Approximation Algorithms for Geometric Intersection Graphs*

Klaus Jansen

Institut für Informatik, Universität zu Kiel, Olshausenstr. 40, D-24098 Kiel, Germany
kj@informatik.uni-kiel.de

Abstract. In this paper we describe together with an overview about other results the main ideas of our polynomial time approximation schemes for the maximum weight independent set problem (selecting a set of disjoint disks in the plane of maximum total weight) in disk graphs and for the maximum bisection problem (finding a partition of the vertex set into two subsets of equal cardinality with maximum number of edges between the subsets) in unit-disk graphs.

For a set V of geometric objects, the corresponding geometric intersection graph is the undirected graph $G = (V, E)$ with vertex set V and an set E of edges between two vertices if the corresponding objects intersect. Assume that we are given a set $\mathcal{D} = \{D_1, \ldots, D_n\}$ of n disks in the plane, where D_i has diameter d_i and center $c_i = (x_i, y_i)$. Disk D_i and D_j intersect if $dist(c_i, c_j) \leq (d_i + d_j)/2$, where $dist(p, q)$ denotes the Euclidean distance between points p and q in the plane. A disk graph is the intersection graph of a set of disks. We assume that the input to our problems is the set $\mathcal{D}$ of disks, not only the corresponding intersection graph. This is an important distinction, because determining for a given graph whether it is a disk graph is known to be NP-hard [10], and hence no efficient method is known for computing a disk representation if only the intersection graph is given. Interestingly, every planar graph is a coin graph, i.e. the intersection graph of a set of interior disjoint disks [13]. Therefore, the class of disk graphs properly contains the class of planar graphs. Applications of geometric intersection graphs are frequency assignment in networks [9,15] and map labelling [1].

We are interested in approximation algorithms for NP-hard optimization problems on disk graphs, in particular for the maximum weighted independent set and maximum bisection problem. The goal of the maximum weight independent set problem (MWIS) is to compute, for a given set of disks $\mathcal{D}$ with certain vertex weights $w_1, \ldots, w_n$, a subset $U \subset \mathcal{D}$ of disjoint non-overlapping disks with maximum total weight $w(U) = \sum_{D_i \in U} w_i$. MIS refers to the unweighted version of this problem (i.e. with $w_i = 1$ for each disk $D_i \in \mathcal{D}$). The goal of the maximum bisection problem (MBS) is to partition the disk set into two subsets

* Research of the author was supported in part by the EU Thematic Network APPOL I + II, Approximation and Online Algorithms, IST-1999-14084 and IST-2001-32007.

A. Brandstädt, D. Kratsch, and H. Müller (Eds.): WG 2007, LNCS 4769, pp. 151–153, 2007.

of the same cardinality (assuming that the number n of disks is even) and to maximize the number of edges across the partition. For unit disk graphs (intersection graphs of disks with equal diameter) and planar graphs, MIS remains NP-hard [4,7]. Recently, it was proved that MBS is also NP-hard for unit disk graphs and planar graphs [5,14].

For a given set $\mathcal{D}$ of disks in the plane, let $OPT(\mathcal{D})$ denote the maximum value (the weight or the number of edges) of an optimum solution for MWIS and MBS, respectively. An algorithm A is a ρ-approximation algorithm for MWIS or MBS (with $\rho \geq 1$) if it runs in time polynomial in the input size (the size of the representation for $\mathcal{D}$) and always computes a solution of total value at least $(1/\rho)OPT(\mathcal{D})$. A polynomial time approximation scheme is a family of approximation algorithms $\{A_\epsilon | \epsilon > 0\}$ where A_ϵ computes a solution of value at least $1/(1+\epsilon)OPT(\mathcal{D})$ and runs in polynomial time in the input size for each fixed $\epsilon > 0$.

Hunt et al. [11] gave a PTAS for MWIS on unit disk graphs, and Baker [3] provided a PTAS for MWIS on planar graphs. For intersection graphs of disks with arbitrary diameters, the best previous known approximation algorithm achieves approximation ratio 5 for MIS [16]. In [11,15], the question was raised whether a PTAS exists for disk graphs. As the class of disk graphs contains the class of unit disk graphs and the class of planar graphs, a PTAS for disk graphs generalizes the result for unit disk graphs due to Hunt et al. [11] and the result for planar graphs due to Baker [3]. We resolve this question by presenting a PTAS for MWIS in disk graphs (with given presentation) [6]. The basic idea of the PTAS is as follows. The plane is partitioned into squares on different levels, and some of the disks are removed from the input so that different squares on the same level yield independent subproblems with respect to all disks that are on this level or on a level with disks of smaller diameter. Furthermore, at most a constant number of disks with larger diameter can be disjoint and intersect a square on the current level. Therefore, all such sets can be enumerated in polynomial time for each square, and a dynamic programming approach becomes feasible. In addition to this result, we [6] propose also a PTAS for the minimum weight vertex cover problem (MWVC).

The complexity and approximability status of MBS on planar graphs have been long-standing open problems. Contrary to the polynomial time algorithm of planar maximum cut (finding a partition into two subsets that maximizes the number of edges with endpoints in both subsets) [8], planar maximum bisection has been proven only recently to be NP-hard in exact setting by Jerrum [14]. The proof of this result can be found in [12]. We [12] have proved that there is a PTAS for MBS in planar graphs. This is obtained by combining (via tree-typed dynamic programming) the original Baker's method [3] of dividing the input graph into families of k-outerplanar graphs with our method of finding maximum partitions of bounded treewidth graphs. For dense graphs, Arora, Karger, and Karpinski [2] gave a PTAS for the maximum bisection problem. We [12] have found a PTAS for the maximum bisection problem on unit disk graphs. It is obtained by combining (again via tree-typed dynamic programming) the idea of Hunt et al. [11] of dividing the input graph defined by plane conditions into family of subgraphs with the aforementioned methods in [2] of finding maximum partitions of dense graphs.

References

1. Agarwal, P.K., van Kreveld, M., Suri, S.: Label placement by maximum independent set in rectangles. Computational Geometry 11, 209–218 (1998)
2. Arora, S., Karger, D., Karpinski, M.: Polynomial time approximation schemes for dense instances of NP-hard problems. Journal of Computer and System Science 58, 193–210 (1999)
3. Baker, B.: Approximation algorithms for NP-complete problems on planar graphs. Journal of the ACM 41, 153–180 (1994)
4. Clark, B.N., Colbourn, C.J., Johnson, D.S.: Unit disk graphs. Discrete Mathematics 86, 165–177 (1990)
5. Diaz, J., Kaminski, M.: Max-CUT and MAX-BISECTION are NP-hard on unit disk graphs. Theoretical Computer Science 377, 271–276 (2007)
6. Erlebach, T., Jansen, K., Seidel, E.: Polynomial-time approximation schemes for geometric graphs. SIAM Journal on Computing 34, 1302–1323 (2005)
7. Garey, M.R., Johnson, D.S., Stockmeyer, L.J.: Some simplified NP-complete graph problems. Theoretical Computer Science 1, 237–267 (1976)
8. Hadlock, F.: Finding a maximum cut of a planar graph in polynomial time. SIAM Journal on Computing 4, 221–225 (1975)
9. Hale, W.K.: Frequency assignment: theory and applications. Proceedings of the IEEE 68, 1497–1514 (1980)
10. Hlineny, P., Kratochvil, J.: Representing graphs by disks and balls. Discrete Mathematics 229, 101–124 (2001)
11. Hunt III, H.B., Marathe, M.V., Radhakrishnan, V., Ravi, S.S., Rosenkrantz, D.J., Stearns, R.E.: NC-approximation schemes for NP-and PSPACE-hard problems for geometric graphs. Journal of Algorithms 26, 238–274 (1998)
12. Jansen, K., Karpinski, M., Lingas, A., Seidel, E.: Polynomial time approximation schemes for max-bisection on planar and geometric graphs. SIAM Journal on Computing 35, 110–119 (2005)
13. Koebe, P.: Kontaktprobleme der konformen Abbildung, Berichteüber die Verhandlungen der Sächsischen Akademie der Wissenschaften. Leipzig, Math.-Phys. Klasse 88, 141–164 (1936)
14. Jerrum, M.: private communication (2000)
15. Malesinska, E.: Graph-theoretical models for frequency assignment problems, PhD thesis, Technische Universität Berlin (1997)
16. Marathe, M.V., Breu, H., Hunt III, H.B., Ravi, S.S., Rosenkrantz, D.J.: Simple heuristics for unit disk graphs. Networks 25, 59–68 (1995)