Comparative Study of Approximate Strategies for Playing Sum Games Based on Subgame Types

Cherif R.S. Andraos, Manal M. Zaky, and Salma A. Ghoneim

Computer and Systems Engineering Department,
Faculty of Engineering, Ain Shams University, Cairo, Egypt
cherif.andraos@ieee.org, manalmourad@yahoo.com, salma_ghoneim@hotmail.com

Abstract. Combinatorial games of the form {{A|B}|{C|D}} can be classified as either left excitable, right excitable, or equitable [2]. Several approximate strategies for playing sums of games of this form have been proposed in the literature [2,3,4]. In this work we propose a new approach for evaluating the different strategies based on the types of the subgames participating in a sum game. While previous comparisons [3,4] were only able to rank the strategies according to their average performance in a large number of randomly generated games, our evaluation is able to pinpoint the strengths and weaknesses of each strategy. We show that none of the strategies can be considered the best in an absolute sense. Therefore we recommend the development of type-based approximate strategies with enhanced performance.

1 Introduction

In [3], several strategies for playing sum games based on combinatorial game theory (cgt) were suggested, namely *BMove*, *MaxMove*, *Sente*, and *SenteQ*. Each of these strategies was compared to the others and to *HotStrat*, *ThermoStrat* [2] and *MiniMax* by allowing them to play against each other a large number of games where each game consisted of a sum of five randomly generated subgames of the form {{A|B}| {C|D}} with D = rnd(50), C = D + rnd(50), B = C + rnd(50) and A = B + rnd(50), where rnd(N) is a function generating a random integer uniformly distributed in the interval [0,N]. The score of each strategy was calculated as the sum of its scores in the different tournaments.

HotStrat was shown to give the best performance [3] according to the evaluation method suggested by the author. Its score even surpassed that of optimal MiniMax search. This is due to the fact that the score used for comparing the strategies had been obtained by playing against other strategies, most of which are not optimal. This is not consistent with the assumption of a perfect opponent used in most game-playing algorithms. From our point of view a more objective test should be used to provide more reliable results.

In [4] another strategy, *HotStrat+*, was proposed for a more generalized game model. As the name implies, this strategy is an enhanced version of HotStrat.

H.J. van den Herik et al. (Eds.): CG 2006, LNCS 4630, pp. 212–219, 2007.

The authors used a more objective criterion for evaluating the new strategy based on the total number of points lost against a perfect opponent in a large number of randomly generated games of varying complexity. It was shown that HotStrat+ performs slightly but consistently better than HotStrat.

We have the following two reservations against the above results.

1. Combinatorial games of the form $\{\{A|B\}|\ \{C|D\}\}$ can be classified [2] as left excitable (sente for left), right excitable (sente for right) or equitable (double sente or gote according to ambient temperature [5]):
   - Left excitable (type 1) if $S > 4B$,
   - Equitable (type 2) if $4B \geq S \geq 4C$,
   - Right excitable (type 3) if $S < 4C$,

   where $S = A+B+C+D$. Skilful human players take into account the types of subgames in selecting the best move in a sum game. For example, in a Go endgame, double sente subgames are played immediately while one-sided sente allows a player to control the course of the game so as to execute a plan of several moves to his[1] advantage. HotStrat+ takes sente games into account in selecting the next move. SenteStrat and SenteQ give special consideration also to double sente games. Thermostrat implicitly takes subgame types into consideration in the process of decision making because of the more complete information available to it, while HotStrat, BStrat, and MaxMove are independent of subgame types. Consequently, the performance of these strategies must depend upon the types of the subgames participating in the sum.
2. In a given sum game, the number of subgames belonging to each type varies. The game model used in the comparison process should be able to produce sum games consisting of the most probable subgames types in real games. We computed the probability density function for sum games of all different type combinations of five subgames by generating a large number of sum games. We found that the game model proposed in [3] yields a sum game with subgames that are all equitable in 43% of the cases, four equitable and one excitable in about 40%, three equitable and two excitable in about 11% with the remaining 5% to cover all the remaining cases. In view of these results, it is obvious that the model used cannot objectively evaluate the performance of the approximate strategies.

No one, as far as we know, has ever considered the effect of subgame types on the performance of different strategies. In this work we investigate how the behavior of the proposed strategies varies when playing sums of games with imposed type patterns.

2 Subgame Types Effect

Experiments were conducted to investigate the effect of the types of subgames upon the performance of HotStrat+, HotStrat, ThermoStrat, MaxMove, Sente,

[1] For brevity and readability, we use 'he' and 'his' wherever 'he or she' and 'his' or 'her' are meant.

SenteQ, and BMove. All the experiments were conducted on sums composed of 4 subgames. The game generator was modified to produce subgames with specified type patterns as follows. Given a list of the required types of the subgames, for each type a random game is generated according to the model described above and has its type checked. If the type matches the desired value, the game is accepted; otherwise the process is repeated until the condition is satisfied.

We also used an alternate method for strategy evaluation. It consists of comparing the move selected by each strategy with the move(s) selected by MiniMax search for a large number of random sums. The performance measure for the strategy is then calculated as the percentage of sums in which it selects the same move (or one of the moves) selected by the optimal strategy. We call this the percentage coincidence. Besides being an objective measure for comparing the strategies, this approach is consistent with the assumption of a perfect opponent used by all game-playing programs. Both this approach and the approach used in [4] are complementary in the sense that the former provides a measure of the number of errors of a certain strategy when compared to the optimal, while the latter provides a measure of the error magnitude. Table 1(a) lists the results for the strategies discussed in [3] using the proposed evaluation method. To arrive at these results we generated 1000 random sums (without forcing any type patterns) and computed the percentage coincidence for each strategy. The results obtained in [3] are shown in Table 1(b) for easy reference.

Table 1. Strategies Performance

(a) Percentage Coincidence

Strategy	% of Optimal
BMove	58.70
SenteQ	64.70
MaxMove	66.80
Sente	72.30
MaxThreat	87.70
ThermoStrat	89.70
HotStrat	91.50
HotStrat+	91.60
Optimal	100.00

(b) Results from [3]

Strategy	% of Optimal
BMove	88.39
MaxMove	95.40
SenteQ	97.63
Sente	97.71
MaxThreat	99.17
ThermoStrat	99.60
Optimal	100.00
HotStrat	100.33

3 Experimental Results

3.1 Objective

Below we study the performance of HotStrat+, HotStrat, ThermoStrat, MaxMove, Sente, SenteQ, and BMove relative to MiniMax for all possible subgames type patterns and players.

3.2 Experimental Setup

The test was performed using sums of 4 random subgames. Since the type of each subgame can take one of the three values 1, 2 or 3, there are exactly 15 possible type combinations. A number of 10,000 sum games of each type combination were generated and the moves selected by the different strategies were compared with the optimal move(s) for the case when it was left's turn to play. The whole process was then repeated for the case when it was right's turn to play. The percentage coincidence of the selected move by each strategy with the one(s) selected by MiniMax was recorded for each type combination, for each player.

3.3 Observations

Table 2 summarizes the obtained results when it is left's turn to start. In these results each type is represented by a number from 1 to 3 where 1 denotes left excitable, 2 equitable, and 3 right excitable games. For example, the pattern "1,1,2,3" represents a sum containing two left excitable games, one equitable, and one right excitable. The same notation is used in the remaining part of this document whenever we want to describe the subgame pattern of a certain sum. Almost equal results were obtained for right and are shown in Table 3.

Table 2. Percentage coincidence of different strategies for 10,000 sums of each type combination when left starts

Pattern	BMove	Sente	SenteQ	Max-Move	Hot-Strat	Hot-Strat+	Thermo-Strat	Optimal
1,1,1,1	92.77	92.92	94.76	89.70	**100**	**100**	99.29	100
2,2,3,3	54.93	83.29	62.96	79.91	**91.74**	**91.74**	89.36	100
1,2,2,3	60.39	82.52	64.74	77.96	**91.72**	**92.14**	90.60	100
1,1,1,2	80.48	84.27	72.90	79.68	**91.01**	**94.25**	89.20	100
1,1,2,2	71.27	81.76	66.66	77.22	**90.67**	**91.60**	90.34	100
1,1,2,3	61.79	84.33	68.90	81.08	**90.36**	**92.12**	88.39	100
2,3,3,3	47.96	87.95	59.91	**91.14**	90.28	90.28	**92.65**	100
1,2,3,3	52.26	85.78	64.17	85.05	**90.13**	**90.64**	88.69	100
2,2,2,3	57.03	76.88	65.43	70.70	**90.13**	**90.13**	88.80	100
1,2,2,2	63.88	76.19	65.76	69.37	**89.94**	**90.06**	89.06	100
2,2,2,2	57.41	70.72	65.93	63.66	**89.25**	**89.25**	88.69	100
1,1,1,3	57.97	77.96	74.40	72.89	**86.89**	77.20	**88.40**	100
1,1,3,3	46.66	73.65	64.30	67.48	**80.23**	65.52	**82.33**	100
1,3,3,3	45.03	**74.08**	55.62	72.90	72.54	61.94	**80.55**	100
3,3,3,3	48.60	79.21	48.76	**93.72**	60.59	60.59	**93.72**	100
Average	59.90	80.77	66.35	78.16	**87.03**	85.16	**89.34**	100
Std. dev.	13.15	**5.99**	10.04	8.94	9.45	12.53	**4.32**	0

Results were sorted in descending order of HotStrat performance. For each type combination, the best and second best strategies are marked with bold.

Table 3. Percentage coincidence of different strategies for 10,000 sums of each type combination when right starts

Pattern	BMove	Sente	SenteQ	Max-Move	Hot-Strat	Hot-Strat+	Thermo-Strat	Optimal
3,3,3,3	63.69	93.09	94.92	90.21	**100**	**100**	99.47	100
2,3,3,3	39.19	85.02	72.90	80.15	**91.36**	**94.55**	89.62	100
1,1,2,2	12.67	82.41	63.46	78.35	**91.32**	**91.32**	88.27	100
1,2,2,3	17.75	82.69	65.14	77.88	**90.98**	**91.72**	90.45	100
2,2,3,3	30.26	82.17	66.25	78.25	90.63	**91.66**	**91.01**	100
1,2,2,2	14.33	76.47	64.61	70.06	**90.47**	**90.47**	88.91	100
2,2,2,3	23.16	76.66	66.48	70.88	**90.44**	**90.54**	89.73	100
1,2,3,3	22.70	84.55	67.44	82.13	**90.37**	**92.29**	88.76	100
1,1,1,2	14.23	87.31	59.75	**91.23**	89.71	89.71	**92.41**	100
1,1,2,3	16.53	85.32	63.75	84.65	**89.42**	**90.04**	89.09	100
2,2,2,2	16.67	71.02	66.44	63.43	**89.13**	**89.13**	88.59	100
1,3,3,3	30.13	77.27	74.24	71.73	**87.80**	78.98	**86.65**	100
1,1,3,3	24.64	73.89	63.76	66.95	**81.09**	67.80	**82.04**	100
1,1,1,3	27.18	**74.24**	54.97	73.36	71.92	63.01	**79.33**	100
1,1,1,1	30.52	78.91	49.24	**93.54**	61.55	61.55	**93.54**	100
Average	25.58	80.73	66.22	78.19	**87.08**	85.52	**89.19**	100
Std. dev.	13.00	**5.93**	10.05	9.02	9.26	11.91	**4.60**	0

Figures 1 and 2 illustrate the results of Tables 2 and 3 respectively. To make visualization easier, the points in the figures are connected by line segments. Below we formulate eight observations.

1. As expected, the subgame types as well as the player affect the performance of the different strategies.
2. The high performance achieved by all the strategies for the combination “1,1,1,1” when it is left’s turn to move and the symmetric case of “3,3,3,3” for right should not be understood as an improvement in their performance. The reason behind the high scores is that when all subgames are of type 1 (left excitable) and it is left’s turn to move, left can select any subgame since the best choice for right will most probably be to respond in the same subgame (since it is left excitable) leaving the B value of that game with left free to choose the next subgame. This will be repeated for all subgames ending in a final value which is the sum of the Bs of all subgames regardless of which game was selected first by left. In fact, in these specific cases MiniMax returns a set of optimal moves instead of a single one. This causes the probability that a move selected by any strategy will coincide with optimal play to be very high, resulting in a high coincidence number for all strategies.
3. ThermoStrat achieves results that are consistently very close to best. It also has the highest average performance. Note however that this is not a true average since the probability of occurrence of each type combination is not the same. A more realistic measure of average performance would be a weighted average of the values listed.

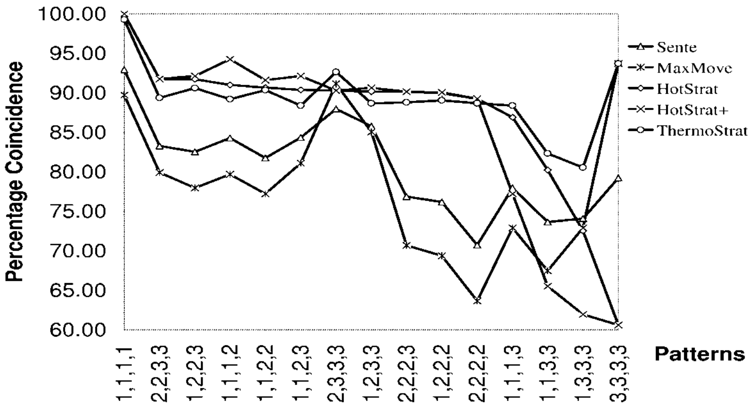

Fig. 1. Percentage coincidence of different strategies for 10,000 sums of each type combination when left starts

4. ThermoStrat's reliability is evident from the standard deviation of its results. It has the smallest standard deviation among all strategies. However, the higher standard deviation associated with the results of HotStrat and HotStrat+ show that they are less reliable than ThermoStrat. This was expected since ThermoStrat implicitly makes use of the complete information about the types of the subgames in the process of move selection.
5. HotStrat and HotStrat+ are sensitive to subgame type patterns. As seen from Tables 2 and 3 their performance varies for from 60.59% to 100%. They achieve the lowest performance for patterns that consist of only type-1 and type-3 subgames or type 3 only.
6. Tables 2 and 3 show that contrary to the results given in [4] the performance of HotStrat+ is not always better than HotStrat. We notice the following.
   - Their results are identical for patterns that do not contain type 1 subgames when it is left's turn to play and for those that do not contain type 3 subgames when it is right's turn.
   - HotStrat performance is much better than HotStrat+ for patterns that contain only type-1 and type-3 subgames where the difference varies between 8.8% & 14.7%.
   - For the rest of the patterns HotStrat+ performs slightly better than HotStrat.
   - Notice that there is no conflict between these results and the result in [4]. As mentioned above, the game generator used in both [3] and [4] is almost incapable of producing the patterns in which HotStrat's performance is better than that of HotStrat+, namely "1,3,3,3", "1,1,3,3", and "1,1,1,3". The total probability of producing these patterns has been computed and found to be equal to 0.05%.

– It is interesting to compare the average percentage coincidence for both HotStrat and HotStrat+ using our results and the result in [4], using the following formula:

$$(Ca)_s = \sum_i P_i.(Ci)_s \tag{1}$$

where **i** $\in$ [1,15] is the pattern number, $(\mathrm{Ci})_s$ is the percentage coincidence for strategy **s** and pattern **i** as given in Table 2. P_i is the probability of the occurrence of pattern **i** and $(\mathrm{Ca})_s$ is the average percentage coincidence for Strategy **s.** Using (1) we get $(\mathrm{Ca})_{HotStrat} = 89.8\%$ and $(\mathrm{Ca})_{HotStrat+} = 89.9\%$, which is consistent with the results in [4] that shows that on the average HotStrat+ is slightly better than HotStrat.

7. An unexpected and very interesting result is that MaxMove, which is a very simple strategy, achieves best performance in the two oppositely symmetric cases: the case where it is left's turn to move in a sum game containing only right excitable (type 3) subgames (93.72%) and the case where it is right's turn to move in a sum of only left excitable (type 1) subgames (93.54%). Its results for these cases are equal to ThermoStrat's. We have developed a mathematical justification of this behavior [1].
8. For the same two cases in 7, HotStrat and HotStrat+ show a considerable degradation in performance.

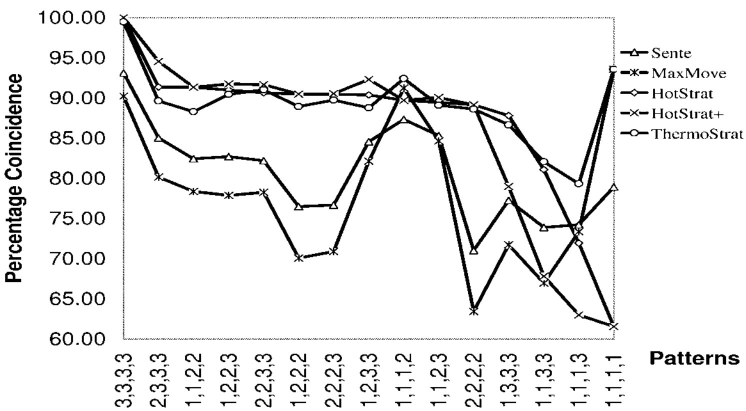

Fig. 2. Percentage coincidence of different strategies for 10,000 sums of each type combination when right starts

4 Conclusions and Future Work

We have shown that the game generator proposed in [3] and generalized in [4] is unable to produce sum games with games of different type patterns with equal probability. In fact, it is almost unable to produce certain patterns, resulting in a biased experimental setup and unreliable results. We proposed a modified version of the game generator that produces subgames with imposed type patterns. The results from our model and those reported in [3,4] agree if we compare the average performance computed as a probabilistic weighted average of the coincidence to previous ones. It would be very interesting to investigate the behavior of the strategies considered using our type-based game generator with different parameters such as the number of games or the interval used for the generation of random games. It would also be very interesting to test these strategies on values extracted from real games or on generators capable of producing realistic sum games with higher probability to sum games that are most likely to occur in real games. More experiments could be executed to compute the error magnitude of each strategy on typed subgames instead of calculating the percentage coincidence only.

We were able to prove for the first time that the performance of approximate strategies for playing sums of combinatorial games is highly dependent on the types of subgames. This dependence is minimumal in the case of ThermoStrat because of its *awareness* of the game types. This was proved experimentally by the low standard deviation in the percentage coincidence for all patterns in case of ThermoStrat. We have also shown the weakness of HotStrat and HotStrat+ in dealing with patterns that contain reverse sente subgames alone or when combined with only sente games. Our experiments were able to determine the patterns where HotStrat+ is better than HotStrat and vice versa. A very significant result is that the MaxMove strategy whose average performance is very low was shown to give the same result as ThermoStrat for the pattern with only reverse sente games. This work could be the basis for the development of type-based approximate strategies with enhanced performance.

References

1. Andraos, C.R.S.: Development of an Integrated Game Strategy Based on Classical AI Game Analysis Techniques and Combinatorial Game Theory. M.Sc. Thesis, Ain Shams University, Cairo, Egypt (2006)
2. Berlekamp, E.R., Conway, J.H., Guy, R.K.: Winning Ways. Academic Press, London, 1982. Revised version published by A.K. Peters, Natlick, MA, USA (2001–2004)
3. Cazenave, T.: Comparative evaluation of strategies based on the values of direct threats. Board Games in Academia V, Barcelona (2002), http://www.ai.univ-paris8.fr/~cazenave/ts.ps
4. Müller, M., Li, Z.: Locally Informed Global Search for Sums of Combinatorial Games. In: van den Herik, H.J., Björnsson, Y., Netanyahu, N.S. (eds.) CG 2004. LNCS, vol. 3846, pp. 273–284. Springer, Heidelberg (2006)
5. Spight, W.: Go Thermography - the 4/21/98 Jiang-Rui Endgame. In: Nowakowski, R. (ed.) More Games of No Chance, pp. 89–105. Cambridge University Press, Cambridge (2002)