Chapter 5
Multiobjective Optimization Methods

5.1 Introduction

Multiobjective optimization methods, or multiobjective decision analysis (MODA), define decision alternatives in terms of a model consisting of a set of objective functions and a set of constraints imposed on the decision variables. Formally, MODA problems are formulated as follows:

$$\text{maximize}\, F(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_n(\boldsymbol{x})\}, \tag{5.1}$$

$$\text{subject to: } \boldsymbol{x} \in \boldsymbol{X}, \tag{5.2}$$

where $F(\boldsymbol{x})$ is the n- dimensional objective function; f_k $(\boldsymbol{x})$ is an objective (criterion) function (k = 1, 2, …, n); $\boldsymbol{X}$ is the set of feasible alternatives, and $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)$ is a vector of decision variables, $x_i \geq 0$, for i = 1, 2, …, m. One can assume without lost of generality that all objective functions in Eq. 5.1 are to be maximized. In spatial optimization problems, there is at least one set of spatially explicit decision variables. The variables can be used in many different ways to define spatial decision alternatives. For example, the concept of location-allocation is often employed for defining a set of spatial alternatives. Specifically, any locational alternative can be defined as a binary vector, $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)$, where a decision variable, x_j, is defined as follows: $x_j = 1$, if an activity (e.g., health service facility) is located at the ith site; and $x_j = 0$, otherwise. Also, a vector of allocation variables associated with the jth location can be defined in terms of a binary variable as follows: $x_{ij} = 1$, if an activity (e.g., demand for health services) at the ith location is allocated to the jth location; and $x_{ij} = 0$, otherwise.

Given that the multiobjective optimization models (5.1)–(5.2) include conflicting and often non-commensurate criteria, the problem involves finding a set of Pareto optimal solutions (which is also known as a set of efficient, non-dominated, and non-inferior solutions). In Sect. 2.2.3.2 we have outlined the concept of Pareto optimal (or non-dominated) alternatives. Here, we define the concept formally.

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A vector of decision variables $\boldsymbol{x}^*$ is said to be Pareto optimal if there exist no other feasible vector $\boldsymbol{x}$ such that $f_k(\boldsymbol{x}) \geq f_k(\boldsymbol{x}*)$ for all $k = 1, 2, \ldots, n$ and $f_k(\boldsymbol{x}) > f_k(\boldsymbol{x}*)$ for at least one k. This implies that $\boldsymbol{x}^*$ is Pareto optimal if there is no feasible vector that would improve some objective without causing a simultaneous deterioration of at least one other objective. The non-dominated set in the objective space is referred to as the Pareto front. In the absence of any preference regarding the objectives, all non-dominated solutions are assumed equivalent or indifferent. However, the multiobjective decision problems often require that a single non-dominated alternative is selected from the set of Pareto optimal solutions. This type of problems has traditionally been handled by combining the objectives into a scalar function and then solving the equivalent single-optimization problem to identify a best-compromise alternative (or a set of non-dominated alternatives). Once the multiobjective problem is specified in terms of single-objective model, it can be solved using conventional mathematical programming algorithms (Cohon 1978; Goicoechea et al. 1982; Huang et al. 2008).

This chapter focuses on the most often used conventional optimization approaches in GIS-MCDA, which can be classified into three groups: (i) methods for generating non-dominated solutions (the weighting and constraint methods), (ii) the distance-based methods (such as compromise programming, goal programming, and reference point methods), and (iii) interactive methods (Hwang and Masud 1979). This classification is based on the ways in which the decision maker's preference information is incorporated into the modeling procedure. Efficient solution generation methods do not require the preference information to be provided before performing the optimization procedure.

These techniques are also referred to as a posteriori methods, because the solution procedure is performed first and the decision maker preferences can then be elicited from the generated set of solutions. In distance-based methods, the preferences are specified a priori; that is, all decision maker preferences are specified before the solution process. The interactive methods assume that the preferences can be provided progressively in the modeling procedure.

5.2 Weighting and Constraint Methods

Several techniques for generating non-dominated solutions are available (Cohon 1978; Goicoechea et al. 1982; Zarghami and Szidarovszky 2011). A common feature of these techniques is that the multiobjective problem is first transformed into a scalar problem and then solved as a single-objective optimization problem. The basic difference among the methods lies in how they make the transformation from a multi- to single-objective model (Cohon 1978). The most often used methods for tackling spatial multiobjective problems are the *weighting* and *constraint methods* (Diamond and Wright 1988; Malczewski and Ogryczak 1995; Church et al. 1992; Maliszewski et al. 2012).

The weighting method involves assigning a weight, w_k ($k = 1, 2, \ldots, n$), to each objective function, $f_k(\boldsymbol{x})$. The multiobjective function (5.1) can then be converted into a single-objective form through the linear combination of the objectives together with the corresponding weights. Thus, the problem (5.1)–(5.2) can be transformed as follows:

$$\text{maximize}\, F(\boldsymbol{x}) = \{w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x}) +, \ldots, + w_n f_n(\boldsymbol{x})\}, \tag{5.3}$$

$$\text{subject to: } \boldsymbol{x} \in \boldsymbol{X}. \tag{5.4}$$

where the weights $w_k \geq 0$ and $w_1 + w_2 +, \ldots, + w_n = 1$. The set of non-dominated solutions to the problem (5.3)–(5.4) is generated by parametric variation of the weights. An approximation of the non-dominated solution set can be generated by systematically varying the weighting coefficients and solving the associated single-objective model. Figure 5.1a illustrates the concept of weighting method for the two objective functions. It shows the feasible solution region and the non-dominated alternatives (or the Pareto-optimal front). For a bi-objective problem, there are two weights, and one of them is independent. Since F is a linear combination of f_1 and f_2, the contour of F in the objective space is a line, l_s. The value of F is the same at any point of the contour line; therefore, the line is referred to as the linear indifference curve. The slope of the line is defined by the value of the weights; specifically, the slope is equal to $-w_1/w_2$. The value of F depends on the location of the line. By changing the values of the weights one can obtain different values of F represented by the parallel indifference curves, l_1, l_2, and l_3. Since model (5.3)–(5.4) involves maximization of the objective functions, the indifference curve

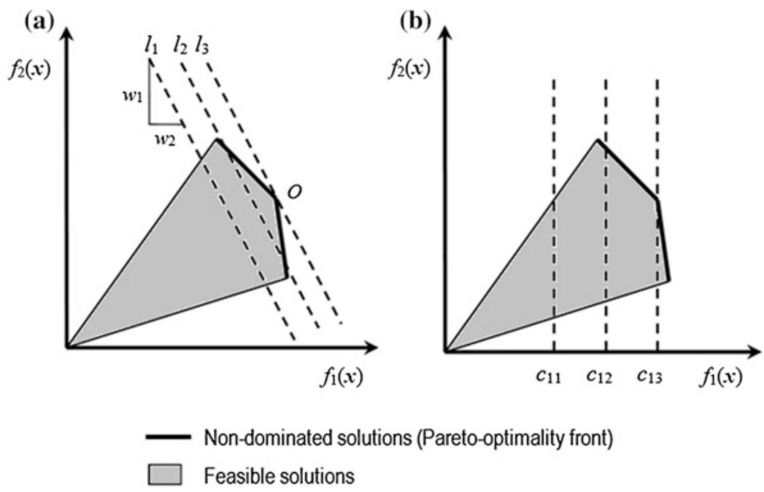


Fig. 5.1 The concept of **a** weighting method, and **b** constraint method (*Note* the objective functions $f_1(x)$ and $f_2(x)$ are maximized; O = optimal solution; w_1, w_2 = weights; l_1, l_2, l_3 = linear indifference curves; c_{11}, c_{12}, c_{13} = constraints imposed on $f_1(x)$)

with the maximum value of F determines the optimal solution. Specifically, the solution to the problem (5.3)–(5.4) is to move the contour line northeastwards in parallel as far as possible until it becomes tangent to the feasible objective space. The point of tangency, O, located on the indifference curve l_3 indicates the optimal solution.

Note that if some value (or utility) functions (see Sect. 2.3.1.1) and associated objective weights are estimated according to the principles described in Sect. 2.3.2, then the weighting method becomes multiobjective *value function* method. Given the value functions, $v(f_k(\boldsymbol{x}))$, for $k = 1, 2, \ldots, n$, the problem (5.1)–(5.2) can be stated with the following value function program:

$$\text{maximize}\, F(\boldsymbol{x}) = \{\Sigma\, w_k\, v(f_k(\boldsymbol{x}))\}, \tag{5.5}$$

$$\text{subject to: } \boldsymbol{x} \in \boldsymbol{X}, w_k \geq 0 \text{ for } k = 1, 2, \ldots, n. \tag{5.6}$$

where w_k is the weight of importance assigned to the kth objective. Note that there is a difference between the value function models (5.5)–(5.6) and the weighting method for generating non-dominated solutions (5.3)–(5.4). The value function method incorporates the decision maker's preferences by assigning weights of importance to the objective functions, while in the weighting method the weights are parameters that may be varied systematically to yield points that are non-dominated solutions. Also, the weighting model (5.3)–(5.4) is used for generating a set of non-dominated alternatives by changing the weighting coefficients, while the problem (5.5)–(5.6) results in a unique non-dominated solution. Thus, strictly speaking, the value function model is not a method for generating a set of non-dominated alternatives. If the objective weights w_1 and w_2 represent the decision makers' preferences with respect the objective functions $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$, and the assumption of a linear value function is accepted, then point O would indicate the best (compromise) solution (see Fig. 5.1a).

One limitation of the weighting method is that certain non-dominated solutions cannot be detected when the Pareto-optimal front is non-convex (Cohon 1978). The constraint method can alleviate this problem. The method involves maximizing only one of the objective functions while all others are converted into inequality constraints. Thus, the multiple objective problem (5.1)–(5.2) can be transformed to the following single-objective problem:

$$\text{maximize}\, f_s(\boldsymbol{x}), \tag{5.7}$$

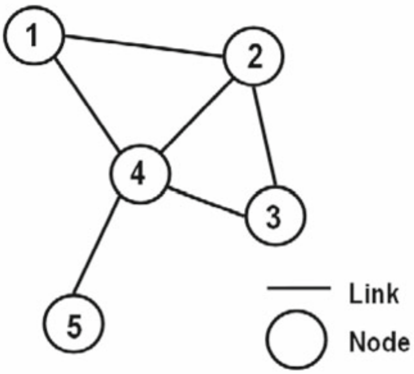
$$\text{subject to: } \boldsymbol{x} \in \boldsymbol{X}, \text{ and } f_k(\boldsymbol{x}) \geq c_k, \text{ for all } k \neq s, \tag{5.8}$$

where c_k is a lower bound on objective k.

The set of non-dominated solutions can be generated by solving the single-criterion problem (5.7)–(5.8) with the parametric variation of the c_k value. Like the weighting method, the constraint problem can be solved with standard mathematical programming techniques (Cohon 1978).

Figure 5.1b demonstrates the concept of the constraint method. It shows situations involving two objective functions, where $f_2(\boldsymbol{x})$ is to be maximized and $f_1(\boldsymbol{x})$ is converted to a constraint $f_1(\boldsymbol{x}) \geq c_{1b}$, for $b = 1$, 2, and 3. The constrain divides the original feasible objective space into two portions: feasible and infeasible; for example, the portion of the original feasible space right from the c_{11} line constrains all feasible solutions, while the left portion becomes infeasible solution space for the problem (5.7)–(5.8). By changing the values of the constraint, c_{1b}, one can obtain different values of the objective function, $f_2(\boldsymbol{x})$. Since the model (5.7)–(5.8) represents a maximization problem, the maximum value of $f_2(\boldsymbol{x})$ determines the optimal solution. A set of non-dominated solutions to the problem can be generated by moving the constraint line eastwards in parallel.

Computational examples of the weighting and constraint methods are given in Goicoechea et al. (1982) and Malczewski (1999). Goicoechea et al. (1982) illustrates the methods by solving resource allocation and watershed management problems. Malczewski (1999) provides a computational example of the methods using a spreadsheet-based solver for tackling a location-allocation problem. Here we give another example of the weighting (value function) method to demonstrate the procedure of generating non-dominated solutions. We consider a hypothetical example of the p-median problem on a network. The problem is to locate p facilities on a network of m nodes and allocate each node to exactly one of them so that the total distance (and other relevant attribute) is minimized (or maximize). We consider a problem of locating two service facilities ($p = 2$) for supplying components to five manufacturers (towns) ($m = 5$) (Fig. 5.2). The demand for the services, z_i, is measured by the number of units required by the ith manufacturer. The problem involves optimizing three objective functions: (i) total distance, (ii) total environmental impact associated with transportation of the components (measured by an index assigned to links of the network), and (iii) total risk of accident. The raw datasets for the attributes (objectives) were normalized using Eq. 2.1. Table 5.1 shows normalized values of the three attributes.



Town i	Demand, z_i (in millions)
1	18
2	12
3	25
4	30
5	15

Fig. 5.2 Network of five demand nodes representing towns and six links representing roads

Table 5.1 Standardized values of (a) distance, d_{ij}, (b) environmental impact, e_{ij}, and (c) risk of accident, r_{ij}, associated with the i,jth link (arc) of the network consisting of five nodes; i = demand node (i = 1, 2, .., 5), and j = node for potential location of facility (j = 1, 2, .., 5)

(a)

i \ j	1	2	3	4	5
1	1.0	0.4	0.2	0.6	0.0
2	0.4	1.0	0.6	0.8	0.2
3	0.2	0.6	1.0	0.8	0.2
4	0.6	0.8	0.8	1.0	0.6
5	0.0	0.2	0.2	0.6	1.0

(b)

i \ j	1	2	3	4	5
1	0.5	0.4	0.1	0.9	0.0
2	0.4	1.0	0.3	0.7	0.7
3	0.1	0.3	0.1	0.6	0.3
4	0.9	0.7	0.6	0.0	0.4
5	0.1	0.7	0.3	0.4	1.0

(c)

i \ j	1	2	3	4	5
1	0.0	0.25	0.25	1.0	0.0
2	0.25	1.0	0.5	1.0	0.75
3	0.25	0.5	1.0	0.5	1.0
4	1.0	1.0	0.5	0.25	0.25
5	0.0	0.75	1.0	0.25	0.0

Formally, the problem can be written as follows:

$$\text{maximize} f_1(\boldsymbol{x}) = \sum_{i=1}^{m} \sum_{j=1}^{n} z_i d_{ij} x_{ij} \tag{5.9}$$

$$\text{maximize} f_2(\boldsymbol{x}) = \sum_{i=1}^{m} \sum_{j=1}^{n} z_i e_{ij} x_{ij} \tag{5.10}$$

$$\text{maximize} f_3(\boldsymbol{x}) = \sum_{i=1}^{m} \sum_{j=1}^{n} z_i r_{ij} x_{ij} \tag{5.11}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \text{for } i = 1, 2, 3, \ldots, m; \tag{5.12}$$

$$x_{ij} - x_{jj} \leq 0, \quad \text{for } i = 1, 2, 3, \ldots, m, \; j = 1, 2, 3, \ldots, n; \tag{5.13}$$

$$\sum_{j=1}^{n} x_{jj} = p; \tag{5.14}$$

$$x_{ij} = 1 \text{ or } 0, \quad \text{for } i = 1, 2, 3, \ldots, m, \; j = 1, 2, 3, \ldots, n; \tag{5.15}$$

where z_i = the number of units demanded by the ith manufacturer; d_{ij} = standardized value of distance between node i and j; e_{ij} = standardized environmental impact index assigned to the link (road) between i and j; r_{ij} = standardized risk of accident associated with the link (road) between i and j.

The objective functions $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{x})$, and $f_3(\boldsymbol{x})$ maximize the total weighted standardized values of distance, environmental impact, and risk of accident, respectively. Equation (5.12) ensures that each demand node (manufacturer) is allocated to a service facility. Inequality (5.13) guarantees that the demand nodes are allocated only to those candidate nodes where facility will be established. Equation (5.14) indicates the number of facilities to be located (that is, p = 2). According to Eq. (5.15), each of the allocation (decision) variables must be equal to 1 or 0; specifically, x_{ij} = 1 if the components required by the ith manufacturer are supplied at the jth facility, and x_{ij} = 0 otherwise;

In order to generate a set of non-dominated solutions, the multiobjective problem (5.9)–(5.15) is converted to the following singe-objective form:

$$\text{maximize } F(\boldsymbol{x}) \left(w_1 \sum_{i=1}^{m} \sum_{j=1}^{n} z_i d_{ij} x_{ij} \right) + \left(w_2 \sum_{i=1}^{m} \sum_{j=1}^{n} z_i e_{ij} x_{ij} \right) + \left(w_3 \sum_{i=1}^{m} \sum_{j=1}^{n} z_i r_{ij} x_{ij} \right), \tag{5.16}$$

$$\text{subject to: } (5.12){-}(5.15). \tag{5.17}$$

This problem can be tackled using a standard mathematical programming solver. We use a spreadsheet based LINDO system (www.lindo.com).

As suggested, a set of non-dominated solutions can be generated by varying the objective weights, w_k. One way of varying the weights is to assign a weight of 1 to one of the objective functions and 0 to all other functions. The problem (5.16)–(5.17) is solved with three different sets of the objective weights; that is, if w_1 = 1, w_2 = 0, and w_3 = 0, then $f_1(\boldsymbol{x})$ is optimized; if w_1 = 0, w_2 = 1, and w_3 = 0, then $f_2(\boldsymbol{x})$ is optimized; and, if w_1 = 0, w_2 = 0, and w_3 = 1, then $f_3(\boldsymbol{x})$ is optimized.

The results are organized in the form of a pay-off matrix (see Table 5.2). The matrix allows us to identify the maximum and minimum values of each objective function; that is, the *ideal* (utopia) and *anti-ideal* (nadir) solutions can be defined. The ideal solution is usually not attainable but it can be presented to the decision maker as a limit to the best numerical values of the objectives; that is, it provides the decision maker with lower limits for minimized criterion functions and upper limits for the functions to be maximized. The anti-ideal point is the worst criterion value. It is the lower limits and upper limits for criterion functions to be maximized and minimized, respectively.

Figure 5.3 shows the optimal location-allocation patterns associated with the results of the three solutions displayed in the pay-off matrix (Table 5.2). The results indicate that there are substantial differences between the three non-dominated solutions. Furthermore, the differences are present in the objective and decision space. Notice that the optimal value of $f_1(\boldsymbol{x})$ and $f_3(\boldsymbol{x})$ are similar. However, the associated location-allocation patterns are considerably different. This remark

Table 5.2 The pay-off matrix for the problem (5.16)–(5.17)

Optimized objective functions	Objective function value		
	$f_1(x)$	$f_2(x)$	$f_3(x)$
$f_1(x)$ ($w_1 = 1, w_2 = 0, w_3 = 0$)	86.6	38.4	35.8
$f_2(x)$ ($w_1 = 0, w_2 = 1, w_3 = 0$)	58.4	66.9	64.0
$f_3(x)$ ($w_1 = 0, w_2 = 0, w_3 = 1$)	71.2	47.2	86.5
Ideal vector	86.6	66.9	86.5
Nadir vector	58.4	38.4	35.8

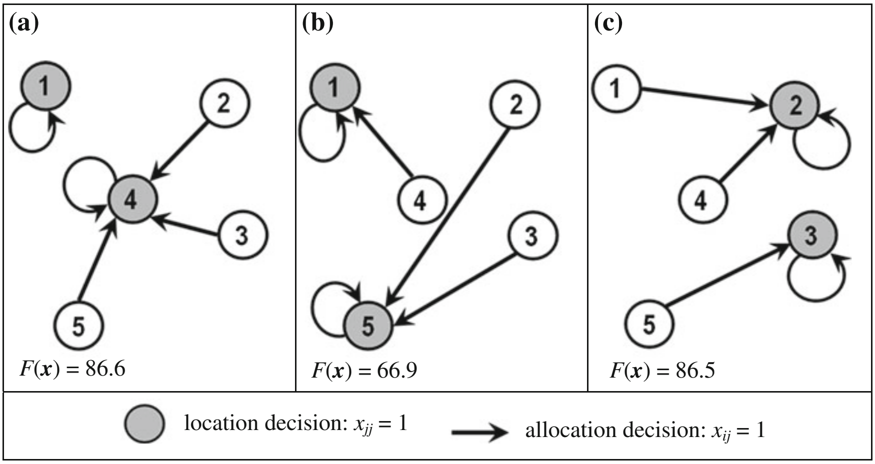


Fig. 5.3 Location-allocation patterns for solution of the multiobjective optimization problem (5.9)–(5.15) for: **a** $w_1 = 1$, and $w_2 = w_3 = 0$; **b** $w_2 = 1$, and $w_1 = w_3 = 0$, and **c** $w_3 = 1$ and $w_1 = w_2 = 0$

underscores the importance of examining the results of spatial multiobjective modeling both in the objective and geographic (decision) space.

In addition to the non-dominated solutions obtained by generating the pay-off matrix, one can solve the problem (5.16)–(5.17) for different sets of the objective weights to analyze the non-dominated set of alternatives. Table 5.3 shows the objective function values for a sample of four sets of weights. The associated location-allocation patterns are given in Fig. 5.4. Notice we obtained the same solutions for $w_1 = 0.5$, $w_2 = 0.25$, $w_3 = 0.25$, and $w_k = 0.33$ (see Fig. 5.4a and d). There are, however, substantial differences between these location-allocation patterns and those displayed in Fig. 5.3b and c. Note also that the values of the objective functions are similar for $w_1 = 0.5$, $w_2 = 0.25$, $w_3 = 0.25$ and $w_1 = 0.25$, $w_2 = 0.25$, $w_3 = 0.50$. This suggests that similar solutions to multiobjective decision problem in the objective space may be substantially different in the geographic space, and vice versa.

The results of the weighting method provide important information about the set of non-dominated alternatives, the range of possible decision outcomes, and the trade-offs involved. In spite of the fact that this information is very useful in searching for the best decision outcomes and corresponding location-allocation pattern, a decision maker would likely find it difficult to choose the best alternative even for a very small spatial (location-allocation) problem. Therefore, an a priori or interactive method has to be applied to identify the best (compromise) alternative (Sect. 5.3).

Several GIS-MODA applications have used the weighting method (e.g., Church et al. 1992; Kao and Lin 1996; Wu and Murray 2005; Farhan and Murray 2008; Herzig 2008; Ligmann-Zielinska and Jankowski 2010; Maliszewski and Horner 2010; Maliszewski et al. 2012). Church et al. (1992) integrated the weighting method into a raster based GIS for generating and exploring spatial alternatives for a corridor location problem. Kao and Lin (1996) also used a raster-based GIS in their spatial analysis of landfill sitting problem with the weighting method. Wu and Murray (2005) integrated the weighting method with GIS to analyze the trade-off between public transit service quality and access coverage in a bus-based transit system. Farhan and Murray (2008) integrated spatial multiobjective model into ArcView GIS and used the weighting method to analyze the trade-offs involved in locating park-and-ride facilities. Maliszewski and Horner (2010) used standard mathematical programming software CPLEX (see www.aimms.com/features/solvers/cplex) and ArcGIS to solve multiobjective problem of sitting critical

Table 5.3 The weighting method: the location-allocation problem results for selected sets of objective weights

Weights			Objective functions			
w_1	w_2	w_3	$f_1(x)$	$f_2(x)$	$f_3(x)$	$F(x)$
0.50	0.25	0.25	71.2	53.2	82.8	69.59
0.25	0.50	0.25	66.0	66.0	65.8	65.94
0.25	0.25	0.50	71.2	47.2	86.5	72.85
0.33	0.33	0.33	71.2	53.2	82.8	69.05

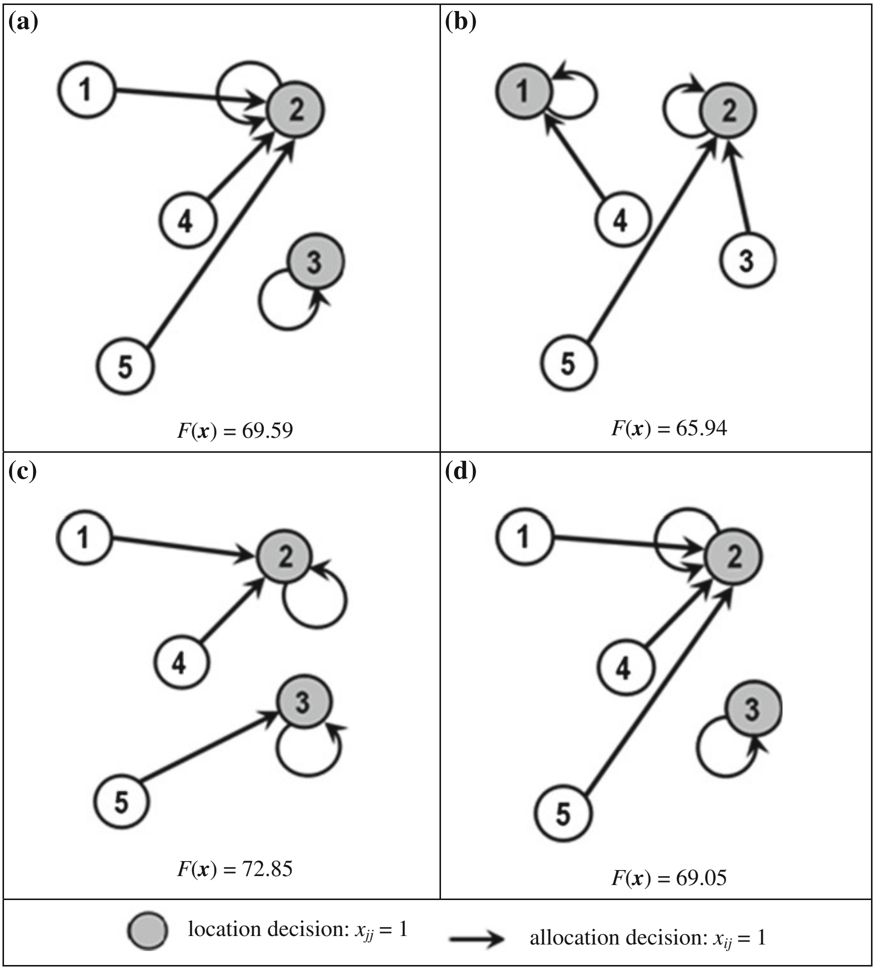


Fig. 5.4 Location-allocation patterns for solution of the multiobjective optimization problem (5.9)–(5.15): **a** $w_1 = 0.5$, $w_2 = 0.25$, $w_3 = 0.25$; **b** $w_1 = 0.25$, $w_2 = 0.5$, $w_3 = 0.25$; **c** $w_1 = 0.25$, $w_2 = 0.25$, $w_3 = 0.5$; and **d** $w_1 = 0.33$, $w_2 = 0.33$, $w_3 = 0.33$

supply facilities (see also Maliszewski et al. (2012)). Herzig (2008) developed LUMASS (Land Use Management Support System), which integrates ArcMap GIS and the open source mixed-integer linear programming system called *lp_solve* (see http://lpsolve.sourceforge.net) for tackling land use allocation problems. The system offers the techniques for generating the set of efficient solutions: the weighting and constrain methods.

One important advantage of the weighting and constraint procedures is that the methods reduce the multiobjective optimization problem to a scalar valued function. This means that the vast body of algorithms, software, and experience that exist for single-objective optimization models can be directly applied to multiobjective problems. This is of major importance considering the extent to which single-objective optimization has influenced the development of spatial analysis methods, such as spatial interaction and location analysis (Thomas and Huggett 1980; Killen 1983).

The weighting and constraint methods are easily used and intuitively appealing. There are, however, some major concerns associated with the use of the methods. They are very intensive computationally. The computational requirements for the weighting and constraint methods depend on the number of objective functions and the number of weights or constraints. There is an exponential relationship between the number of objective functions and computational burden (Cohon 1978). Since the resulting subset of efficient solutions depends on the particular weights or constraints applied, the methods may not generate a good representation of the entire non-dominated set. One possible way of handling this problem is to reduce the scale of weights or the intervals of the constraints. However, this will increase the computational burden. There is no generic rule for varying the weights or constraint intervals for generating a representative subset of non-dominated solutions.

5.3 Distance Metric Based Methods

The distance metric based MODA methods aim at minimizing a function of the distance between the desired (usually unachievable) and achieved solutions (Jones and Tamiz 2010; Zarghami and Szidarovszky 2011). The desired solution (target values) can be defined as an ideal point, some reference point, or a set of goals. The most often used distance metric approaches include: goal programming (Charnes and Cooper 1961), compromise programming (Zeleny 1982), and the reference point method (Wierzbicki 1982). These methods are also the most popular distance metric procedures implemented in the GIS environment (e.g., Church et al. 1992; Antoine et al. 1997; Agrell et al. 2004; Zeng et al. 2007; Huang et al. 2008; Meyer et al. 2009; Li and Leung 2011; Coutinho-Rodrigues et al. 2012).

The distance based methods are also referred to as the L_p-norm approaches. Indeed, the definition of distance metric is the main procedural difference between the different types of those methods. A generic form of the distance metric model can be written as follows (Jones and Tamiz 2010):

$$L_p = \left[\sum_{k=1}^{n} \left(\frac{|f_k(x) - a_k|}{h_k} \right)^p \right]^{\frac{1}{p}}, \tag{5.18}$$

where $f_k(\boldsymbol{x})$ is the achieved value of the kth objective ($k = 1, 2, \ldots, n$); a_k the target value; h_k is the normalisation constant associated with the kth objective; and p is a power parameter ranging from 1 to ∞ (see Sect. 4.4.1).

5.3.1 Goal Programming

The goal programming methods require the decision maker to specify the most desirable value (goal) for each objective (criterion) as the aspiration level or target value. The objective functions (5.1) are then transformed into goals as follows:

$$f_k(\boldsymbol{x}) + d_k^- - d_k^+ = a_k, \text{ for } k = 1, 2, \ldots, n, \tag{5.19}$$

$$d_k^-, d_k^+ \geq 0, \ (d_k^-, d_k^+) = 0, \tag{5.20}$$

where a_k is the aspiration level for the kth objective, d_k^-, d_k^+ are negative and positive goal deviations, respectively; that is, non-negative state variables that measure deviations of the achieved value of the kth objective function from the corresponding aspiration level. Thus, two types of variables are part of any goal programming formulation: the decision variables, x_i, and the deviational variables, d_k.

A number of measures of multidimensional deviations (achievement functions) and corresponding goal programming forms have been proposed by Jones and Tamiz (2010). The achievement function, $g(\boldsymbol{d}^+, \boldsymbol{d}^-)$, can be formulated in terms of the weighted L_p norm as follows:

$$g(d^+, d^-) = \left[\sum_{k=1}^{n} \left(w_k^- d_k^- + w_k^+ d_k^+\right)^p\right]^{\frac{1}{p}}, \tag{5.21}$$

where w_k^- and w_k^+ are weights corresponding to the kth goal deviations. The weights represent additional information reflecting the decision maker's preferences with respect to the deviation variables. One can generate a number of models by changing the value of p. The *weighted goal* and *Chebyshev goal programming* have been the most often used goal programming methods in the GIS environment (see Malczewski 2006a). For $p = 1$, the achievement function (5.21) takes the form of the weighted goal programming:

$$g(\boldsymbol{d}^+, \boldsymbol{d}^-) = \sum_{k=1}^{n} \left(w_k^- d_k^- + w_k^+ d_k^+\right), \tag{5.22}$$

The weighted goal programming assumes that the positive deviations and negative deviations of the criterion outcomes from the aspired goals are equally undesirable.

One can also use the L_p norm to develop weighted Chebyshev goal programming. Specifically, for $p = \infty$ the achievement function of the Chebyshev goal programming takes the following form:

$$g(\boldsymbol{d}^+, \boldsymbol{d}^-) = \max_{k=1,2,..,n} \left(w_k^- d_k^- + w_k^+ d_k^+\right), \tag{5.23}$$

This type of goal programming minimizes the deviation from those aspiration levels so that the worst deviation from any single-goal aspiration level is minimized.

It is important to note that the models (5.22) and (5.23) are related to other L_p based multiobjective methods. Section 5.3.2 provides an example of a spatial multiobjective optimization problem to demonstrate the relationship between the distance metric based models. A computational example of goal programming is given in Malczewski (1999). He considers a location-allocation problem in the context of transporting and disposing hazardous waste. There have been a number of studies on integrating goal programming methods into GIS. The weighted Chebyshev goal programming was used by Church et al. (1992) for tackling multiobjective corridor location problem. November et al. (1996) integrated TransCAD GIS and goal programming for analyzing alternative patterns of school districting. Ghosh (2008) used a loose coupling approach for integrating a goal programming method into SPANS GIS to analyze alternative patterns of land use. The location-routing problem using standard mixed integer linear programming modeling have been tackled in several studies, including Coutinho-Rodrigues et al. (1997), Alçada-Almeida et al. (2009), and Coutinho-Rodrigues et al. (2012). For example, Coutinho-Rodrigues et al. (2012) used the weighted goal programming method within GIS environment to solve a location-routing problem in the context of designing urban evacuation plans. Meyer et al. (2009) and Cisneros et al. (2011) applied GIS-based goal programming approaches for analyzing spatial patterns of agricultural land use. Meyer et al. (2009) developed a weighted goal programming model for analyzing alternative spatial patterns of farming systems. The distance metric based methods, including the weighted goal programming model, have been employed by Cisneros et al. (2011) to analyze the conflicts and trade-off among environmental, economic, and social interests in the context of agricultural land use.

The major advantage of goal programming is its computational efficiency. While dealing with the multi-objective decision problems, goal programming approaches allow us to stay within an efficient linear programming computational environment. There are, however, several conceptual and technical problems with using goal programming methods for tackling spatial multicriteria optimization problems. The standard goal programming methods require the decision maker to specify fairly detailed a priori information about his/her aspiration levels, and the importance of goals in the form of weights. One can expect that in a complex spatial decision situation, the decision maker will find it difficult (or even impossible) to provide the precise information required by these methods. Another weakness of weighted goal programming is its poor control over the interactive process in the case of discrete problems. For example, in the case of multiobjective location problems, this may mean some efficient locational decisions are likely to be selected for various aspiration levels and weights, whereas other decisions, despite being efficient, are selected only for aspiration levels defined very close to their outcomes (Malczewski and Ogryczak 1996). This problem associated with a priori information required by standard goal programming methods can be overcome, at least partially, by an interactive approach (see Sect. 5.4).

5.3.2 Compromise Programming

The compromise programming method is based on the assumption that the performance of decision alternatives can be evaluated with respect to a point of reference (Zeleny 1982). The obvious choice for a point of reference is the ideal solution (or ideal point), which defines the optimal value for each objective considered separately. The method identifies the non-dominated solution closest to the ideal point using various weighted L_p norms as follows:

$$\text{minimize}\left\{L_p(x) = \left[\sum_{k=1}^{n} w_k^p\left(\frac{f_k^+ - f_k(x)}{f_k^+ - f_k^-}\right)^p\right]^{\frac{1}{p}}\right\}, \tag{5.24}$$

$$\text{subject to: } \boldsymbol{x} \in \boldsymbol{X}, w_k \geq 0 \text{ for } k = 1, 2, \ldots, n. \tag{5.25}$$

where $L_p(\boldsymbol{x})$ is the distance metric; w_k is the weight associated with the kth objective function ($k = 1, 2, \ldots, n$); $f_k(\boldsymbol{x})$ is the value of the kth objective function; f_k^+ is the ideal value of the kth objective function; f_k^- is the nadir or anti-ideal value of the k-the objective function; and p is a power parameter ranging from 1 to ∞. The compromise set consists of all compromise solutions determined by solving (5.24)–(5.25) for a given set of weights (w_1, w_2, …, w_n) and for $p \geq 1$. The parameter p reflects the importance of the maximum deviation from the ideal point (see Sect. 4.3). In general, larger values of p reflect greater concern for minimizing the maximum deviation. For $p = 1$, all deviations are weighted equally; for $p = 2$, each deviation is weighted in proportion to its magnitude. For the value of $p = \infty$, the problem involves minimizing the maximum deviation, which is known as the min-max problem or the weighted Chebyshev problem. Note that the compromise programming approach involves a double-weighting scheme (Karni and Werczberger 1995). The parameters w_k and p reflect the importance of the maximal deviation and the relative importance of the kth objective, respectively. The weights, w_k, weigh deviations according to objectives but irrespective of their magnitudes. The parameter p weights the individual deviations according to their magnitudes and across the objectives.

It is general practice to use compromise programming models for p = 1, 2, and ∞ (Goicoechea et al. 1982). In order to identify the compromise set, we need to determine the pay-off matrix. Let us illustrate the concept of compromise programming for p = 1, 2, and ∞, and $w_1 = w_2 = w_3 = 0.33$ using the location-allocation problem (see Sect. 5.2). Given the pay-off matrix (see Table 5.2), the location-allocation problem (5.9)–(5.15) for p = 1 can be written as follows:

$$\text{minimize}\left\{L_1(x) = \left(0.33\frac{86.6 - f_1(x)}{86.6 - 58.4}\right) + \left(0.33\frac{66.9 - f_2(x)}{66.9 - 38.4}\right) + \left(0.33\frac{86.5 - f_3(x)}{86.5 - 35.8}\right)\right\}, \tag{5.26}$$

subject to: (5.12)–(5.15).

Likewise, compromise programming models for $p = 2$ and ∞ can be formulated. Given the operational definition of the compromise programming models for $p = 1$, 2 and ∞, the solution of the problem always results in a non-dominated point for $1 \leq p < \infty$. However, for $p = \infty$, one can obtain a dominated solution (Goicoechea et al. 1982). This general remark is confirmed by the results shown in Table 5.4 Also, the value of $L_p(\boldsymbol{x})$ suggest that the compromise programming model for $p = 2$ generates a non-dominated solution closest to the ideal point.

Tables 5.3 and 5.4 show the same values of the objective functions for the compromise programming model for $p = 1$ and the weighting methods for $w_k = 0.333$. Also, the corresponding location-allocation patterns are identical (see Figs. 5.4d and 5.5a). This finding can be generalized. Indeed, it can be shown that the compromise programming model for $p = 1$ and the weighting (value function) methods (see Sect. 5.2) result in an equivalent solutions for the same set of objective weights (see Li and Leung 2011). Furthermore, the compromise programming resembles goal programming (Goicoechea et al. 1982; Jones and Tamiz 2010). The solution for the weighted goal programming (see Sect. 5.3.1) corresponds to the solution of compromise programming for $p = 1$ if the same weights

Table 5.4 Compromise programming: the location-allocation problem for $p = 1$, 2, and ∞, and $w_1 = w_2 = w_3 = 0.333$

p	$L_p(x)$	$f_1(x)$	$f_2(x)$	$f_3(x)$
1	0.367	71.2	53.2	82.75
2	0.234	72.0	60.0	65.75
∞	0.432	50.0	46.5	44.50

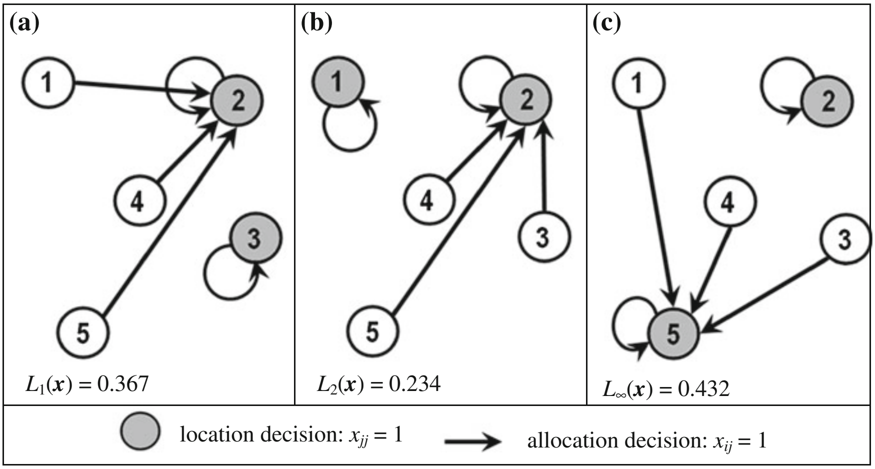


Fig. 5.5 The location-allocation patterns generated by the compromise programming method for $w_1 = w_2 = w_3 = 0.333$ and selected p values: **a** $p = 1$, **b** $p = 2$, and **c** $p = \infty$

and the same ideal (aspiration) levels are chosen. Also, the Chebyshev goal programming model is equivalent to compromise programming for $p = \infty$ (Jones and Tamiz 2010).

There have been several applications of GIS-based compromise programming methods for solving spatial multiobjective optimization problems (e.g., Church et al. 1992; Chang et al. 1997; Shih and Lin 2003; Huang et al. 2008; Li and Leung 2011). All the studies focus on tackling problems in the transportation sector, such as locating transportation corridors and routing problems. Church et al. (1992) have used the weighted Chebyshev distance model as a method for dealing with some of the concerns associated with the use of the weighting method (see Sect. 5.2). They used the method for analyzing a corridor location problem. This study is of particular significance because of its approach for exploring spatial alternatives in both decision space and objective space. Chang et al. (1997) developed an ArcGIS-based compromise programming model for tackling vehicle routing and scheduling problems. The limitations of the weighting method, in the context of spatial multiobjective optimization, have also been highlighted by Huang et al. (2008), and Li and Leung (2011). They demonstrated the relationship between the utility/value function approach and compromise programming, and used the weighted Chebyshev model for tackling routing problems using GIS. Shih and Lin (2003) used GIS and a combination of multiobjective methods, including compromise programming, for tackling routing and scheduling problem.

One advantage of the compromise programming approach is its simple conceptual structure. In addition, the set of preferred compromise solutions can be ordered between the extreme criterion outcomes, and consequently, an implicit trade-off between criteria can be performed. A disadvantage of this approach is that, except for the two extremes (that is, when $p = 0$ and ∞), there is no clear interpretation of the various values of the parameter p. Therefore, the selection of the "best" alternative within the reduced set of compromise alternatives must be made based on a further insight into the compromise set of non-dominated alternatives. One way to achieved this is using the approach as a component of an interactive procedure (see Sect. 5.4).

5.3.3 Reference Point Method

From the perspective of behavioural decision theory, the reference point method can be recognized as an approach that combines the classical optimizing and satisficing decision rules (Wierzbicki 1982, 1983). It is argued that an individual has some tendency toward maximization of his/her utility even if he/she behaves according to satisficing rationality principles; that is, he/she forms aspiration levels as a guide for decision making (Malczewski and Ogryczak 1996). Such type of behaviour is referred to as quasi-satisficing rationality. The concept of quasi-satisficing rationality can be considered as an attempt to generalize the underlying behavioural principles of the distance based multiobjective methods. Indeed, the compromise programming approach is based on the optimizing rationality principle, while the satisficing behaviour is underlying philosophy of goal programming (Romero et al. 1998).

The key element of the quasi-satisficing decision framework is the relationship between the non-dominated set of solutions and aspired goals. According to the quasi-satisficing principle, the decision maker should identify the best (most preferred) alternative as the one which belongs to the set of non-dominated solutions, irrespective of the attainability of his/her aspiration levels. Although the aspired levels may not be achievable, they can be projected onto the Pareto optimal front by using the achievement scalarizing function (Wierzbicki 1982; Romero et al. 1998). Using the achievement scalarizing function, the reference point model can be written as follows:

$$\text{minimize}\left\{\max_k\left[\frac{w_k}{h_k}(a_k - f_k(x))\right] - \boldsymbol{\varepsilon}\sum_{k=1}^{n}\frac{w_k}{h_k}f_k(x)\right\} \tag{5.27}$$

$$\text{subject to: } \boldsymbol{x} \in \boldsymbol{X}, w_k \geq 0 \text{ for } k = 1, 2, \ldots, n. \tag{5.28}$$

where ε is an arbitrary sufficiently small positive number; it guarantees a non-dominated solution of the problem (5.1)–(5.2). The objective function (5.27) has two components: (i) the difference between the weighted Cherbyshev norm of the discrepancies between reference levels, a_k, and the achieved value of the k-th objective $f_k(\boldsymbol{x})$, and (ii) a small regularization term of the weighted sum of the n objectives.

Malczewski and Ogryczak (1996) provide a computational example of the reference point method using a hypothetical plant location problem. They also demonstrate the use of the method within the framework of goal programming. Specifically, the reference point approach can be operationalized within a goal programming framework as an initial Chebyshev goal programme followed by the $L_1(\boldsymbol{x})$ Pareto restoration phase (Romero et al. 1998). Zeng et al. (2007) integrated reference point based systems into ArcGIS for tackling forest planning and management. Antoine et al. (1997) developed a decision support system called Aspiration-Reservation Based Decision Support (ARBDS) (see also Malczewski and Ogryczak 1996). The system integrates the FAO Agro-Ecological Zones/GIS package and the reference (aspiration-reservation) point method (see also Agrell et al. 2004). Antoine et al. (1997) and Agrell et al. (2004) used the system for land use planning. Maniezzo et al. (1998) and Rozakis et al. (2001) employed the reference point method as a component of spatial decision support systems for locating waste management facilities and bio-energy projects, respectively.

One advantage of the reference point method is that it has the capability to capture every Pareto optimal solution by using appropriate aspiration levels. For this reason, the method is especially suitable as a component of interactive multiobjective modeling. However, the reference point model shares some of the drawbacks associated with the other distance metric based approaches. The method requires the decision maker to specify fairly detailed a priori information regarding the reference point(s) and the objective weights. This information may be difficult to elicit for the decision maker. This problem can be alleviated, at least partially, by using the method within the framework of an interactive modeling (see Sect. 5.4).

5.4 Interactive Programming Methods

The main idea behind interactive multiobjective programming methods is to determine the best (compromise or satisficing) decision outcome among the set of efficient solutions by means of a progressive communication process between the decision maker and the computer based system (Nijkamp 1979; Steuer 1986). Interactive multiobjective programming methods do not require a priori information about the decision maker's preference structure. The existence of a utility/value function is implicitly assumed and the function is maximized by means of a formal mechanism that involves an interactive exchange of information between a substantive model of the decision situation (computer-based decision support system) and the user. An interactive procedure consists of two phases: (i) in the dialogue phase, the decision maker analyzes and evaluates information provided by a computer-based system and articulates his/her preferences, and (ii) in the computational phase, a solution (or a group of solutions) that meets the decision maker's requirements specified in the dialogue phase, is generated. This interactive exchange of information is continued until an outcome is deemed acceptable to the decision maker.

Although there is a number of interactive multiobjective programming methods available (Steuer 1986; Korhonen and Wallenius 2010), the interactive approaches to spatial decision problems have been mostly limited to distance metric base methods (see Sect. 5.3). Examples of integrating GIS and interactive goal programming approaches are given in Coutinho-Rodrigues et al. (1997), Roettera et al. (2005), Santé and Crecentea (2007), and Alçada-Almeida et al. (2009). The reference point method is the core of spatial interactive decision support system developed by Antoine et al. (1997) and Agrell et al. (2004). Malczewski and Ogryczak (1996) provide a computational example of an interactive multiobjective approach to plant location problem (see also Malczewski and Ogryczak 1990).

Since the decision maker is an essential part of the multicriteria decision making process, an interactive method is a natural approach for tackling multiobjective decision problems (Korhonen and Wallenius 2010). Also, the methods are amenable to the use of graphical representation of alternative solutions to support the interactive process of decision making. This feature of interactive procedures is of particular significance as a component of spatial decision support (Church et al. 1992; Malczewski and Ogryczak 1996). There is evidence to show that GIS-based interactive methods provide valuable support for understanding and analyzing complex spatial decision problem (Alçada-Almeida et al. 2009).

5.5 Conclusion

This chapter discussed classic multiobjective optimization methods. It focused on those approaches that have been most often integrated into GIS: methods for generating non-inferior solutions, distance metric-based methods, and interactive methods. We overviewed GIS-based applications of multiobjective optimization methods, and signified relationships between different methods. First, we indicated that the weighting method for generating non-dominated solution can be considered the value/utility function method providing that suitable value/utility functions and associated objective weights have been elicited from the decision maker. Second, we demonstrated the links between the distance metric based methods. Specifically, compromise programming models with the $L_1(\boldsymbol{x})$ and $L_\infty(\boldsymbol{x})$ distance metrics are equivalents to the weighted and Chebyshev goal programme methods with the target values set at ideal levels. Also, the reference point method can be considered within the framework of goal programming as a Chebyshev goal programme, along with the $L_1(\boldsymbol{x})$ Pareto restoration procedure. Third, we indicated that distance metric based methods are often used as components of interactive approaches for tackling spatial decision problems. We emphasised the importance of displaying alternative solutions using GIS within the framework of interactive decision support procedures.

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