IBE setting.

We can specialize the HIBE protocol described in this paper to obtain a PKE and an IBE. With some natural simplifications, the PKE turns out to be the key encapsulation mechanism (KEM) proposed by BMW [9] composed with a one-time secure data encapsulation mechanism (DEM). On the other hand, the IBE is different from previous work. Kiltz-Galindo [24] had proposed an IB-KEM. Composed with a suitable symmetric encryption algorithm, this provides an IBE. The decryption algorithm of our IBE is faster than the IBE obtained from the KEM given in [24].

Our construction has a security degradation of approximately q^h (where q is the number of queries and h is the number of levels). This is better than a degradation of q^{h+1} which is what one would obtain by a straightforward application of the known techniques. Another advantage is that by instantiating the AE protocol with a single pass algorithm [27,22,20,12], it is possible to obtain a speed-up by a factor of two for both encryption and decryption of the symmetric part of the hybrid encryption. Also, by using the authentication aspect of the AE protocol for verifying the well formedness of the ciphertext we can avoid a number of pairing based verifications. This leads to a faster decryption algorithm.

We make a few remarks on the proof. Since the new protocol is obtained by augmenting the protocol in [14], the proof of the new protocol is also obtained by augmenting the proof in [14] (which is actually based on the construction and proof in [31]). We do not repeat the aspects of the proof that already appear in [14]. Incorporating the length of the identity in the ciphertext is required to avoid certain attacks as we discuss later. Verifying ciphertext well formedness using symmetric authentication requires us to adapt the proof technique (especially the method of deferred analysis) of [1] to the identity based setting. The combination of different techniques introduces several subtleties in the proof.

1.2 Related Work

The construction in [19] is based on the random oracle assumption and does not constitute a solution to the problem considered in this paper. A generic technique [11,7] is known which converts an $(h + 1)$ -level CPA-secure HIBE protocol into an h -level CCA-secure HIBE protocol while preserving the other features (security model, with/without random oracle, hardness assumption) of the original CPA-secure protocol. This technique is based on one-time signatures and requires prepending each identity component by a bit. Applying this technique directly to the protocol in [14] does not provide a protocol which is more efficient than the protocol we describe in this paper.

The BMW paper [9] provided a method of constructing a PKE from an IBE. They also mentioned that the technique can be used for constructing (H)IBE. Later work by Kiltz-Galindo [24] built on the BMW paper and described an efficient CCA-secure IB-KEM. The KG paper suggested a method for extending their IB-KEM to a HIB-KEM. Details were provided in [3]. Our work also uses the BMW technique, but introduces several other ideas to obtain a more efficient (H)IBE compared to previous work.

In an interesting paper, Boneh-Boyen-Goh [6] have shown how to construct a constant size ciphertext (H)IBE based on the weak decisional bilinear Diffie-Hellman exponent problem which is a variant of the DBDH problem. Their protocol is CPA-secure in the selective-ID model. Using the technique of Waters, this protocol can be made CPA-secure in the full model. Further, using the techniques of Boyen-Mei-Waters this can be converted into a CCA-secure protocol. For details of this conversion and also for a protocol secure in a different model see [15]. The work [23] also considers the same problem.

The main difference between the current work and that of [15,23] is that the hardness assumptions are different. This makes a direct comparison difficult. We, however, note that the ciphertext expansion in the later is constant while in the former it increases linearly with the number of components in the identity. This is due to the fact that the assumption used in [15,23] is tailored to ensure constant size ciphertext. On the other hand, the number of public parameters in the current construction is significantly less than the number of public parameters in [15,23]. This is due to the fact that the current protocol is built using the protocol in [14] which significantly reduces the number of public parameters.

On Security Degradation of HIBE Protocols. All known HIBE protocols which are secure against adaptive-ID attacks have a security degradation which is exponential in the depth of the HIBE. This is true, even if the random oracle heuristic is used in the security proof. In view of this, all such protocols can be considered to have a valid security bound only for a small number of levels. Currently, the most important open problem in the construction of HIBE protocols is to avoid (or reduce) this exponential security decay.

2 Preliminaries

2.1 HIBE Protocol

Following [21,19], a hierarchical identity based encryption (HIBE) scheme is specified by four algorithms: Setup, KeyGen, Encrypt and Decrypt. For a HIBE of height h (henceforth denoted as h -HIBE) any identity \mathbf{v} is a tuple (v_1, \dots, v_j) where $1 \leq j \leq h$.

- HIBE.Setup: Takes as input a security parameter and outputs (pk, sk) , where pk is the public parameter of the PKG and sk is the master secret of the PKG. It also defines the domains of identities, messages and ciphertexts.
- HIBE.KeyGen($\mathbf{v}, d_{\mathbf{v}|_{j-1}}, pk$): Takes as input a j -level identity \mathbf{v} , the secret $d_{\mathbf{v}|_{j-1}}$ corresponding to its $(j-1)$ -level prefix and pk and returns as output $d_{\mathbf{v}}$, the secret key corresponding to \mathbf{v} . In case $j=1$, $d_{\mathbf{v}|_{j-1}}$ is equal to sk , the master secret of the PKG.
- HIBE.Encrypt(\mathbf{v}, M, pk): Takes as input \mathbf{v} , the message M and pk , and returns C , the ciphertext obtained by encrypting M under \mathbf{v} and pk .
- HIBE.Decrypt($\mathbf{v}, d_{\mathbf{v}}, C, pk$): Takes as input \mathbf{v} , the secret key $d_{\mathbf{v}}$ corresponding to \mathbf{v} , a ciphertext C and pk . Returns either *bad* or M , the message which is the decryption of C .

As usual, for soundness, we require that $\text{HIBE.Decrypt}(\mathbf{v}, d_{\mathbf{v}}, C, pk) = M$ must hold for all $\mathbf{v}, d_{\mathbf{v}}, C, pk, sk$ and M associated by the above four algorithms.

2.2 Security Model for HIBE

Security is defined using an adversarial game. An adversary \mathcal{A} is allowed to query two oracles – a decryption oracle and a key-extraction oracle. At the initiation, it is provided with the public parameters of the PKG. The game has two query phases with a challenge phase in between.

Query Phase 1. Adversary \mathcal{A} makes a finite number of queries where each query is addressed either to the decryption oracle or to the key-extraction oracle. In a query to the decryption oracle it provides a ciphertext as well as the identity under which it wants the decryption. It gets back the corresponding message or *bad* if the ciphertext is invalid. Similarly, in a query to the key-extraction oracle, it asks for the private key of the identity it provides and gets back this private

key. Further, \mathcal{A} is allowed to make these queries adaptively, i.e., any query may depend on the previous queries as well as their answers. The adversary is not allowed to make any useless queries, i.e., queries for which it can compute the answer itself. For example, the adversary is not allowed to ask for the decryption of a message under an identity if it has already obtained a private key corresponding to the identity.

Challenge. At this stage, \mathcal{A} outputs an identity $\mathbf{v}^* = (v_1^*, \dots, v_j^*)$ for $1 \leq j \leq h$, and a pair of messages M_0 and M_1 . There is the natural restriction on the adversary, that it cannot query the key extraction oracle on \mathbf{v}^* or any of its proper prefixes in either of the phases 1 or 2. A random bit δ is chosen and the adversary is provided with C^* which is an encryption of M_δ under \mathbf{v}^* .

Query Phase2. \mathcal{A} now issues additional queries just like Phase 1, with the (obvious) restrictions that it cannot ask the decryption oracle for the decryption of C^* under \mathbf{v}^* , nor the key-extraction oracle for the private key of \mathbf{v}^* or any of its prefixes.

Guess. \mathcal{A} outputs a guess δ' of δ .

The advantage of the adversary \mathcal{A} is defined as:

$$\text{Adv}_{\mathcal{A}}^{\text{HIBE}} = |\Pr[(\delta = \delta')] - 1/2|.$$

The quantity $\text{Adv}_{\mathcal{A}}^{\text{HIBE}}(t, q_{\text{ID}}, q_{\text{C}})$ denotes the maximum of $\text{Adv}_{\mathcal{A}}^{\text{HIBE}}$ where the maximum is taken over all adversaries running in time at most t and making at most q_{C} queries to the decryption oracle and at most q_{ID} queries to the key-extraction oracle. A HIBE protocol is said to be $(\epsilon, t, q_{\text{ID}}, q_{\text{C}})$ -CCA secure if $\text{Adv}_{\mathcal{A}}^{\text{HIBE}}(t, q_{\text{ID}}, q_{\text{C}}) \leq \epsilon$.

In the above game, we can disallow the adversary \mathcal{A} from querying the decryption oracle. $\text{Adv}_{\mathcal{A}}^{\text{HIBE}}(t, q)$ in this context denotes the maximum advantage where the maximum is taken over all adversaries running in time at most t and making at most q queries to the key-extraction oracle. A HIBE protocol is said to be (t, q, ϵ) -CPA secure if $\text{Adv}_{\mathcal{A}}^{\text{HIBE}}(t, q) \leq \epsilon$.

2.3 Cryptographic Bilinear Map

Let G_1 and G_2 be cyclic groups having the same prime order p and $G_1 = \langle P \rangle$, where we write G_1 additively and G_2 multiplicatively. A mapping $e : G_1 \times G_1 \rightarrow G_2$ is called a cryptographic bilinear map if it satisfies the following properties.

- Bilinearity : $e(aP, bQ) = e(P, Q)^{ab}$ for all $P, Q \in G_1$ and $a, b \in \mathbb{Z}_p$.
- Non-degeneracy : If $G_1 = \langle P \rangle$, then $G_2 = \langle e(P, P) \rangle$.
- Computability : There exists an efficient algorithm to compute $e(P, Q)$ for all $P, Q \in G_1$.

Since $e(aP, bP) = e(P, P)^{ab} = e(bP, aP)$, $e(\cdot)$ also satisfies the symmetry property. The modified Weil pairing [8] and Tate pairing [2,18] are examples of cryptographic bilinear maps.

Known examples of $e()$ have G_1 to be a group of Elliptic Curve (EC) points and G_2 to be a subgroup of a multiplicative group of a finite field. Hence, in papers on pairing implementations [2,18], it is customary to write G_1 additively and G_2 multiplicatively. On the other hand, some “pure” protocol papers such as [5,31] write both G_1 and G_2 multiplicatively though this is not true of the initial protocol papers [8,19]. Here we follow the first convention as it is closer to the known examples.

2.4 Hardness Assumption

The decisional bilinear Diffie-Hellman (DBDH) problem in $\langle G_1, G_2, e \rangle$ [8] is as follows: Given a tuple $\langle P, aP, bP, cP, Z \rangle$, where $Z \in G_2$, decide whether $Z = e(P, P)^{abc}$ (which we denote as Z is real) or Z is random. The advantage of a probabilistic algorithm \mathcal{B} , which takes as input a tuple $\langle P, aP, bP, cP, Z \rangle$ and outputs a bit, in solving the DBDH problem is defined as

$$\text{Adv}_{\mathcal{B}}^{\text{DBDH}} = |\Pr[\mathcal{B}(P, aP, bP, cP, Z) = 1 | Z \text{ is real}] - \Pr[\mathcal{B}(P, aP, bP, cP, Z) = 1 | Z \text{ is random}]| \quad (1)$$

where the probability is calculated over the random choices of $a, b, c \in \mathbb{Z}_p$ as well as the random bits used by \mathcal{B} . The quantity $\text{Adv}_{\mathcal{B}}^{\text{DBDH}}(t)$ denotes the maximum of $\text{Adv}_{\mathcal{B}}^{\text{DBDH}}$ where the maximum is taken over all adversaries \mathcal{B} running in time at most t . By the (ϵ, t) -DBDH assumption we mean $\text{Adv}_{\mathcal{B}}^{\text{DBDH}}(t) \leq \epsilon$.

2.5 Components (AE, KDF, UOWHF)

We briefly introduce and state the security notions for AE, KDF and UOWHF.

An AE protocol consists of two deterministic algorithms – **Encrypt** and **Decrypt**. Both of these use a common secret key k . The **Encrypt** $_k$ algorithm takes as input a nonce IV and a message M and returns (C, tag) , where C is the ciphertext corresponding to M (and is usually of the same length as M). The **Decrypt** $_k$ algorithm takes as input IV and a pair (C, tag) and returns either the message M or \perp (indicating invalid ciphertext).

An AE algorithm possesses two security properties – privacy and authenticity. For privacy, the adversarial game is the following. The adversary \mathcal{A} is given access to an oracle which is either the encryption oracle instantiated with a random key k or is an oracle which simply returns random strings of length equal to its input. After interacting with the oracle the adversary ultimately outputs a bit. The advantage of \mathcal{A} is defined to be

$$|\text{Prob}[\mathcal{A} = 1 | \text{real oracle}] - \text{Prob}[\mathcal{A} = 1 | \text{random oracle}]|.$$

In the above game, the adversary is assumed to be nonce-respecting, in that it does not repeat a nonce. The requirement that IV is a nonce can be replaced by the requirement that IV is chosen randomly. This leads to an additive quadratic degradation in the advantage.

The security notion defined above is that of pseudorandom permutation. This provides the privacy of an AE protocol. In particular, it implies the following notion of one-time security. The adversary submits two equal length messages M_0 and M_1 . A random (IV^*, k^*) pair is chosen and a random bit δ is chosen. The adversary is given (C^*, tag^*) which is the encryption of M_δ using IV^* and k^* . The adversary then outputs δ' and its advantage is

$$\left| \text{Prob}[\delta = \delta'] - \frac{1}{2} \right|.$$

We say that an AE protocol satisfies (ϵ, t) one-time encryption security if the maximum advantage of any adversary running in time t in the above game is ϵ .

The authenticity property of an AE protocol is defined through the following game. A nonce respecting adversary \mathcal{A} is given access to an encryption oracle instantiated by a secret key k . It submits messages to the oracle and receives as output ciphertext-tag pairs. Finally, it outputs a “new” ciphertext-tag pair and a nonce, which can be equal to a previous nonce. The advantage of \mathcal{A} in this game is the probability that the forgery is valid, i.e., it will be accepted as a valid ciphertext.

As before, we can replace the requirement that IV be a nonce by the requirement that IV is random without significant loss of security. By an (ϵ, t) -secure authentication of an AE protocol we mean that the maximum advantage of any adversary running in time t in the above game is ϵ .

A KDF is a function $\text{KDF}()$ which takes an input K and produces (IV, dk) as output. The security notion for KDF is the following. For a randomly chosen K , the adversary has to distinguish between $\text{KDF}(K)$ from a randomly chosen (IV, dk) .

A function family $\{H_k\}_{k \in \mathcal{K}}$ is said to be a universal one-way hash family if the following adversarial task is difficult. The adversary outputs an x ; is then given a randomly chosen $k \in \mathcal{K}$ and has to find $x' \neq x$ such that $H_k(x) = H_k(x')$. We say that the family is (ϵ, t) -secure if the maximum advantage (probability) of an adversary running in time t and winning the above game is ϵ .

3 CCA-Secure HIBE Protocol

In this section, we modify the CPA-secure HIBE protocol in [14] to obtain a CCA-secure HIBE protocol. We provide an explicit hybrid protocol. This allows us to improve the decryption efficiency as we explain later. The modification consists of certain additions to the set-up procedure as well as modifications of the encryption and the decryption algorithms. *No changes are required in the key generation algorithm.*

The additions are based on the technique used by Boyen-Mei-Waters [9] and are also based on the IBE construction by Boneh-Boyen [4] (BB-IBE). Some new ideas – incorporating length of the identity into the ciphertext and using symmetric key authentication to verify ciphertext well formedness – are introduced. Also, an AE protocol is used to combine the two tasks of symmetric key encryption and authentication.

A Useful Notation. Let $v = (v_1, \dots, v_l)$, where each v_i is an (n/l) -bit string (where l divides n) and is considered to be an element of $\mathbb{Z}_{2^{n/l}}$. For $1 \leq k \leq h$ we define,

$$V_k(v) = U'_k + \sum_{i=1}^l v_i U_i. \quad (2)$$

The modularity introduced by this notation allows an easier understanding of the protocol, since one does not need to bother about the exact value of l . When v is clear from the context, we will write V_k instead of $V_k(v)$.

Cost of Computing $V_k(v)$. This consists of computing the individual $v_i U_i$ s and then summing the l points. Each v_i is a bit string of length n/l . Consequently, the time for computing $V_k(v)$ is approximately equal to the time for computing a scalar multiplication of the form mP , where m is an n -bit string and P is a point on the curve.

In the protocol, we will be dealing with identities of the form $\mathbf{v} = (v_1, \dots, v_j)$ with $j \in \{1, \dots, h\}$, $\mathbf{v}_k = (v_1^{(k)}, \dots, v_l^{(k)})$ and $v_i^{(k)}$ is an (n/l) -bit string. In this context, $V_k(\mathbf{v}_k)$ is obtained by replacing v_k for v in (2).

3.1 Construction

The description of the construction is given in Figure 1 and the approximate costs of the different algorithms are given in Table 1. In these costs we do include symmetric encryption or authentication.

The following things should be noted while going through Figure 1.

1. Maximum depth of the HIBE is h .
2. Identities are of the form $\mathbf{v} = (v_1, \dots, v_j)$, $j \in \{1, \dots, h\}$, $\mathbf{v}_k = (v_1^{(k)}, \dots, v_l^{(k)})$ and $v_i^{(k)}$ is an (n/l) -bit string.
3. $\langle G_1, G_2, e \rangle$ is as defined in Section 2.3.
4. The notation $V_k()$ is given in (2).
5. The standard way to avoid the computation of $e(P_1, P_2)$ in HIBE.Encrypt is to replace P_2 with $e(P_1, P_2)$ in the public parameters.
6. Key generation is essentially the same as in [31,14].

The bold portions of Figure 1 provide the additional points required over the CPA-secure HIBE construction from [14]. We provide some intuition of how decryption queries are answered. (Key extraction can be answered using the technique from [14] which is built on the work of Waters [31].) First, let us consider what happens if we attempt to simulate decryption queries by key extraction queries. The idea is that we use a key extraction query to derive the private key of the identity which is provided as part of the decryption query. Then this private key is used to decrypt the ciphertext. This idea works fine except for the situation where a decryption query is made on a prefix of the challenge identity. Since, it is not allowed to query the key extraction oracle on

<p>HIBE.SetUp</p> <ol style="list-style-type: none"> 1. Choose α randomly from \mathbb{Z}_p. 2. Set $P_1 = \alpha P$. 3. Choose $P_2, U'_1, \dots, U'_h, U_1, \dots, U_l$ randomly from G_1. 4. Choose \mathbf{W} randomly from \mathbf{G}_1. 5. Let $\mathbf{H}_s : \{\mathbf{1}, \dots, \mathbf{h}\} \times \mathbf{G}_1 \rightarrow \mathbb{Z}_p$ be chosen from a UOWHF and made public. 6. Public parameters: $P, P_1, P_2, U'_1, \dots, U'_h, U_1, \dots, U_l$ and \mathbf{W}. 7. Master secret key: αP_2.
<p>HIBE.KeyGen: Identity $\mathbf{v} = (v_1, \dots, v_j)$.</p> <ol style="list-style-type: none"> 1. Choose r_1, \dots, r_j randomly from \mathbb{Z}_p. 2. $d_0 = \alpha P_2 + \sum_{k=1}^j r_k V_k(v_k)$. 3. $d_k = r_k P$ for $k = 1, \dots, j$. 4. Output $d_v = (d_0, d_1, \dots, d_j)$. <p>(Key delegation, i.e., generating d_v from $d_{v _{j-1}}$ can be done in the standard manner as shown in [31,14].)</p>
<p>HIBE.Encrypt: Identity $\mathbf{v} = (v_1, \dots, v_j)$; message M.</p> <ol style="list-style-type: none"> 1. Choose t randomly from \mathbb{Z}_p. 2. $C_1 = tP, B_1 = tV_1(v_1), \dots, B_j = tV_j(v_j)$. 3. $K = e(P_1, P_2)^t$. 4. $(IV, dk) = \text{KDF}(K)$. 5. $(\text{cpr}, \text{tag}) = \text{AE.Encrypt}_{dk}(IV, M)$. 6. $\gamma = \mathbf{H}_s(\mathbf{j}, \mathbf{C}_1); \mathbf{W}_\gamma = \mathbf{W} + \gamma \mathbf{P}_1; \mathbf{C}_2 = t\mathbf{W}_\gamma$. 7. Output $(C_1, \mathbf{C}_2, B_1, \dots, B_j, \text{cpr}, \text{tag})$.
<p>HIBE.Decrypt: Identity $\mathbf{v} = (v_1, \dots, v_j)$; ciphertext $(C_1, \mathbf{C}_2, B_1, \dots, B_j, \text{cpr}, \text{tag})$; decryption key $d_v = (d_0, d_1, \dots, d_j)$.</p> <ol style="list-style-type: none"> 1. $\gamma = \mathbf{H}_s(\mathbf{j}, \mathbf{C}_1); \mathbf{W}_\gamma = \mathbf{W} + \gamma \mathbf{P}_1$. 2. If $e(\mathbf{C}_1, \mathbf{W}_\gamma) \neq e(P, \mathbf{C}_2)$ return \perp. 3. $K = e(d_0, C_1) / \prod_{k=1}^j e(B_k, d_k)$. 4. $(IV, dk) = \text{KDF}(K)$. 5. $M = \text{AE.Decrypt}_{dk}(IV, C, \text{tag})$. (This may abort and return \perp). 6. Output M.

Fig. 1. CCA-secure HIBE

Table 1. Cost of different operations. The variable j refers to the number of components in the input identity tuple. Here $1 \leq j \leq h$, where h is the maximum depth of the HIBE. Cost of symmetric key operations are not shown. [SM]: cost of one scalar multiplication in G_1 ; [P]: cost of one pairing operation; [VP]: cost of one pairing verification of the type $e(Q_1, Q_2) = e(R_1, R_2)$; [e]: cost of one exponentiation in G_2 ; [i]: cost of inversion in G_2 .

No. of public parameters	$(3 + h + l)$ elements of G_1 and one element of G_2
Secret key size	$j + 1$ elements of G_1
Cost of key generation	$3j$ [SM]
Cost of encryption	$(2j + 3)$ [SM]+1[e]
Cost of decryption	1 [SM]+1[VP]+ $(j + 1)$ [P]+1[i]

prefixes of the challenge identity, the above simulation technique will not work. We need an additional mechanism to answer such decryption queries.

The mechanism that we have used is primarily based on the BMW technique. The parameter W along with P and P_1 define an instance of a BB-IBE protocol. During encryption, an ‘‘identity’’ $\gamma = H_s(j, C_1)$ for this protocol is generated from the randomizer $C_1 = tP$ and the length j of the identity tuple. Using this identity, a separate encapsulation of the key $e(P_1, P_2)^t$ is made. This encapsulation consists of the element C_2 (and C_1). In the security proof, if a decryption query is made on the challenge identity, then this encapsulation is used to obtain the private key of γ and answer the decryption query.

The use of the function $H()$ is different from its use in [9]. In [9], the function $H()$ maps G_1 to \mathbb{Z}_p . On the other hand, in the HIBE protocol in Figure 1, $H()$ maps $\{1, \dots, h\} \times G_1$ to \mathbb{Z}_p . Our aim is to include information about the length of the identity into the output of $H()$. Without this information, an encryption for a $(j + 1)$ -level identity can be converted to an encryption for its j -level prefix by simply dropping the term corresponding to the last component in the identity. (This was pointed out by a reviewer of an earlier version of this work, who, however, did not provide the solution described here.)

The other aspect is that of checking for the well formedness of the ciphertext. A well formed ciphertext requires verifying that $C_1 = tP$, $C_2 = tW_\gamma$ and $B_1 = tV_1(v_1), \dots, B_j = tV_j(v_j)$. In other words, we need to verify the following.

$$\log_P C_1 = \log_{W_\gamma} C_2 \text{ and } \log_P C_1 = \log_{V_1(v_1)} B_1 = \dots = \log_{V_j(v_j)} B_j.$$

In Figure 1, the first equality is explicitly verified, whereas the second equality is not. The idea is that if the second equality does not hold, then the key K that will be reconstructed will be improper and indistinguishable from random (to the adversary). Correspondingly, the quantities (IV, dk) will also be indistinguishable from random and symmetric authentication with this pair will fail (otherwise the adversary has broken the authentication of the AE protocol). Thus, instead of using j pairings for verifying the second equality, we use symmetric authentication to reject invalid ciphertext. This leads to a more efficient decryption algorithm. Note that the use of hybrid encryption is very crucial in

the current context. This is similar to the Kurosawa-Desmedt PKE, which provides improved efficiency over the Cramer-Shoup protocol for hybrid encryption.

The additional requirements of group elements and operations for attaining CCA-security compared to the protocol in [14] consists of the following.

1. One extra group element in the public parameters.
2. Two additional scalar multiplications during encryption.
3. One additional scalar multiplication and one pairing based verification during decryption.

3.2 Security Statement

The security statement for the new protocol is given below.

Theorem 1. *The HIBE protocol described in Figure 1 is $(\epsilon_{hibe}, t, q_{ID}, q_C)$ -CCA secure assuming that the (t', ϵ_{dbdh}) -DBDH assumption holds in $\langle G_1, G_2, e \rangle$; H_s is an (ϵ_{uowhf}, t) -UOWHF; KDF is (ϵ_{kdf}, t) -secure; and the AE protocol possesses (ϵ_{auth}, t) -authorization security and (ϵ_{enc}, t) one-time encryption security; where*

$$\epsilon_{hibe} \leq 2\epsilon_{uowhf} + \frac{\epsilon_{dbdh}}{\lambda} + 4\epsilon_{kdf} + 2\epsilon_{enc} + 2hq_C\epsilon_{auth}. \quad (3)$$

where $t' = t + O(\tau q) + \chi(\epsilon_{hibe})$ and

$$\begin{aligned} \chi(\epsilon) &= O(\tau q + O(\epsilon^{-2} \ln(\epsilon^{-1}) \lambda^{-1} \ln(\lambda^{-1}))); \\ \tau &\text{ is the time required for one scalar multiplication in } G_1; \\ \lambda &= 1/(2h(2\sigma(\mu_l + 1))^h) \text{ with } \mu_l = l(2^{n/l} - 1), \sigma = \max(2q, 2^{n/l}) \text{ and} \\ q &= q_{ID} + q_C. \end{aligned}$$

We further assume $2\sigma(1 + \mu_l) < p$.

The proof can be found in the expanded version of this paper [28]. The statement of Theorem 1 is almost the same as that of Theorem 1 in [14] with the following differences.

1. The above theorem states CCA-security where as [14] proves CPA-security.
2. The value of λ is equal to $1/(2h(2\sigma(\mu_l + 1))^h)$ in the above statement where as it is equal to $1/(2(2\sigma(\mu_l + 1))^h)$ in [14], i.e., there is an additional degradation by a factor of h .
3. The value of q in the expression for σ is the sum of q_{ID} and q_C whereas in [14] it is only q_{ID} . The reason for having q_C as part of q is that it may be required to simulate decryption queries using key extraction queries.

For $2q \geq 2^{n/l}$ (typically l would be chosen to ensure this), we have

$$\epsilon_{hibe} \leq 2\epsilon_{uowhf} + 2h(4lq2^{n/l})^h \epsilon_{dbdh} + 4\epsilon_{kdf} + 2\epsilon_{enc} + 2hq_C\epsilon_{auth}.$$

The corresponding bound on ϵ_{dbdh} in [14] is $2(4lq_{ID}2^{n/l})^h \epsilon_{dbdh}$. Thus, we get an additional security degradation of ϵ_{dbdh} by a factor of h while attaining CCA-security. Since h is the maximum number of levels in the HIBE, its value is small

and the degradation is not significant. Also, q in the present case includes both key extraction and decryption queries.

The statement of Theorem 1 is a little complicated. The complexity is inherited from the corresponding security statement in [14]. These arise from the requirement of tackling key extraction queries and providing challenge ciphertexts. In particular, λ is a lower bound on the probability of not abort by the simulator and $O(\epsilon^{-2} \ln(\epsilon^{-1}) \lambda^{-1} \ln(\lambda^{-1}))$ is the extra runtime introduced due to the artificial abort requirement. In [14], the security degradation is worked out in more details and much of these also hold for Theorem 1. Hence, we do not repeat the analysis in this paper.

The technique for showing security against chosen plaintext attacks is taken from [14] and is based on the works of Waters [31] and Boneh-Boyen [4]. Since these details are already given in [14], we do not repeat them in the proof of Theorem 1. The proof technique for answering decryption queries is based on the work of Boyen-Mei-Waters [9]. Also relevant is the work of Kiltz-Galindo [24]. The basic idea of using symmetric authentication to verify ciphertext well formedness is taken from the paper by Kurosawa-Desmedt [25]. A proof of the KD protocol using the so called method of “deferred analysis” is given in [1]. This proof is in the PKE setting which we had to adapt to fit the (H)IBE framework.

4 Comparison to Previous Work

The construction in Figure 1 can be specialized to obtain CCA-secure PKE and IBE as special cases. We show that when specialized to PKE, the protocol in Figure 1 simplifies to yield the BMW construction. On the other hand, when specialized to IBE, we obtain a more efficient (actually the decryption algorithm is more efficient) IBE protocol compared to the previously best known construction of Kiltz-Galindo [24].

Public Key Encryption. In this case there are no identities and no PKG. It is possible to make the following simplifications.

SetUp:

1. The elements $U'_1, \dots, U'_h, U_1, \dots, U_l$ are no longer required.
2. The UOWHF H_s can be replaced by an injective embedding from G_1 to \mathbb{Z}_p .
3. A random w in \mathbb{Z}_p is chosen and W is set to be equal to wP .
4. The secret key is set to be equal to $(\alpha P_2, \alpha, w)$.
5. The AE protocol can be replaced with a one-time secure data encapsulation mechanism (DEM).

KeyGen: This is not required at all.

Encrypt:

1. The elements B_1, \dots, B_j are not required.
2. Encryption with a DEM will not produce a tag.

Decrypt:

1. The purpose of the pairing verification $e(C_1, W_\gamma) = e(P, C_2)$ is to ensure that $C_1 = tP$ and $C_2 = tW_\gamma$, where $W_\gamma = W + \gamma P_1$. With the knowledge of w and α , this can be done as follows. Compute $w' = w + \gamma\alpha$ and verify whether $w'C_1 = C_2$. This requires only one scalar multiplication as opposed to one pairing verification.
2. The value of K is reconstructed as $K = e(C_1, \alpha P_2)$.
3. Since the AE protocol is replaced with a DEM, symmetric authentication will not be done.

With these simplifications, the protocol becomes the BMW protocol.

Identity Based Encryption. In this case $h = 1$. The protocol in Figure 1 remains unchanged except for one simplification. In a HIBE, the length of the identity tuple can vary from 1 to h . For an IBE, the length is always one. Hence, in this case, we can restrict the domain of H_s to be G_1 . Since, G_1 has cardinality p , the domain and range of H_s are the same and we can also take H_s to be an injective embedding from G_1 to \mathbb{Z}_p as has been done in the BMW construction.

Let us now compare the resulting IBE construction with the previous construction of Kiltz-Galindo [24]. In both cases, the public key portion of the ciphertext is of the form (C_1, C_2, B_1) . During decryption, KG protocol verifies that $C_1 = tP$, $C_2 = tW_\gamma$ and $B_1 = tV_1(v_1)$. This requires two pairing based verifications of the type $e(P, C_2) = e(C_1, W_\gamma)$ and $e(P, B_1) = e(C_1, V_1(v_1))$. The cost of one such verification is less than the cost of two pairing operations. Recall from Table 1 that by $[VP]$ we denote the cost of one such verification. Also, let $[P]$, $[SM]$, and $[i]$ respectively denote the costs of one pairing operation, one scalar multiplication in G_1 , and one inversion in G_2 . The total cost of decryption in the KG protocol with the pairing based verification technique is $1[SM] + 2[VP] + 2[P] + 1[i]$.

Implicit Rejection. KG [24] suggests a method of implicit rejection. This provides a KEM which cannot explicitly reject a ciphertext. More precisely, the notion of KEM used by KG [24] is the following. In the adversarial game, the adversary queries the decryption oracle. If the query is valid, then the adversary gets the corresponding secret key, while if the query is invalid, then the adversary gets a random value for the secret key. In particular, the adversary is not told whether the decryption failed.

First, we would like to point out that this is a restricted notion of KEM. The original notion of KEM as conceived by Shoup [30] allows the simulator to inform the adversary whether the decryption failed. We quote from [30, Page 15, Lines 5–6] (the bold font appears in the cited reference).

“if the decryption algorithm **fails**, then this information is given to the adversary”

In view of this, we consider the notion of KEM used by KG to be restricted-KEM. Apart from the difference mentioned above, such a restricted-KEM is not really

sufficient for constructing a complete encryption protocol. When combined with a one-time secure DEM (as envisaged by Shoup [30] and later used by many authors), a restricted-KEM provides an encryption protocol which *cannot* reject invalid ciphertexts. Clearly, such an encryption protocol is also more restricted compared to the currently accepted notion. (On the other hand, we do note that the notion of restricted-KEM may be sufficient for some applications.)

In the identity based setting, KG [24] suggests a method of implicit rejection leading to a restricted-KEM. The idea is the following. The pairing based verifications are not done; instead two random elements r_1 and r_2 are chosen and K is computed as

$$\frac{e(C_1, d_0 + r_1 W_\gamma + r_2 V_1(v_1))}{e(B_1, d_1 + r_2 P)e(r_1 P, C_2)}.$$

If the ciphertext is proper, then the proper K is computed, while if the ciphertext is improper, then a random K is computed. *Note that an invalid ciphertext is not explicitly rejected and combining such a KEM with a one-time secure DEM will result in a IBE which cannot reject invalid ciphertexts.* The cost of decryption with implicit rejection is $5[SM] + 3[P] + 1[i]$.

In contrast, the cost of verification in our case is $1[SM] + 1[VP] + 2[P] + 1[i]$. The costs of decryption using our algorithm and also that of KG algorithm (for both explicit and implicit rejections) are shown in Table 2. Clearly, the cost of decryption algorithm given in this work is significantly lower than the KG protocol with explicit pairing based verification. Compared to the implicit rejection technique, our cost will be lower when $1[VP] < 1[P] + 4[SM]$. Based on the current status of efficient pairing based algorithms, this seems to be a reasonable condition.

The reason for obtaining this lower cost is that we do not verify $e(P, B_1) = e(C_1, V_1(v_1))$ either explicitly or implicitly. In other words, we do not verify whether $\log_P C_1 = \log_{V_1(v_1)} B_1$. If this does not hold, then an incorrect session key will be generated and ultimately the authentication of the AE protocol will fail. In a sense, this is also an implicit verification, but the verification is done using the symmetric component which reduces the total cost of decryption. Also, an invalid ciphertext will always be rejected.

In summary, the IBE version of the protocol in Figure 1 is the currently known most efficient CCA-secure IBE protocol in the full model without the random oracle heuristic and based on the DBDH assumption.

Hierarchical Identity Based Encryption. Based on the work by BMW [9], the KG paper [24] sketches a construction of a HIBE. The details are worked out in [3]. Compared to this approach, there are several advantages of our protocol. First, the ciphertext verification procedure in this approach requires the verification of $\log_P C_1 = \log_{V_1(v_1)} B_1 = \dots = \log_{V_j(v_j)} B_j$ either explicitly using pairing based verifications or implicitly (but, without being able to reject invalid ciphertexts) as suggested by Kiltz-Galindo. On the other hand, our approach does not require these verifications. If any of these equalities do not hold, then an improper value of K will be obtained and as a result the authentication of the AE protocol will fail. This significantly reduces the cost of the decryption

Table 2. Comparison of decryption algorithms of KG-IBE with our algorithm

Protocol	Decryption Cost	Reject Invalid Ciphertexts
KG (explicit rej.)	$1[\text{SM}] + 2[\text{VP}] + 2[\text{P}] + 1[\text{i}]$	Yes
KG (implicit rej.)	$5[\text{SM}] + 3[\text{P}] + 1[\text{i}]$	No
This work	$1[\text{SM}] + 1[\text{VP}] + 2[\text{P}] + 1[\text{i}]$	Yes

Table 3. Comparison of decryption algorithms of KG-HIBE with our algorithm. The quantity j below refers to the number of components in the identity tuple. Here $1 \leq j \leq h$, where h is the maximum depth of the HIBE.

Protocol	Decryption Cost	Reject Invalid Ciphertexts
KG (explicit rej.)	$1[\text{SM}] + (j + 1)[\text{VP}] + (j + 1)[\text{P}] + 1[\text{i}]$	Yes
KG (implicit rej.)	$(2j + 1)[\text{SM}] + (j + 2)[\text{P}] + 1[\text{i}]$	No
This work	$1[\text{SM}] + 1[\text{VP}] + (j + 1)[\text{P}] + 1[\text{i}]$	Yes

algorithm. Second, we use an AE algorithm to perform simultaneous encryption and authentication which can be twice as fast as separate encryption and authentication. Table 3 shows the costs of decryption algorithms for our method and that of the KG method with explicit and implicit rejection. As mentioned earlier, due to the security degradation being exponential in h , the value of j has to be small, at most around 4. Even for small values of j , the cost of the new decryption algorithm is smaller than that of the KG-HIBE.

An earlier work [11,7] showed a generic construction for converting an $(h + 1)$ -level CPA-secure HIBE into an h -level CCA-secure HIBE. The construction used one-time signatures, which make it quite inefficient. It was suggested (without details) in [11] that a MAC based construction can be used to remove the inefficiency of the one-time signature based approach. Also, the efficiency of the resulting protocol is less than that of Figure 1. A problem with the approach in [11] is that the identity components of the CCA-secure HIBE are prepended with a bit to obtain identity components of the underlying CPA-secure HIBE. This can cause difficulties in implementation. Typically, the n -bit identity will be obtained by hashing an arbitrary length string such as an email address. Suppose, $n = 160$. Hashing gives us a 160-bit identity for the underlying CPA-secure HIBE. Then the length of the identity string for the CCA-secure HIBE is 161. This value of length will not align with byte boundaries and will cause implementation difficulties.

The currently known techniques (both generic and non-generic) for converting a CPA-secure HIBE protocol to a CCA-secure HIBE protocol, starts with an $(h + 1)$ -level CPA-secure HIBE and then converts it to an h -level CCA-secure HIBE. The security degradation thus correspond to the $(h + 1)$ -level HIBE. If we apply this technique to the protocol in [14], then the security degradation for the obtained h -level CCA-secure HIBE will be $2(4lq2^{n/l})^{h+1}$. Compared to this, the security degradation given by Theorem 1 is $2h(4lq2^{n/l})^h$. In other words, we have managed to reduce the exponent from $(h + 1)$ to h and have introduced

a multiplicative factor of h . From the viewpoint of concrete security analysis, a typical value of q is 2^{30} . Assuming this value of q , we are able to prevent approximately a 30-bit security degradation compared to previous work.

5 Conclusion

In this paper, we have provided a construction of a hybrid HIBE protocol. The protocol is secure against adaptive adversaries (making both key extraction and decryption queries) without using the random oracle hypothesis. Security is reduced from the computational hardness of the DBDH problem. To the best of our knowledge, in this setting, the HIBE protocol described in this paper is the currently known most efficient construction.

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