8

Dimensional Analysis and Scaling Rules

One might ask, here at the outset, "Why bother with 'scaling rules' when one can simulate practically any size and type cyclone of interest with a good model?" While the latter is generally true, scaling—when based on the performance of a sufficiently large, geometrically similar laboratory model—can predict the performance of an industrial cyclone installation considerably more accurately than the models. It is also the writers' experience that scaling rules are important for two additional reasons:

- Certain simplified scaling rules and dimensionless quantities allow the designer or practitioner to make quick, 'back-of-the-envelope' type calculations and decisions pertaining to cyclone design and performance.
- The scaling formulae allow one to better 'see' the effects of changes in one variable upon another—both qualitatively and quantitatively.

In this chapter we shall derive and present relationships or formulae that will allow us to predict a cyclone's cut-point diameter, grade-efficiency curve, overall or 'gross' efficiency, and pressure drop on the basis of measurements taken on a geometrically similar cyclone. These formulae should also allow us to evaluate the performance of an operating cyclone and, if necessary, assist us in troubleshooting its design, mechanical condition, or mode of operation.

When scaling cyclones we have to consider not only the fluid but also the particle dynamics. This might lead us to expect complicated scaling laws, but in the end we shall find that simple rules can provide a wealth of useful information.

In scaling we wish to predict the performance of one unit, which we will call the 'prototype', from that of another, the 'model'. We do this by identifying all parameters determining the unit's performance. We may not know the effect of each parameter, but we do know that the equations expressing the performance in terms of the parameters must be *dimensionally consistent*. This allows us to reduce the number of parameters by bundling them in dimensionless groups. Making these groups the same between model and prototype, we know that their dimensionless performance will also be the same (Perry, 1997).

To derive the dimensionless groups for cyclones, we can proceed along two lines of inquiry:

a) classical dimensional analysis, or
b) inspection of the equations of motion for gas and particles.

In a) we list the variables influencing the cyclone performance, and arrange them in dimensionless groups. In b) we arrive at the groups by making the equations of motion for the gas and the particles dimensionless. Both lines of enquiry are enlightening in their own way, so we shall follow both, the latter in Appendix 8.A.

8.1 Classical Dimensional Analysis

8.1.1 Separation Efficiency

The separation efficiency in a cyclone depends on a series of physical and operational parameters, which we can subdivide as follows:

- Parameters related to the individual particle
  - the particle size x,
  - the particle density ρ_p. We see in the equation of motion for the particle, Eq. (2.2.1), that both ρ_p itself and the density difference with the gas $(\rho_p - \rho)$ can be included, the former for the unsteady terms, the latter for the steady terms.
  - the particle shape, which we express as Wadell's sphericity ψ, defined as *the surface area of a volume equivalent sphere divided by the surface area of the actual particle.*
- Parameters related to the feed solids as a whole
  - the solids loading at the inlet, c_o.
  - the particle size distribution (PSD) of the feed solids, which can influence the grade-efficiency of the cyclone. In reality, these authors cannot confirm that the PSD influences the cyclone cut-point or grade-efficiency, but it has been claimed in the literature that large particles in the feed will 'sweep' smaller ones to the wall in the inlet region, so we include this parameter initially. If a mathematical distribution function is fitted to the feed, the size distribution can be characterized by a mean size $\langle x\rangle$, and a spread σ.
- Parameters related to the gas
  - the gas density ρ,
  - the gas viscosity μ,
  - a characteristic velocity v_{ch}. In practice, the inlet velocity v_{in} or the mean velocity in the vortex finder v_x are often preferred; some prefer the mean axial velocity in the cyclone body $\langle v_z\rangle$.

  - the gas relative humidity, RH. Components such as water vapour or ammonia can influence particle agglomeration or dispersion and, hence, grade and overall collection efficiency.
- Parameters related to the configuration of the cyclone
  - the cyclone size, which we can represent by the body diameter D,
  - geometry of the cyclone (H, a, b, D_x etc.),
  - roughness of the cyclone wall, k_s.
- Parameters related to conservative force fields
  - the gravitational acceleration g,
  - the Coulomb potential of an electrical field ϕ,
  - parameter giving the strength of other fields present; we assume there are none.

This is a large number of parameters, the full list is:

$$\begin{aligned}\eta(x) = \mathrm{f}(&x, \rho_p, \Delta\rho, \psi, c_o, \langle x\rangle, \sigma, \rho, \mu, v_{ch}, g, RH, \phi, D, H, a, b,\\ &D_x, \text{more geometrical parameters}).\end{aligned} \tag{8.1.1}$$

In order to render the process more tractable, we must make some simplifications and assumptions:

The simplifications:

- We ignore the effects arising from particle agglomeration, and, therefore, also the effects of the composition (humidity) of the gas.
- As is standard in scaling, we assume that the model and the prototype are geometrically similar. This means that all dimensionless numbers describing the cyclone geometry, for example the ratio of the vortex finder diameter to the body diameter: D_x/D, are the same between model and prototype.
- The particle sphericity ψ mainly enters the analysis because it influences the particle terminal velocity. We can account for its effect if we use the Stokesian diameter as a measure of particle size x rather than, for instance, a volume or mass equivalent diameter. We recall from Chap. 2 that the Stokesian (or "dynamically equivalent") diameter is the diameter of a sphere having the same terminal settling velocity and density as the particle under consideration.

The assumptions:

- The gravitational field represented by g is so small compared to the centrifugal field that its effect can be ignored. We need to qualify this: at higher solids loadings, the motion of the strands formed on the cyclone wall can be influenced by gravitational forces. This influence may or may not be beneficial depending upon cyclone orientation in the gravitational field.

- The particles are always at their terminal velocity (Chap. 2). In this case, ρ_p need not be included explicitly, but only the density difference: $\rho_p - \rho \equiv \Delta\rho$.

These simplifications and assumptions do not significantly limit the range of application of our analysis. We now make some additional assumptions, however, that do limit the range of systems to which our analysis can be applied. The reader confronted with a specific scaling problem should check which, if any, of the assumptions below are warranted for his or her system.

- We assume that no electrostatic field is present. So our analysis is not valid for electrostatically-enhanced cyclone separators.
- We assume that the solid loading is no more than 2–3 g dust per kg feed gas (*i.e.*, mass loading ratio of less than about 0.003 kg solids/kg gas). Then we can ignore the effect of the particles on the gas flow pattern, which is determined mainly by c_o, and, to a much lesser extent, by $\langle x\rangle$ and σ. This limits the range of validity of this analysis in practice to lightly-loaded cyclones and/or to second and third-stage units. Many first-stage or 'rough-cut' cyclones work at elevated solids loadings.
- Finally, we ignore the effect of the wall roughness, k_s. Wall roughness is due not only to the material of construction (e.g. metal, refractory lining) but also to the solids loading. Solids rolling and sliding along the wall give rise to an 'equivalent' wall roughness of their own (see Chaps. 4 and 6). Thus, in this chapter, we shall limit our discussion to cyclones with smooth walls and low solids loadings. Note that *cyclone wall roughness is important in many practical situations.* We look at the effect of wall friction in Chaps. 4, 5, 6 and 9. As a final comment on roughness, it is actually the difference in *relative* roughness between the model and the prototype that we are ignoring here. This relative roughness is defined as the absolute surface roughness k_s divided by the radius of the cyclone body (upper cylindrical section), $D/2$. A large commercial-scale cyclone with some wall deposits or surface erosion, for example, may have a relative roughness no larger than a small, 'smooth walled', laboratory cyclone. If this is the case, then the wall friction and, hence, wall shear stress, imposed on the gas flow is the same in both model and prototype at comparable cyclone Reynolds numbers or in the fully developed turbulent flow regime of the friction factor versus Reynolds number chart (described in Chap. 6). This is exactly analogous to ordinary flow in pipes. Recalling that the Reynolds number is the ratio of inertial to viscous forces, at high Reynolds numbers the flow within a cyclone becomes turbulent because the viscosity of the fluid is unable to dampen out the effects of any local disturbance.

Making all these simplifications, we state that the cyclone's separation efficiency, $\eta(x)$ is:

$$\eta\,(x) = \mathrm{f}(x, \Delta\rho, \rho, \mu, v_{ch}, D), \tag{8.1.2}$$

and by the classical techniques for dimensional analysis, for instance the *Buckingham Pi method* (Perry, 1997), we obtain:

$$\eta(x) = \mathrm{f}\left(\underset{\mathrm{I}}{\frac{\mu}{D\rho\, v_{ch}}}, \underset{\mathrm{II}}{\frac{x}{D}}, \underset{\mathrm{III}}{\frac{\Delta\rho}{\rho}}\right). \tag{8.1.3}$$

Group I is $1/Re$. It is a classical result in fluid mechanics that the Reynolds number Re determines the gas flow pattern.

One thing missing from Eq. (8.1.3) is a dimensionless group that relates directly to the movement or separation of the particles. This can be achieved by introducing the well-known 'Stokes number', Stk. We can create this new dimensionless number by multiplying together powers of the existing numbers as follows:

$$\frac{1}{18} \times \mathrm{III} \times \mathrm{II}^2 \times \mathrm{I}^{-1} = \frac{\Delta\rho\, x^2 v_{ch}}{18\mu D} \equiv Stk. \tag{8.1.4}$$

This new number can replace any one of the numbers from which it was made without loss of information. If we replace group II with Stk, we obtain:

$$\eta(x) = \mathrm{f}\left(Re, Stk, \frac{\Delta\rho}{\rho}\right). \tag{8.1.5}$$

This is as far as classical dimensional analysis can take us. However, in Appendix 8.A we obtain more physical insight by inspecting the equations of motion for the gas and the particles. One important result of this is that the density ratio in (8.1.5) need not appear separately, as the effect of the particle density is accounted for in Stk. This fact allows us to simplify (8.1.5) even further so that it becomes:

$$\eta(x) = \mathrm{f}(Re, Stk). \tag{8.1.6}$$

Often the designer or investigator is interested only in the cut size x_{50}, when applying scaling rules to cyclones. Then $\eta(x)$ in (8.1.6) can be set equal to 0.5 and, denoting the Stokes number corresponding to x_{50} by Stk_{50}, Eq. (8.1.6) gives:

$$Stk_{50} = \mathrm{f}(Re). \tag{8.1.7}$$

8.1.2 Pressure Drop

A similar analysis can be made for the cyclone pressure drop Δp. If we include all the variables influencing Δp we obtain:

$$\Delta p = \mathrm{f}(\langle x\rangle, \sigma, c_o, \rho, \mu, v_{ch}, D, k_s, g, \text{geometrical parameters}). \tag{8.1.8}$$

Here also we assume geometrical similarity and a low solids loading, so that the effect of the geometrical variables, $\langle x\rangle$, σ and c_o can be neglected. We also neglect k_s. After these simplifications, performing the dimensional analysis gives:

$$\frac{\Delta p}{\rho v_{ch}^2} = \mathrm{f}\left(\frac{\rho v_{ch} D}{\mu}, \frac{gD}{v_{ch}^2}\right). \tag{8.1.9}$$

The group on the left-hand side we recognize as one half of the Euler number, Eu. The groups on the RHS are the Reynolds number Re, and the Froude number Fr, respectively. Thus:

$$Eu = \mathrm{f}\left(Re, Fr\right). \tag{8.1.10}$$

Fr describes the influence of gravity on the flow field. This can be dismissed directly, referring to the classical result that there is no effect of gravity in the absence of free fluid surfaces or stratification in the system. Thus:

$$Eu = \mathrm{f}\left(Re\right). \tag{8.1.11}$$

We may note that this result is identical to that which we would have obtained had we applied the above analysis to the flow of a fluid through a smooth-walled pipe.

We should reiterate that the dimensional analyses in these sections only apply to cyclones operating at low solid loading, where the effect of the particles on the gas flow can be ignored.

This completes our treatment of dimensional analysis in cyclones. In the following section we look at applying it in practice.

8.2 Scaling Cyclones in Practice

The formal rules for scaling of cyclones have thus provided us with some fairly simple scaling rules. In practice, even further simplification is possible, and we shall discuss this below.

8.2.1 Approximately Constant Stk_{50} over a Wide Range of Re

Equations (8.1.7) and (8.1.11) suggest that if we arrange experiments in a geometrically similar model to have the same Re as the prototype, then Stk_{50} and Eu will be the same as well. From Stk_{50} and Eu we can then calculate the cut size and the pressure drop in the prototype. Thus, although the pressure drop or the cut size for the prototype and the model are different in general, one can use the pressure drop and cut size obtained from model tests to predict these same quantities for the prototype.

Still, there is a practical problem. Due mainly to their difference in the characteristic size (D), it is not always easy to obtain Reynolds number similarity in a laboratory model with an industrial-scale cyclone without operating the model at very high velocities or constructing a very large 'model'. The first example presented in Appendix 8.B is included to help illustrate this point.

To avoid the problem of having to deal with extremely high velocities in the model experiments, it is possible to perform such studies with water as the carrier fluid, rather than a gas.

An additional problem in achieving Reynolds-number similarity is that, when comparing the performance of one industrial cyclone with that of another, obtaining data at the same Re for the two is often not possible.

But is it really necessary to scale-up cyclones on the basis of Re similarity? A redeeming feature is that, in many cases, *Reynolds number similarity is not very critical.* This has long been known, but the issue has only been studied quantitatively recently, by Overcamp and Scarlett (1993), among others. They defined Re and Stk in terms of the inlet velocity. We shall use the symbols Re_{in} and Stk_{in}, respectively. Figure 8.2.1 shows a plot of the square root of Stk_{in50} against Re_{in} for a wide range of cyclones, taken from their paper.

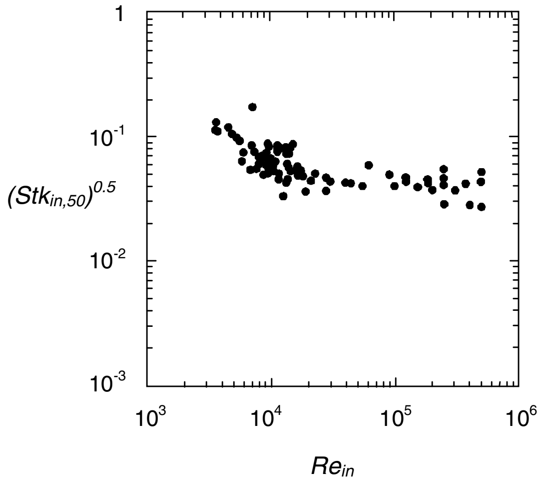

Fig. 8.2.1. Stk_{in50} vs. Re_{in} for a range of cyclones, taken from Overcamp and Scarlett (1993)

Most commercial-sized cyclones operate at Re_{in} values between 10^5 and 10^6. Thus, for the commercially important range of Re_{in} greater than 2×10^4, Stk_{in50} is seen to be reasonably independent of Re_{in}. This lack of, or weak, dependency of Stk_{50} (or Stk corresponding to some other efficiency) upon Re is the basis for the widely used 'Stokesian scaling' of cyclones. We therefore arrive at the important conclusion that, in Stokesian scaling, the same separation efficiency is assumed for the same value of Stk in geometrically similar cyclones. We can see this more clearly if we examine the equation for the 50% collection efficiency point as computed by means of our equilibrium-orbit model from Chap. 5. Therein, we recall, we performed a simple force balance (centrifugal force in equilibrium with Stokes' drag force) on a particle orbiting on an imaginary cylinder of diameter D_x. This led to the following expression for the cut size (here we include the gas density via the $\Delta\rho$ term)

$$x_{50} = \sqrt{\frac{9\mu v_{rCS} D_x}{\Delta\rho v_{\theta CS}^2}}, \tag{8.2.1}$$

which can be written as,

$$\frac{x_{50}^2 v_{\theta CS} \Delta\rho}{9\mu D_x} \equiv Stk_{50} = \frac{v_{rCS}}{v_{\theta CS}} \tag{8.2.2}$$

where we here have chosen $2 \times v_{\theta CS}$ as the characteristic velocity v_{ch} in Stk_{50}. Thus, from the equilibrium orbit model point of view, when we apply Stokesian scaling for the computation of x_{50}, this is equialvent to stating that the ratio of radial to tangential velocity,

$$\frac{v_{rCS}}{v_{\theta CS}}$$

at the equilibrium orbit position is the same for both of our geometrically similar cyclones. From our general knowledge of fluid flow in various equipment types, a constant velocity ratio is what we would expect for fully developed, gas phase turbulent flow conditions.

It is interesting to note that the Stokes number that we present above, and which we can readily derive from an elementary force balance on an equilibrium orbiting particle, multiplied by the velocity ratio $\frac{v_{\theta CS}}{v_{rCS}}$ is simply equal to the ratio of drag to inertial forces acting upon the particle.

In a stricter sense, however, Stk is also the ratio of the response or relaxation time of the particle, $\Delta\rho/18\mu$ (see Chap. 2), to a characteristic time scale of the fluid, D/v_{ch}. In addition the Stokes number shown in Eq. (8.2.2) can also be expressed in the form:

$$Stk_{50} = \frac{x_{50}^2 v_{\theta CS} \Delta\rho}{9\mu D_x} = \frac{\frac{x_{50}^2 v_{\theta CS} \Delta\rho}{9\mu}}{D_x} \equiv \frac{d_s}{D_x}, \tag{8.2.3}$$

which is the ratio of the x_{50} particle's "*stopping distance*" to some characteristic cyclone dimension, in this case, the diameter of the vortex finder, D_x. This stopping distance is the distance that particle of size x_{50} would travel against fluid drag if the fluid surrounding the particle was to suddenly stop its motion. Thus when we apply Stokesian scaling, we are also implicity stating that the dimensionaless stopping distance ratio, d_s/D_x, reamins constant between model and prototype. If, for example, we were to increase the size of a cyclone so that D_x increases, in order for the Stokes number to remain constant, the equilbrium particle's stopping distance, d_s, would also have to increase. If we hold $v_{\theta CS}$, $\Delta\rho$ and μ constant and double D_x, then x_{50}^2 would have to double, causing x_{50} to increase by the square root of 2. Thus, scaling-up a cyclone always results in an increase in cut size if the other variables (comprising the stopping distance) are fixed. It is for this reason, and from this perspective, that one sometimes encounters the statement “cyclones do not scale-up”.

For $Re_{in} < 2 \times 10^4$, Stk_{in50} can be seen to increase, showing that small cyclones are less efficient than one would expect from Stokesian scaling. Our defining equation, (8.1.4), for the Stokes number,

$$\frac{\Delta\rho x^2 v_{ch}}{18\mu D} \equiv Stk \tag{8.2.4}$$

shows that an increasing Stk_{50} means an increasing x_{50}.

We can give an idea of the scale where this becomes a consideration. In a cyclone with a tangential inlet of circular cross section working with ambient air at an inlet velocity of 15 m/s, $Re_{in} = 2 \times 10^4$ would correspond to an inlet diameter of 2 cm. The reduced efficiency is therefore only a feature in small sampling cyclones or in very small ‘multicyclone’ banks. Cyclones in ‘small’ industrial units comprising multicyclone installations are seldom less than 15 cm in diameter.

We should mention that Lidén and Gudmundsson (1997) found some variation in Stk_{50} with Re, even at larger Re. Their study mostly concentrated on small cyclones, typically a few centimeters in diameter.

Figure 8.2.1 includes data from cyclones of different geometries. Although we can clearly see the trend in the figure, there is considerable scatter. For $Re_{in} > 2 \times 10^4$, $(Stk_{in50})^{0.5}$ varies by a factor of 4, and so does the cut-point diameter. Trying to predict the performance of all cyclones, irrespective of the geometry, from a Stk_{50}-Re plot is therefore not a worthwhile exercise. Even so, the plot does give a ‘ball-park’ estimate of the Stk_{in50} of cyclones.

Since separation performance, as measured by Stk or Stk_{in50}, is virtually independent of Re it is not necessary to maintain a constant Re between the model and the prototype when attempting cyclone scale-up. In most such laboratory studies, it is far preferable to use air rather than water. This also makes it easier to find a test dust, since many dusts give problems with solubilization or swelling or incomplete particle wetting, among other things, when dispersed in water.

8.2.2 *Eu* Only Weakly Dependent on *Re*

As we found for separation performance described above, Re-similarity is not critical for the pressure drop, either. In Chap. 4 we found that many of the empirical models for cyclone pressure drop only contain the ratio of inlet to outlet areas, implying that Eu will be the same between geometrically similar cyclones, irrespective of Re-similarity. Obviously, as was the case for separation efficiency, this is only valid when Re is high enough that the friction factor is essentially independent of Re. This should come as no real surprise since the same situation holds true for most flow devices (such as pipes, elbows, orifices, contractions and expansions, etc.) that operate in fully developed turbulent flow. In such cases, pressure loss can be characterized by the formula:

$$\Delta p = K\frac{1}{2}\rho v^2 \tag{8.2.5}$$

where K is recognized as the familiar Eu number which we defined earlier. For such devices it is well known that K is very weakly dependent upon Re and can generally be treated as a constant in matters of practical interest.

This principle does have its limitation. The scaling rule of constant Eu and the models in Chap. 4 have, in the authors' experience, significantly overpredicted pressure loss in very large-scale cyclones. There is a weak but definite Re-number effect on the pressure loss coefficient Eu. This variation in Eu is only with Re to the power of -0.17 to -0.2 but, when scaling up small scale lab data by a factor of 10 or more, one can easily overpredict pressure loss by 50% or more.

In most cases, such an error on the conservative side in predicting pressure drop is perfectly acceptable since, if anything, the plant will experience less pressure drop through the cyclone installation and this seldom creates an operational problem. However, if there is some delicate pressure balance across a slide valve, for example, which relies on an accurate knowledge of the cyclone's pressure drop, then one should try to acquire the most accurate estimate possible.

8.2.3 Some other Considerations

The above scaling rules have been found by experience to be valid in cyclones of somewhat conventional designs operating at low solids loadings and normal inlet velocities.

But cut size, grade-efficiency and pressure drop in such conventional designs are not the only items one must consider when designing a cyclone or evaluating its performance. Other factors will often play a crucial role; these may include:

- the design configuration and operating environment at the gas outlet and the solids discharge,
- the position and action of the vortex natural turning point,
- the effectiveness of the underflow seal in preventing gas upflow,
- the effect of solids loading upon overall separation efficiency and pressure loss,
- the effect of wall roughness upon separation performance and pressure loss,
- the effects of physical damage, poor construction, wall deposits, wall flow disturbances and other such nonideal conditions.

Most of these factors are discussed elsewhere in this book. The effects of the natural turning point and solids loading are discussed in Chap. 9.

In geometries where the position of the end of the vortex is known to affect performance (for instance if the cyclone length is near the 'critical length'), one would require Re-similarity between the model and the prototype if one were to have confidence in the scale-up.

8.2.4 *Stk*-*Eu* Relationships

In process engineering work the viewpoint is often taken that an improved quality of separation or purification is achieved at a correspondingly higher cost. Empirical relationships are developed which relate the quality of separation achieved to the cost. Not surprisingly this includes both cyclones and swirl tube separators. The measure of the quality of separation is the cut size, x_{50} or the dimensionless cut size, Stk_{50}, and the 'cost' is the pressure drop required to achieve this, or its dimensionless measure: Eu.

It follows from Eqs. (8.1.7) and (8.1.11):

$$Stk_{50} = \mathrm{f}\left(Eu\right). \tag{8.2.6}$$

Svarovsky (1984) found that for all 'reasonable' cyclone designs:

$$Eu_b\sqrt{Stk_{b,50}} = \sqrt{12}. \tag{8.2.7}$$

The mean axial velocity in the cyclone body $\langle v_z \rangle$ was used to evaluate the Reynolds and Euler numbers. This is signified with the subscript b. His plot, featuring the line representing Eq. (8.2.7) and his supporting data, is shown in Fig. 8.2.2.

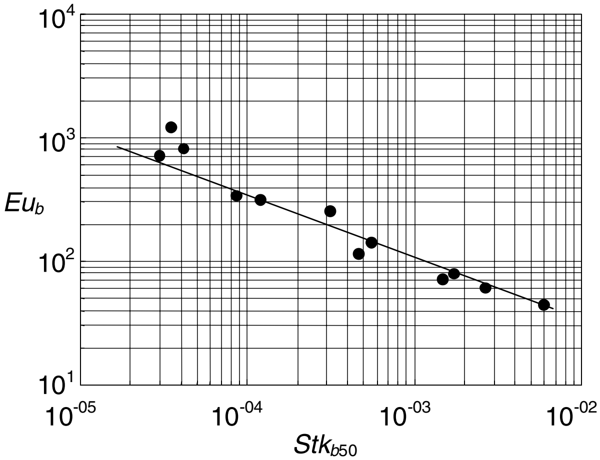

Fig. 8.2.2. Eu_b vs. Stk_{b50} for a variety of cyclones according to Svarovsky (1984). The line represents Eq. (8.2.7)

Others have presented similar correlations. Karpov and Saburov (1998) give:

$$Eu_b\ Stk_{b50} = 0.8, \tag{8.2.8}$$

which lies considerably above the line of Svarovsky, and thus gives a more pessimistic view of cyclone performance.

It is noteworthy that if one makes the reasonable assumption: $H \cong 3D$, then Rietema's criterion $Cy_{50} = 3.5$ (see Chap. 5) can be written $Eu_b Stk_{b50} = 0.92$, which is very close to Eq. (8.2.8). However, as we saw in Chap. 5, Rietema's model predicts the effect of cyclone length correctly, while Eqs. (8.2.7) and (8.2.8) do not.

We should also mention the work of Bürkholz (1989), who, by a variant of dimensional analysis, derived a general separation parameter applicable to various types of separation equipment based on impaction, including cyclones, mist-mats, lamellar fibre filters and packed beds. Bürkholz' derivation begins with a dimensional analysis including, in addition to the variables in Eq. (8.1.2), a height H[1], the gravitational acceleration, g, and *the pressure drop,* Δp. The two former variables are quickly eliminated, while inclusion of the latter is an important aspect of the analysis, and allows Bürkholz to arrive directly at a relationship between Stk_{in50} and Eu_{in}. We note that including Δp in the analysis is not quite *kosher* in terms of dimensional analysis, since Δp is not an independent, but a dependent, variable.

This procedure initially leads Bürkholz to a relation:

$$\eta(x) = f(Stk_{in}, Re_{in}, Eu_{in}) \tag{8.2.9}$$

where the subscript *in* signifies that Bürkholz used v_{in} as a characteristic velocity. Note that Bürkholz' definitions of Eu_{in} and Stk_{in} miss the factors 1/2 and 18, respectively, in the numerators relative to our definitions.

Various further simplifications, partly based on experimental evidence and similar in nature to the simplification that earlier allowed us to eliminate the density ratio $\Delta\rho/\rho$ before, allow Bürkholz to reduce these three dimensionless groups to one:

$$\eta(x) = f(\psi_A), \qquad \psi_A \equiv \frac{1}{4}\frac{\rho_s \Delta p^{\frac{2}{3}} x}{\rho^{\frac{1}{3}} \mu^{\frac{4}{3}} D^{\frac{2}{3}}} = Stk_{in} Re_{in}^{\frac{1}{3}} Eu_{in}^{\frac{2}{3}} \frac{1}{4}\frac{18}{2^{\frac{2}{3}}} \tag{8.2.10}$$

where the last numerical factor on the right-hand-side arises from the different definitions of Stk_{in} and Eu_{in}. Some of the simplifications leading to Eq. (8.2.10) are easy to follow, while the reasoning behind one of them remains obscure, at least to these authors.

Bürkholz found by experiment that in cyclones $\sqrt{\psi_A} \approx 1.7$ for $\eta(x) = 0.5$, i.e. by the cutsize of the cyclone. Substituting this in (8.2.10), and solving for Eu_{in} gives:

$$Eu_{in} = 2\left(\frac{1.7^2}{\frac{1}{4} 18 Stk_{in50} Re_{in}^{\frac{1}{3}}}\right)^{\frac{2}{3}}. \tag{8.2.11}$$

which is also a *Stk*-*Eu* relation.

[1] This is a geometrical variable. Bürkholz' initial derivation relates to a mist mat, and H is its thickness

These authors' experience is that Svarovsky's relation, Fig. 8.2.2 or Eq. (8.2.7) works very well. We have presented the other relations for completeness, and for the reader to recognize them should he or she encounter them in other contexts. It is possible that the difference between the various relations to some extent reflects a difference in the experimental methods on which they are based, namely a difference in the method of measuring the outlet pressure, as discussed in Chap. 4.

The line fitting the experimental results by Svarovsky (see Fig. 8.2.2 or Eq. (8.2.7)) tells us that the Stokes number varies as the reciprocal of the square of the Euler number. Furthermore, since the Stokes number varies as the square of the cut size, x_{50}, and the Euler number varies directly with pressure loss, Δp, it follows that the cut size is inversely proportional to pressure loss. Thus, other factors unchanged, any attempt to decrease the cut size will be accompanied by an increase in pressure loss.

Figure 8.2.2 also serves as a sort of benchmark for comparing cyclone designs or for evaluating the performance of a working cyclone. For example, if we were to gather test data on a lightly loaded cyclone system and plot its Stokes number versus Euler number on Figure 8.2.2, the resulting line *should* lie close to or below Svarovsky's line. If, on the other hand, the point were to lie significantly above his line, this would be a good indication that something is hampering the cyclone's performance. This could include any number of factors: physical damage, poor constuction, wall deposits, wall flow disturbances, blockage, gas upflow and even poor design practice. For cyclones operating in parallel, connected by common plenums, an operating point significantly above Svarovsky's line may indicate problems arising from cross-talk (see Chap. 16).

We may also use Svarovsky's graph or equation to get a rough estimate of the cut size that we could expect at some maximum allowable pressure loss. In this case, we would first compute the Euler number knowing the allowable pressure loss, the gas density and the superficial axial velocity (based on the cyclone's cross sectional area). We would then use Fig. 8.2.2 or Eq. (8.2.7) to compute the corresponding Stokes number and, from this, the cyclone's cut size (knowing the gas properties, cyclone diameter and, again, the superficial axial velocity). Clearly, we could reverse this process and obtain an estimate of pressure loss corresponding to a given cut size.

The relationships derived in this chapter allow us to predict a cyclone's cut-point diameter, grade-efficiency curve, overall or 'gross' efficiency, and pressure drop on the basis of measurements taken on another, geometrically similar, cyclone. They also allow us to assess the performance of an operating cyclone and determine whether or not there is something wrong with its design, or with its physical/mechanical condition, or in the way in which it is operated. We will look at an example in Appendix 8.B.

8.A Inspecting the Equations of Motion

In this appendix we make the equations of motion for gas and particles dimensionless, so that the parameters form dimensionless groups. If these groups are equal for model and prototype, the governing equations are identical in the two.

8.A.1 Equation of Motion for the Gas

The flow pattern of the gas can be determined by solving the Navier-Stokes equations if there is no influence from the particles. If we assume a Newtonian viscosity in Eq. (2.A.2), it becomes:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}. \tag{8.A.1}$$

Using a characteristic velocity v_{ch}, along with D and ρ as scaling parameters, we get the following dimensionless parameters:

$$\mathbf{v}^* \equiv \frac{\mathbf{v}}{v_{ch}};\ p^* \equiv \frac{(p - p_o)}{\rho v_{ch}^2};\ t^* \equiv \frac{t v_{ch}}{D};\ \nabla^* \equiv D\nabla;\ \nabla^{*2} \equiv D^2 \nabla^2; \\ \frac{D}{Dt^*} \equiv \frac{D}{v_{ch}} \frac{D}{Dt}.$$

Introducing these in Eq. (8.A.1), we get, after some work (Bird et al., 2002)

$$\frac{D\mathbf{v}^*}{Dt^*} = -\nabla^* p^* + \left(\frac{1}{Re}\right) \nabla^{*2} \mathbf{v}^* + \left(\frac{1}{Fr}\right) \frac{\mathbf{g}}{g}. \tag{8.A.2}$$

The dimensionless velocity and pressure of the gas are thus determined by Re and Fr. As mentioned in the main text, gravity only affects a flow pattern if the system contains free surfaces or stratification layers. Thus, Reynolds number similarity with geometric similarity is enough to ensure dynamic similarity for a gas cyclone with low solids loading.

This is the formal requirement for dynamic similarity, and is consistent with the results of the classical dimensional analysis in the main text. As we mentioned there, experience teaches us that over a wide range of operating conditions Reynolds number similarity is not all that critical for Stokes number similarity between cyclones, and this indicates that, in this range, it is not all that critical for dynamic similarity.

8.A.2 Equation of Motion for a Particle

The Lagrangian equation of motion of a particle rotating at the radial position r in a centrifugal field with circumferential velocity v_θ is (Eq. 2.2.4 resolved in the radial direction)

$$\left(\frac{\pi x^3}{6}\right)\rho_p\frac{dU_r'}{dt} = -3\pi x\mu U_r' + \left(\frac{\pi x^3}{6}\right)\Delta\rho\frac{v_\theta^2}{r} \tag{8.A.3}$$

where we have assumed that the particle is sufficiently small for Stokes drag law to apply. Making this equation dimensionless, using the same scaling parameters as above, v_{ch}, D and ρ, gives:

$$Stk\frac{\rho_p}{\Delta\rho}\frac{dU_r'^*}{dt} = -U_r'^* + Stk\frac{v_\theta^{*2}}{r^*}. \tag{8.A.4}$$

If the motion is steady, the left-hand side is zero. The steady motion of the particle is therefore determined by Stk, and ρ_p does not need to be included explicitly in the analysis, as mentioned in the discussion following Eq. (8.1.4). Also the density ratio $\Delta\rho/\rho$ does not occur in the equations when the added mass and Basset terms are neglected.

Inspection of the equations of motion of the gas and particle phases has thus confirmed the results of classical dimensional analysis, simplified the results of the analysis further, and has, we trust, increased our understanding of the physical significance of the dimensionless groups.

8.B Sample Cyclone Scaling Calculations

8.B.1 Calculating the Inlet Velocity in a Scale Model Required for *Re* Similarity

Determine the velocity at which we would have to operate a 6" (152 mm) diameter model cyclone to obtain Reynolds number similarity to a 48" (1220 mm) industrial cyclone.

Operating Conditions:

Model: $\rho_m = 1.19$ kg/m^3
$D_m = 0.152$ m (ID)
$\mu_m = 1.8\ 10^{-5}$ Pa s
$v_{in,m} =$ (to be determined)

Prototype: $\rho = 1.30$ kg/m^3
$D = 1.22$ m
$\mu = 3.710^{-5}$ Pa s
$v_{in} = 22.4$ m/s

Herein, the subscript m refers to the model, no subscript to the prototype.

Solution

Since $Re_{in,m} = Re_{in}$, it follows that,

$$v_{in,m} = \rho v_{in} D \mu_m / (\rho_m D_m \mu)$$

and, upon substitution, we find that,

$$v_{inm} = 82.4\text{m/s!}$$

This is an extremely high velocity and, for an inlet area of 5×10^{-3} m^2 (8 in^2) for the model (corresponding to 0.33 m^2 or 512 in^2 for the prototype), the model cyclone would require 0.42 m^3/s or 15 ft^3/sec of blower capacity (at a considerable pressure loss). Such a flow condition would normally exceed the limits of most laboratory facilities. Additionally, a laboratory-sized cyclone operating at such high inlet velocities would exhibit such a small cut-point diameter that it *could* prove difficult to collect sufficient overhead solids to permit an accurate determination of the cut size and the grade-efficiency curve. Static charge effects created at such high velocities, along with particle agglomeration, could also complicate the analysis.

Although the 1.2 m diameter cyclone used in the preceding example may seem rather large, many industrially important processes utilize cyclones of at least this size. Some are as large as 4 m or 13 feet.

8.B.2 Predicting Full-Scale Cyclone Performance using a Scale Model

A model cyclone of diameter D = 0.2 m operating at an inlet velocity v_{in} = 15 m/s at ambient conditions on a chalk powder of density 2700 kg/m^3, at low solids loading, is by testing found to have the grade-efficiency curve shown in Fig. 8.B.1. The cyclone pressure drop was found to be 950 Pa.

We wish to predict the performance of a geometrically similar cyclone of diameter D = 1.5 m operating with an inlet velocity v_{in} of 20 m/s, separating catalyst particles from a gas consisting of light hydrocarbons at elevated temperature and pressure in an FCC reactor installation.

Solution

We note from the experimental data reported above that the model's cut-point diameter, x_{50}, is 0.98 μm. We do not have data at the same Re for model and prototype, so we will make use of the approximations mentioned in the main text. Re_{in} is large enough in both model and prototype to assume that the grade-efficiency is about the same in the two cyclones for the same value of Stk, and that their Eu values are the same.

We look up the physical properties of gas and particles in the industrial unit. They are summarized together with the relevant geometrical data in Table 8.B.1

We begin by scaling the entire grade-efficiency curve. We calculate Stk corresponding to the particle sizes in the model data. These values of Stk

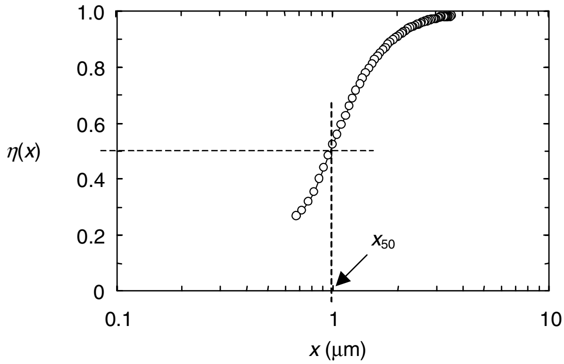

Fig. 8.B.1. Experimental grade-efficiency data for a laboratory model cyclone

Table 8.B.1. Relevant physical and operational data for model and prototype

	Model	Industrial unit
$\Delta\rho$	2700 kg/m^3	1500 kg/m^3
v_{in}	15 m/s	20 m/s
μ	$1.8.\times 10^{-5}$ kg/ms	$1.5.\times 10^{-5}$ kg/ms
ρ	1.2 kg/m^3	1.0 kg/m^3
D	0.2 m	1.5 m
a	0.1 m	0.75 m
b	0.04 m	0.3 m

must correspond to the same efficiencies in the prototype, and we can back-calculate the corresponding particle sizes. Using subscript i as an index for the experimental points on the grade-efficiency curve, the scheme can be written:

$$\text{Given } (x_i, \eta_i)_{\text{model}} \rightarrow \text{ calculate } (Stk_i, \eta_i)_{\text{model}} = (Stk_i, \eta_i)_{\text{prototype}} \rightarrow \text{ calculate } (x_i, \eta_i)_{\text{prototype}}.$$

For instance, one of the points on the curve in Fig. 8.B.1 is: $(x_i,\ \eta_i) = (1.15,\ 0.637)$. Since we are assuming approximate dynamic similarity between the model and prototype we can use *Stk* based on any characteristic velocity, such as v_{in}, which is the velocity we have been given. We then calculate:

$$Stk_{in,i} = \frac{\Delta\rho x_i^2 v_{in}}{18\mu D} = \frac{2700 \times \left(1.15 \times 10^{-6}\right)^2 \times 15}{18 \times (1.8 \times 10^{-5}) \times 0.2} = 8.27 \times 10^{-4}.$$

This is also the value of $Stk_{in,i}$ for the same η_i in the prototype, so we can back-calculate the corresponding value of x_i in the prototype by solving for x_i in:

$$8.27 \times 10^{-4} = \frac{1500 \times x_i^2 \times 20}{18 \times (1.5 \times 10^{-5}) \times 1.5} \text{ giving: } x_i = 3.34 \times 10^{-6}\text{m} = 3.3\ \mu\text{m}.$$

Repeating this for all the points gives the data for the prototype shown by the solid black points in Fig. 8.B.2.

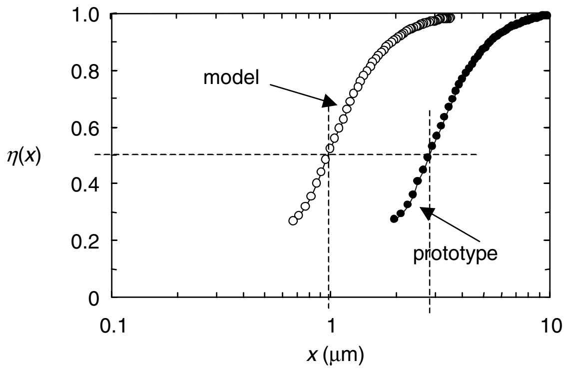

Fig. 8.B.2. G-E data for the model, with the calculated G-E data for the prototype

We can read off the cut size of the prototype in the figure, or we can calculate it directly by setting Stk_{in50} equal in the two cyclones:

$$\left(\frac{\Delta\rho x_{50}^2 v_{in}}{18\mu D}\right)_{\text{model}} = \left(\frac{\Delta\rho x_{50}^2 v_{in}}{18\mu D}\right)_{\text{prototype}},$$

and solve for x_{50} in the prototype. The result is:

$$x_{50} = 2.85 \times 10^{-6}\text{m} = 2.8\mu\text{m}.$$

We see from the figure that we can expect all catalyst particles greater than about 10 μm to be completely captured in the industrial unit. We also see that the grade-efficiency curve for the prototype has the same s-shape form as that of the model on the logarithmic scale.

One may quickly obtain a rough 'back of the envelope' estimate of the overall or gross separation performance of a lightly-loaded cyclone as follows (see also Sect. 3.2.3):

First, we 'fit' the experimental G-E data with a simple step function or stair-step curve, as shown in Fig. 8.B.3, for example. This step function representation of the s-shaped G-E curve has the properties:

$$\eta(x) = 0 \quad \text{for} \quad x < x_{50} \quad \text{and} \quad \eta(x) = 1 \quad \text{for} \quad x > x_{50}.$$

Next, we note the weight percent of the feed particles $> x_{50}$. This is our estimate of the overall or gross collection efficiency. The weight percent $< x_{50}$ comprises the losses.

The authors have found this technique to be quite useful in practice, especially in situations where one is anticipating the effect a change in cut-point

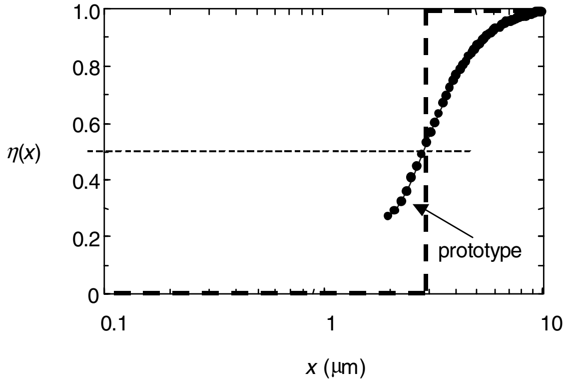

Fig. 8.B.3. Grade-efficiency data for the prototype showing a stair-step type of approximation

diameter may have upon total collection performance, namely, the loss fraction.

When viewed from this 'step function' perspective the cyclone behaves in a manner that is completely analogous to a perfect sieve: all particles entering the 'sieve' (the cyclone) that are greater than the sieve openings (the cyclone cut-point) are retained (captured). Likewise, all particles that enter but are less than the sieve openings (cut-point) pass on though (exit with the overhead gases). Even for the particle that may become 'stuck' in the sieve openings there is an analogy—this is the cyclone's cut size or, as it is sometimes called, its cut-point.

Let us now try to estimate the overall pressure drop in the commercial reactor cyclone. The discussion in the main text of this chapter and in Chap. 4 leads us to expect the Eu number to be the same for the two. In addition, we can calculate Eu on basis of the inlet velocity:

$$
\begin{aligned}
(Eu_{in})_{\mathrm{model}} = \left(\frac{\Delta p}{\frac{1}{2}\rho v_{in}^2}\right)_{\mathrm{model}} &= \frac{950}{\frac{1}{2}\times 1.2\times 15^2} = 7.03 \\
= (Eu_{in})_{\mathrm{prototype}} &= \frac{(\Delta p)_{\mathrm{prototype}}}{\frac{1}{2}\times 1.0\times 20^2} .
\end{aligned}
$$

Solving for the pressure drop over the industrial prototype, we obtain a value of 1400 Pa. We note that, even though the gas density was lower in the commercial unit, the overall pressure drop for the commercial unit increased, relative to the model, due the increase in inlet velocity.

It should also be pointed out that the Eu number of the commercial cyclone is essentially constant and independent of variations in any of the variables comprising this dimensionless number, such as gas density or inlet velocity. Thus, if we were to know its pressure drop for any operating condition then, knowing also the gas density and its inlet velocity, we could quite easily compute its Euler number. From this, we could then compute its pressure drop at any gas density and/or inlet velocity of interest. In the above example, if the inlet velocity were to be increased to 30 m/sec, the pressure drop would increase to $(30/20)^2 \times 1410 = 3170$ Pa. Such simple scaling methods can be of very practical value to the plant or support engineer.

We will leave it to the reader to locate the point corresponding to Stk and Eu on the plot of Svarovsky, to see how well this cyclone works. Remember to change the scale velocity from v_{in} to $\langle v_z \rangle$. You will find the necessary information for this in Table 8.B.1.