2

Basic Ideas

In order to understand the practical working of cyclones, it is necessary to master a number of topics, which span a range of different disciplines. Fluid mechanics, particularly relating to swirling flows, particle motion in a fluid, and different aspects of particle properties, such as size and size distribution, shape, and density, are all topics relevant to the later chapters.

This chapter contains a series of short discussions of these topics. Due to the nature of the subject matter, it can be difficult to recognize a 'red thread' in this chapter, but the account of each particular topic should be sufficient for appreciating the material in the subsequent chapters. Literature references are given for the reader wishing to study the disciplines more broadly.

2.1 Gas Flow

This section discusses some aspects of fluid mechanics that are particularly relevant to cyclones and swirl tubes.

2.1.1 Swirling Flow

Swirling flow, or vortex flow, occurs in different types of equipment, such as cyclones, hydrocyclones, spray dryers and vortex burners. Swirling flow also plays a central role in the developing fields of fluidics and process intensification. It is also the basis for the operation of foam-breaking or 'defoaming' separators that have received significant industrial attention in recent years.

We derive the equations for the tangential velocity distribution in two types of ideal swirling flows:

1. forced vortex flow, which is swirling flow with the same tangential velocity distribution as a rotating solid body, and
2. free vortex flow, which is the way a frictionless fluid would swirl. The tangential velocity in such a swirl is such that the moment-of-momentum of fluid elements is the same at all radii.

The tangential velocity distribution in real swirling flows is intermediate between these two extremes.

In order to derive these equations, we start by considering the forces acting on a fluid element in a swirling flow, as shown in Fig. 2.1.1. We use a cylindrical coordinate system (r, θ, z) (Weisstein, 1999) fixed in space with the z-axis, the axis of rotation, pointing out of the paper.

As the element rotates, it accelerates toward the center. If it did not accelerate it would continue in a linear path tangent to the orbit toward the axis of rotation. This acceleration is the 'centripetal acceleration'.

If we observe the element from a coordinate system, which is not fixed in space by rotating with the element, the centripetal acceleration will not be observed, but will appear as an *apparent force* directed away from the axis of rotation, the 'centrifugal force' (Fig. 2.1.1 b). This latter force is similar in nature to the gravity force, and acts away from the axis of rotation with a magnitude equal to the mass of the element times the centripetal acceleration.

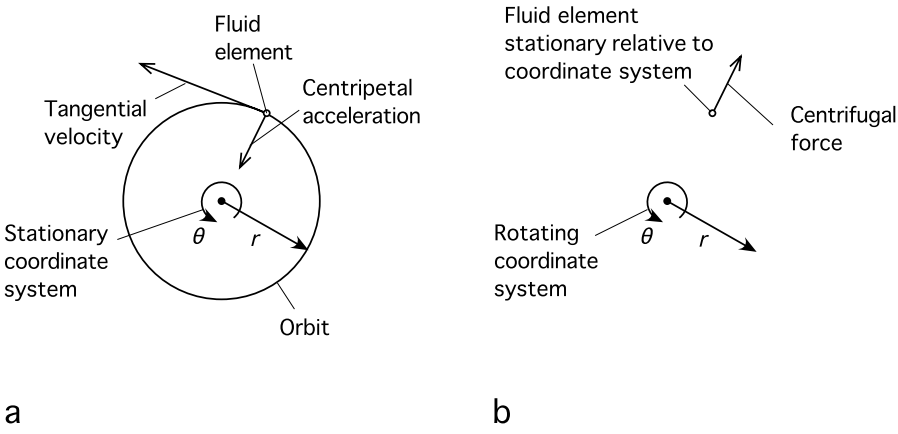

Fig. 2.1.1. A fluid element in a swirling flow, from two different points of view **a** a fixed coordinate system, and **b** a coordinate system rotating with the element

Strictly speaking, Newton's equations of motion apply only in a coordinate system that is not accelerating (in this case, rotating). Nevertheless, for mathematical simplicity, scientists and engineers often use an accelerating coordinate system (a rotating one, for instance), and then devise a non-physical or "pseudo force" (such as the "centrifugal force") in order to apply or preserve the equation of motion. We say "non-physical" because, in a rotating coordinate system, it is not possible to identify a physical object which produces the force needed to satisfy Newton's laws of motion. Also, real forces always occur in pairs yet, if we were to shrink in size and hitch a ride atop a tiny ball connected to a central post by a string and spin around the post, the one and only *force* we would experience would be the inward force (or

tension) exerted by the string. To then explain why the string does not pull us inward toward the post, we create an outwardly directed "pseudo force", called the "centrifugal force", whose magnitude is the mass of the spinning object times the inwardly directed centripetal acceleration, or mv_θ^2/r, where v_θ is the tangential velocity of the ball, and r the length of the string.

For a fluid element (as opposed to a solid or liquid particle, which we will discuss later), the so-called "centrifugal force" is balanced by a force created by a gradient in the static pressure. This pressure gradient acting over the surface of the particle is the "string" tension in our spinning ball example mentioned above. Thus, this latter force acts toward the axis of rotation and keeps the element in its path. This is sketched in Fig. 2.1.2. Depending on our point of view[1], we could also say that this pressure force *gives rise to* the centripetal acceleration or "centrifugal force". As we shall see in Appendix 2.A the pressure in a swirling flow increases with the distance from the axis of rotation.

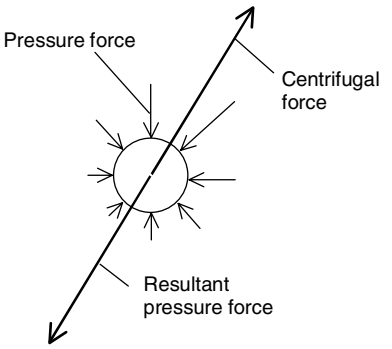

Fig. 2.1.2. Detail of a rotating fluid element in a rotating coordinate system, with the forces acting on it indicated

Now imagine first that the swirling fluid has an *infinite viscosity* (behaves like a solid body). Hence, no shearing motion exists between fluid layers at different radii. In this case fluid elements at all radial positions are forced to have the same angular velocity. The angular velocity, Ω, is measured in radians per unit of time, usually seconds, and therefore has units s^{-1}. It equals v_θ/r, with v_θ the tangential velocity, measured in m/s. Swirl with constant Ω is called 'forced vortex flow' or 'solid-body rotation':

$$v_\theta = \Omega r \tag{2.1.1}$$

This is the first ideal swirl flow.

In the other extreme, if the swirling fluid has *no viscosity*, the motion of a given fluid element is not influenced by the neighboring elements at smaller

[1] In principle, cause and effect cannot be identified in this type of flow

and larger radii. If we, in such a fluid, bring an element to a smaller radius, its tangential velocity will increase, since its moment-of-momentum (mass times tangential velocity times radius of rotation: $mv_\theta r$) will be conserved. We call a vortex where moment-of-momentum is conserved in this way, 'loss free', or 'frictionless'. In such a flow we have $rv_\theta = C$, with C a constant, so that:

$$v_\theta = \frac{C}{r}. \tag{2.1.2}$$

This is the second ideal swirl flow. We should point out that the quantities v_θ, Ω and r are vectors since they have both magnitude and direction. Here, however, we are only interested in their magnitudes and, for this reason, we dispense with the vectorial notation.

These two ideal flow patterns are derived from the fundamental equations of fluid mechanics in Appendix 2.A. This derivation is useful for a fuller understanding of the flow, but it is not essential for appreciating the material in this book as a whole.

A real fluid will have some finite viscosity, which will cause transfer of moment-of-momentum between layers at different radii. An additional transport of moment-of-momentum will be caused by any turbulence present, due to exchange of fluid elements between the layers.

A real swirling flow normally has a core of near solid-body rotation surrounded by a region of near loss-free rotation as sketched in Fig. 2.1.3. This is called a 'Rankine vortex'.

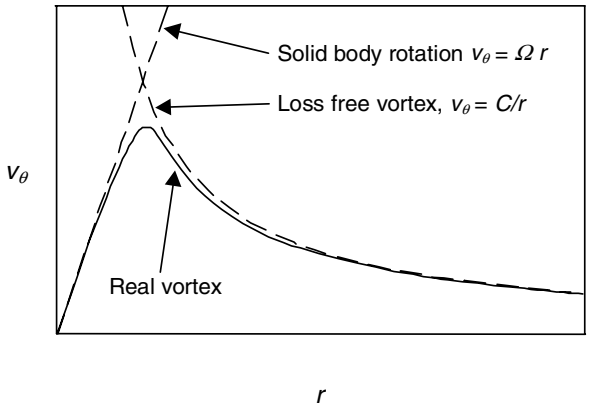

Fig. 2.1.3. Sketch showing the two ideal vortex flows, and the tangential velocity distribution in a real vortex

2.1.2 Static and Dynamic Pressure

The flow and pressure distribution within cyclones and swirl tubes is more easily understood if we make clear the relation between *static* and *dynamic*

pressures: p and $1/2\rho v^2$, respectively, with ρ the density. The well-known Bernoulli equation for steady flow of a frictionless, constant density fluid, which can be derived from the Navier-Stokes equations (Bird et al., 2002), states that:

$$\frac{p}{\rho} + gh + \frac{1}{2}v^2 = \text{constant along a streamline.} \tag{2.1.3}$$

In this equation, we recognize the static and dynamic pressures—the latter is often called the 'velocity head'—as the first and third terms on the left-hand side. They have been divided by the fluid density.

This equation shows that static and dynamic pressures can be interchanged in the flow field. In areas where the velocity is high, the static pressure will be low and *vice versa*. This is the principle used in many flow meters, for instance pitot tubes and venturi meters. It is especially important to appreciate this interdependence between static and dynamic pressure when dealing with swirling flows.

The left-hand side of Eq. (2.1.3) is sometimes called 'Bernoulli's trinomial'. The second term is unimportant relative to the two others when discussing gas cyclones and swirl tubes, since the fluid density is relatively low, and height differences not very large.

In an actual flow situation, the fluid is not frictionless. Frictional dissipation of mechanical energy will therefore cause Bernoulli's trinomial to decrease in the flow direction, *i.e.* the trinomial is no longer constant, but decreases along a streamline.

Frictionless flow is, nevertheless, a reasonably good approximation in the outer part of the swirl in a cyclone, Bernoulli's trinomial does not change very much there.

Friction is taken into account in the 'extended Bernoulli equation', sometimes called the 'frictional form of the Bernoulli equation' or the 'engineering Bernoulli equation' (Bird et al., 2002).

2.2 Particle Motion

We now look at the motion of a solid or liquid particle in a fluid, starting with a general discussion and focusing on the particle motion in swirling gas flows toward the end of the section.

In a gas cyclone or swirl tube, the particles of interest are almost always moving relative to the gas at their *terminal velocity*, and the terminal velocity of a given particle determines whether it will be captured or lost. This terminal velocity is exactly analogous to that of a particle settling in the earth's gravitational field, g, under steady-state conditions except that, for a cyclone, the radially directed centrifugal force, mv_θ^2/r replaces the gravitational one. This will be discussed in detail later.

We are therefore interested in calculating the particle terminal velocity in the swirling flow. We begin with the equation of motion of a particle in a fluid.

Applying Newton's law to a particle moving in a fluid, equating its mass times acceleration to the sum of the forces acting on it, we obtain

$$\begin{pmatrix}\text{mass}\\ \text{times}\\ \text{acceleration}\end{pmatrix} = \begin{pmatrix}\text{body}\\ \text{force}\end{pmatrix} + \begin{pmatrix}\text{fluid}\\ \text{drag}\end{pmatrix} + \begin{pmatrix}\text{unsteady}\\ \text{force}\\ \text{terms}\end{pmatrix}$$

where the body force is normally due to a gravitational field and/or a centrifugal force. Following our earlier discussion, in using the term 'centrifugal force' we are implying that the above force balance is being performed in a reference coordinate systems that is rotating with the particle. The fluid drag is the drag acting on the particle if it moves with a steady velocity relative to the fluid, while the unsteady terms account for the effects of acceleration of the particle relative to the fluid. With appropriate substitution into the above expression the general equation of motion for a particle in a Newtonian fluid becomes Clift et al. (2005):

$$\begin{aligned}\left(\frac{\pi x^3}{6}\right)\rho_p\frac{d\mathbf{U}'}{dt} = \left(\frac{\pi x^3}{6}\right)(\rho_p-\rho)\,\mathbf{a} - C_D\left(\frac{1}{2}\rho\mathbf{U}'\,\|\mathbf{U}'\|\right)\left(\frac{\pi x^2}{4}\right)\\ -\begin{pmatrix}\text{added}\\ \text{mass}\end{pmatrix} - \begin{pmatrix}\text{Basset}\\ \text{term}\end{pmatrix}\end{aligned} \tag{2.2.1}$$

where $\mathbf{U}$' is the particle velocity vector relative to the gas and has cylindrical coordinate components (U_r', U_θ', U_z'); $\mathbf{a}$ is the acceleration vector of an external force field (equal to $\mathbf{g}$ for a gravitational field); ρ_p and ρ are the particle and fluid densities, respectively, and t is time. $\|\cdot\|$ denotes the absolute value (the length) of the vector. Throughout this book we shall represent the particle diameter with the symbol x.

The first term on the right-hand side represents the body force, and the second term the drag $\mathbf{F}_D$ acting on the particle when the flow around it is fully developed, C_D is the drag coefficient.

The two last terms on the right-hand side of (2.2.1) relate to fast, unsteady motion. The added mass term accounts for the fact that when accelerating a particle from rest, the surrounding fluid must also be accelerated. This appears to 'add mass' to the particle. The Basset integral says that the drag will, by rapidly changing motion, depend not only on its instantaneous velocity relative to the fluid, but also on the previous motion since the fluid flow pattern may not have had time to adjust, due to the fluid inertia. These two terms are zero in steady movement.

Clift et al. (2005) showed that ignoring these two unsteady terms (in particular the Basset integral) can lead to errors for a rapidly changing motion in liquid. Fortunately, in the case of gas cyclones we can safely ignore them, even when calculating the rapid, small-scale turbulent motion, since the gas inertia is relatively small. In fact, it turns out that this is true even for the case

of hydrocyclones, where the carrier fluid is a liquid rather than a gas. Also practical plant experience with their design and operation indicates that it is not necessary to include either the added mass or the Basset terms appearing on the right-hand side of (2.2.1).

The second term on the right-hand side in (2.2.1) can be simplified. In gas cyclones we are concerned with small particles (small x) moving in a fluid of low density (small ρ), so that the 'particle Reynolds number':

$$Re_p \equiv \frac{\rho \, \|\mathbf{U}'\| \, x}{\mu} \tag{2.2.2}$$

is relatively low, in spite of the relatively low viscosity μ. For low Re_p, the equations of motion—Eqs. (2.A.1) and (2.A.2) in Appendix 2.A—for the fluid moving around the particle can be solved, and $\mathbf{F}_D$ calculated. If there is no slip between fluid and particle surface (that is: the fluid velocity is equal to the velocity of the surface at the surface), the result is 'Stokes drag law' (Bird et al., 2002):

$$\mathbf{F}_D = -3\pi x \mu \mathbf{U}'. \tag{2.2.3}$$

Comparing this with the expression for the fluid drag term of Eq. (2.2.1) and, by using Eq. (2.2.2), we see that $C_D = 24/Re_p$, which is the particle's drag coefficient under conditions of laminar flow.

These simplifications produce the following equation of motion for the particle:

$$\left(\frac{\pi x^3}{6}\right) \rho_p \frac{d\mathbf{U}'}{dt} = -3\pi x \mu \mathbf{U}' + \left(\frac{\pi x^3}{6}\right) (\rho_p - \rho) \, \mathbf{a}. \tag{2.2.4}$$

If we solve this differential equation in one direction indicated by the index i (replacing the vectors with their components in the i-direction, where i is a Cartesian coordinate), assuming $U_i' = U_{i,0}'$ at $t = 0$, the particle velocity relative to the gas becomes:

$$\begin{aligned} U_i' = \frac{x^2 \, (\rho_p - \rho) \;\; a_i}{18\mu} \left(1 - \exp\left[-\frac{18\mu t}{x^2 \rho_p}\right]\right) + U_{i,0}' \exp\left[-\frac{18\mu t}{x^2 \rho_p}\right] = \\ \left(\frac{\rho_p - \rho}{\rho_p}\right) \tau a_i \left(1 - e^{-t/\tau}\right) + U_{i,0}' e^{-t/\tau} \end{aligned} \tag{2.2.5}$$

where τ is called the 'particle relaxation time':

$$\tau \equiv \frac{x^2 \rho_p}{18\mu}. \tag{2.2.6}$$

For large t the exponential terms go to zero, and the particle reaches its terminal velocity. If $\rho_p \gg \rho$, as it is in gas cyclones, the terminal velocity is (dropping the index i):

$$U_t' = U_{Stk}' = \tau a = \frac{x^2 \rho_p}{18\mu} a \text{ for } \rho_p \gg \rho \tag{2.2.7}$$

where the subscript Stk signifies that this is the 'Stokes' velocity', *i.e.* the terminal velocity of a particle when Stokes' drag law applies. Small particles in gas cyclones reach their terminal velocity quickly. We can see this from Eq. (2.2.5): τ is small for small x (in the cases we are considering it is of the order 10^{-3} s), so the exponential term goes to zero quickly. We may put this into perspective for commercial cyclones for which the particle residence time within the cyclone typically lies within the range of about 50 milliseconds (ms) for small, high velocity cyclones to 1 to 2 seconds for large industrial units such as those in use in large coal conversion units or fluid catalytic cracking (FCC) units. This means that we can ignore the unsteady part of Eq. (2.2.5), even for the rapid, small-scale velocity fluctuations caused by the gas turbulence, and assume the particle will always be at its terminal velocity relative to the gas.

In Fig. 2.2.1 velocity is plotted against time for a 10 μm particle of density 2700 kg/m^3 (a typical density, close to, for instance, those of chalk or sand) dropped in air in the field of gravity. The terminal velocity and τ are indicated. The particle approaches its terminal velocity within a couple of milliseconds.

As mentioned, Stokes' drag law is valid for low particle Reynolds numbers. Another requirement for Stokes' law to apply is that the surrounding fluid can be *considered a continuous medium*. This is not so for very small (sub-micron size) particles in gases, especially for sub-micron particles feeding cyclones operating under high vacuum conditions. In some applications, for example, vacuum assists with the drying of moist or solvent-laden incoming solids. Here we have to take into account the fact that the gas consists of individual molecules. This has two effects:

1. Collisions with gas molecules give rise to a *fluctuating particle motion* ('Brownian motion'). This can be neglected compared to any large-scale turbulent particle dispersion.
2. A *slip* takes place between the gas and the particle as the free space between the gas molecules becomes comparable to the particle size. This is accounted for in the drag law by multiplying the terminal particle velocity calculated from Stokes' law, U'_{Stk}, by the 'Cunningham correction factor', C_c (Allen, 1990):

$$U' = U'_{Stk} C_c = U'_{Stk} \left(1 + \frac{2\lambda}{x}\right) \tag{2.2.8}$$

where λ is the molecular mean free path. This correction factor can be viewed also as a correction to the viscosity term appearing in the denominator of (2.2.7). Thus, the 'effective' viscosity becomes the gas viscosity times the Cunningham correction factor. It has the effect of always decreasing the effective viscosity and, hence, the drag that all particles—not just the the sub-micron particles—experience.

We now turn our attention to the behavior of a particle in a swirling flow. Here, we can apply some of what we have previously discussed about

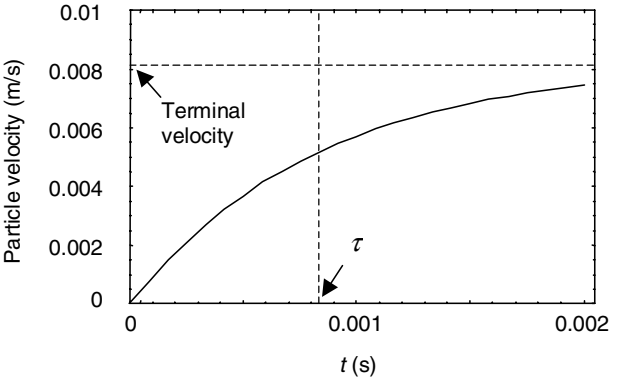

Fig. 2.2.1. The velocity of a particle (x=10 μm, ρ_p=2700 kg/m^3) dropped in air. Calculation is according to Eq. (2.2.5)

a rotating fluid element to a solid or liquid particle. One difference, though, is that the force arising from the pressure gradient in the fluid, which in this context is akin to the buoyancy force acting in a gravitational field, will not keep the particle in its path unless it has the same density as the fluid. In gas cyclones the particle density is much higher than that of the carrier gas, so the 'buoyancy' is low and the particle will move radially outward in the vortex. This, then, becomes and defines the primary mechanism for separation of particles within a cyclone. As we shall see, we can even ignore the buoyancy when calculating the particle's velocity, so that the only significant force opposing the particle's outward radial motion is a drag force.

If the particle moves with the same tangential velocity as the gas, and we choose a coordinate system rotating with the particle, we can consider the centrifugal force as analogous to the force of gravity. This allows us to replace the acceleration a in Eq. (2.2.5) with the magnitude of the centripetal acceleration: v_θ^2/r, and we can say that a centrifugal force equal to $m_p v_\theta^2/r$, where m_p is the mass of the particle, acts on the particle (compare with Eq. (2.A.12) in Appendix 2.A).

When $\rho_p \gg \rho$, the particle will thus be centrifuged outward (see Fig. 2.2.2), resisted by drag, and will move with a terminal velocity relative to the gas of:

$$U_r' = (U_r - v_r) = \frac{x^2 \rho_p}{18\mu}\left(\frac{v_\theta^2}{r}\right) = \tau\left(\frac{v_\theta^2}{r}\right) \text{ for } \rho_p \gg \rho. \tag{2.2.9}$$

This outward movement of the particle is, as mentioned, the principle of separation in all centrifugal separators, both for dedusting and demisting.

In addition to its mean movement, a particle in a cyclone will also have a small scale, fluctuating motion in response to the local turbulence in the gas. This is more severe for smaller particles, which are most affected by the fluctuations in the gas velocity. The turbulent motion gives rise to some dispersion and mixing of the particles.

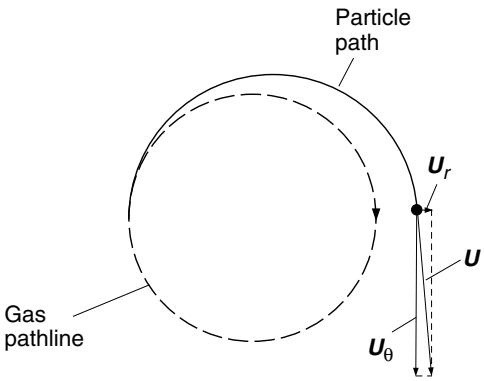

Fig. 2.2.2. Sketch showing gas and particle pathlines in a swirling flow field where the gas has no radial velocity component, that is: $v_r = 0$ in Eq. (2.2.9). The vectors represent the radial, tangential and resultant particle velocity components

2.3 Particle Size

2.3.1 Definitions of Particle Size

The motion of a particle, and its separation in a cyclone, obviously depends on its size, amongst other important factors, such as its density, shape, and tangential velocity. By the term 'size' we normally mean the diameter. The particle diameter can be defined in different ways, and one should be aware which one is used in a given context. Clift et al. (2005) and Allen (1990) review this issue. We mention here the definitions that are most relevant for cyclones.

The 'volume equivalent' diameter is the *diameter of a sphere with the same volume as the actual particle*[2]. The 'surface equivalent' diameter is the *diameter of a sphere with the same surface area as the actual particle*. The 'surface/volume diameter' is the *diameter of a particle with the same surface-to-volume ratio as the actual particle*.

To illustrate this, a cylindrical particle with height $2L$ and diameter L is shown in Fig. 2.3.1, together with its equivalent spheres.

Very central to cyclone technology is the 'dynamically equivalent' particle diameter. This is the *diameter of an equi-dense sphere that has the same terminal velocity as the actual particle*. Calculating this can be difficult in the range of intermediate Reynolds numbers, or when the Cunningham correction is significant. In the region where Stokes drag law applies, we call it the 'Stokesian' diameter.

[2] If all the particles are of the same density, then the volume equivalent diameter is the same as the mass equivalent diameter, since their mass is then proportional to their volume.

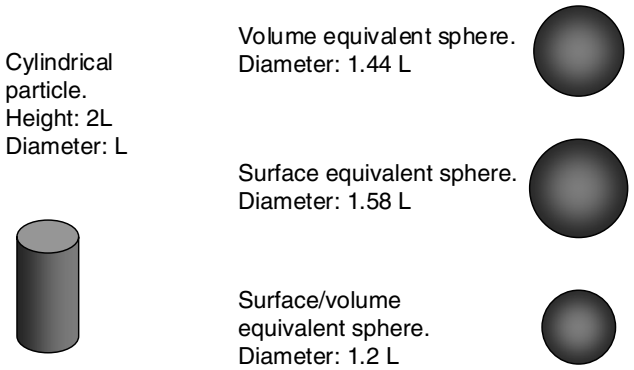

Fig. 2.3.1. Sketch of a cylindrical particle with the different equivalent spheres

A similar measure, which is widely used in aerosol science, is the 'aerodynamic particle size'. This is the *diameter of a sphere of density 1000 kg/m*^{3} *that has the same terminal velocity as the actual particle in air at normal temperature and pressure in a gravity field.*

Figure 2.3.2 from Kaye (1995) shows silhouettes of dynamically equivalent particles. The more nonspherical the actual particle, the larger it needs to be in order for it to settle with the same terminal velocity. The spheres to the right are Stokes diameters, those to the left aerodynamic diameters. Since uranium dioxide is far denser than 1000 kg/m^3, the two diameters differ the most for this type of particle.

2.3.2 Particle Size Distribution

The particle size distribution of a given dust or mist can be reported as a number, length, surface, volume or mass (weight) distribution. Figure 2.3.3 shows number and volume distribution curves for a sample powder. The curves in the figure are *density curves*: the function values $f(x)$ represent *the fraction of particles in a given interval divided by the width of that interval.* The definition of the number density distribution $f_N(x)$ is thus:

$f_N(x)dx$ = the number fraction of particles with a diameter between $x - 1/2\ dx$ and $x + 1/2\ dx$,

and the definition of the volume density distribution $f_V(x)$ is:

$f_V(x)dx$ = the volume fraction of particles with a diameter between $x - 1/2\ dx$ and $x + 1/2\ dx$.

Since the particle volume is proportional to x^3, the larger particles contribute much more to the volume distribution than to the number distribution. This can be seen in the shapes of the curves in Fig. 2.3.3. The larger particles contribute negligibly to the number distribution, which appears to go

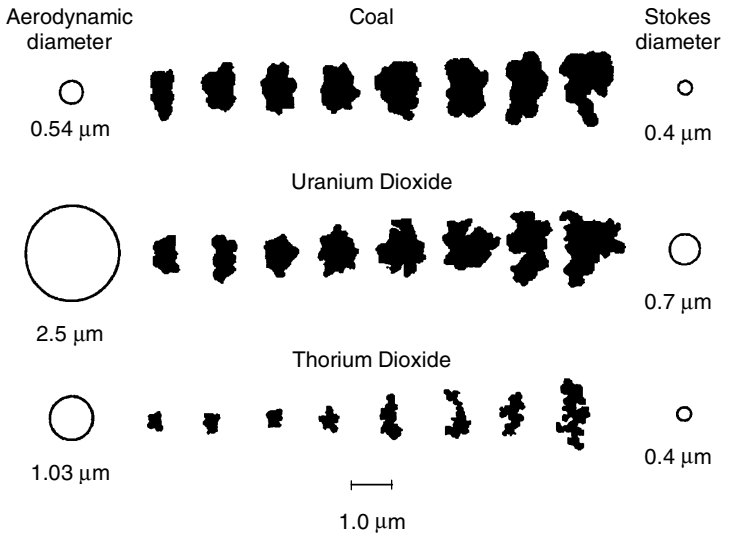

Fig. 2.3.2. Silhouettes of several different particle types along with their equivalent aerodynamic and Stokesian diameters from Kaye (1995)

to zero, while they contribute substantially to the volume distribution. The same holds true for the mass or weight distribution. For this reason it is also difficult to obtain a statistically satisfactory volume distribution from sizing methods based on particle counting if the particle size distribution is wide (many small particles need to be counted for each large one).

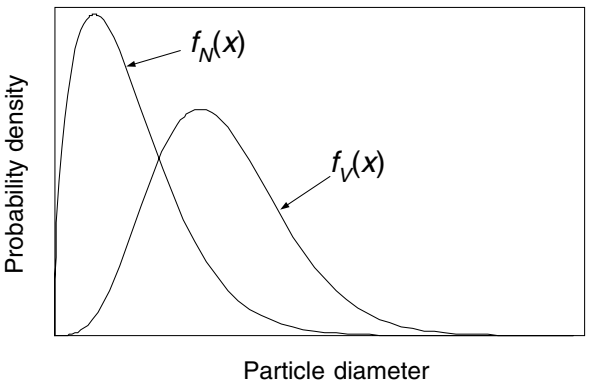

Fig. 2.3.3. Number and volume density distributions for a sample powder

If one of the distributions is known, the others can be calculated; at least this is the case if one assumes the particles to be spherical (Allen, 1990). For instance, we can calculate the volume density distribution from the number density distribution. Since the number fraction of particles in the diameter interval: $x - 1/2\ dx$ and $x + 1/2\ dx$ is $f_N(x)dx$, then:

$$f_V\,(x) \propto \frac{\pi x^3}{6} f_N\,(x)\,dx. \tag{2.3.1}$$

We have to choose the proportionality constant so that the area under our volume density distribution becomes unity:

$$f_V\,(x)\ \mathrm{d}x = \frac{\frac{\pi x^3}{6} f_N\,(x)\ dx}{\int\limits_0^\infty \frac{\pi x^3}{6} f_N\,(x)\ dx}. \tag{2.3.2}$$

In addition to density distributions, a very widely used method of reporting a particle size distribution is through the use of its 'cumulative undersize distribution' $F(x)$, defined as *the fraction of particles with a diameter less than* x. $F(x)$ is related to the density function $f(x)$ by:

$$F\,(x) = \int\limits_0^x f\,(z)\,dz, \qquad f\,(x)\,dx = dF\,(x) \tag{2.3.3}$$

where we have used z as the dummy variable of integration.

In Fig. 2.3.4 the cumulative undersize distributions corresponding to the density functions in Fig. 2.3.3 are shown.

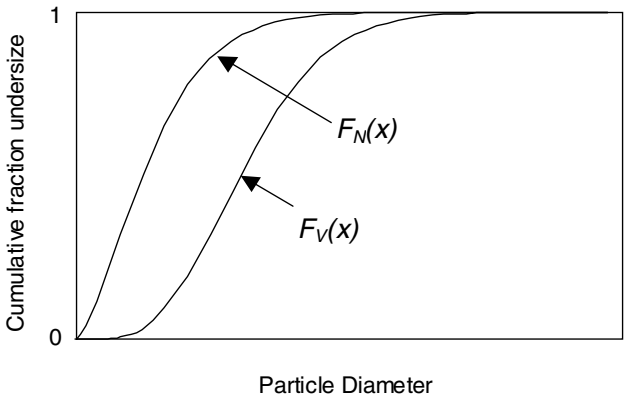

Fig. 2.3.4. Cumulative number and volume distributions for the powder in Fig. 2.3.3

Throughout this book, we shall be using the volume distributions and, in order to simplify the notation, we drop the subscript V from now on. In practice, one frequently encounters the terms 'mass' or 'weight' distributions such

as that obtained from sieve analysis. It is important to recall that, irrespective of the way in which the particle sizes are measured or reported, if the density of the particles comprising the distribution does not change as a function of particle size, then the particles' mass and weight distributions are identical to their volume distribution. This is so because the volume of any one particle or fraction of particles is directly proportional to its mass or weight if its density remains constant.

A number of *model distribution functions* exist, some of which fit the size distributions of many powders quite well. The model functions used most frequently are the 'normal' (or 'Gaussian') distribution, the 'log-normal' distribution and the 'Rosin-Rammler' distribution (Allen, 1990). These are given in Appendix 2.B for reference. The latter two can be fitted particularly well to the volume distributions of a wide range of powders. The Rosin-Rammler distribution was used to produce Figures 2.3.3 and 2.3.4.

It should be added, however, that it is generally not necessary or even necessarily desirable to represent a particle distribution by 'fitting' it to any particular distribution function, *a priori*. Computer programs for designing and evaluating cyclone performance normally utilize the 'raw' particle size distribution data (often in cumulative form) in their internal computations. This technique eliminates any errors pertaining to any differences that may exist between the mathematical 'fit' of the data and the actual data that is being fitted. Often it is observed that some distribution function does 'fit' the majority of the measured distribution data but may, for example, fail to fit the smallest particle size fraction. Under such conditions, if it were important to know about the collection or losses of the 'fines', one would not want to use such a distribution model in practice. Instead, the actual measured distribution data would be utilized for cyclone simulation purposes.

Finally, we consider the mean and spread of a particle size distribution. The mean size can be defined in different ways, depending on which property of the powder is important. A review of this can be found in Allen (1990). In this book, we use the volume distribution, and the 'volume mean' particle size, which is equivalent to the mass mean particle size, is defined as the first moment of the volume distribution around zero:

$$\langle x \rangle = \int_0^\infty x f\,(x)\,dx. \tag{2.3.4}$$

Other characteristic sizes are: the 'median size', x_{med}, defined as the size at which $F(x)$=0.5, and the 'mode', defined as the size where $f(x)$ takes its maximum value.

We note here that the mean and the median sizes are often close in practice, and that, of these two, the median size is much easier to determine by reading directly off the cumulative size distribution. The median is therefore often taken as the 'mean' particle size for a given powder in practice, while in the model distribution functions the mean is that defined in Eq. (2.3.4). We shall

follow common practice by using the median diameter as a measure of the 'mean' particle size in this book, we will refer to it as the '(mass) average' or the 'median' size, but always use the symbol x_{med} when we mean the median.

We may also characterize the particle size distribution through a parameter that characterizes its spread, σ, the square of which is the second moment around the mean :

$$\sigma^2 = \int_0^\infty (x - \langle x \rangle)^2 \, f(x) \, dx. \tag{2.3.5}$$

The above parameters are very useful to characterize powders with only two parameters, for instance for controlled laboratory experiments in which results need to be related to the particle size distribution. However, they rarely enter the considerations in engineering design. Those whose job it is to design and/or troubleshoot cyclone systems in industry are generally faced with designing or evaluating the performance based on what the upstream process delivers, using a measured particle size distribution as a basis for calculations.

2.4 Particle Density

In addition to size, one more particle property plays an important role in determining particle motion in fluids, and therefore also in cyclones: the particle density.

If the particle is a nonporous solid, its density is unequivocal, but if it is porous, we need to distinguish the *density of the solid material* comprising the particle (often called the 'skeletal' density) and the *overall* or *effective* particle density, including both the solid material and the pores. The latter is often called the 'envelope density' or 'the density in a Stokes-settling sense'. In practice, it is the envelope density that determines the behavior of the particle in a fluid, and is therefore the density we wish to determine.

Particle density is often determined by some sort of *pycnometry*. If a liquid is used as the pycnometric fluid, this is mostly done in a so-called 'density bottle', where the masses are determined of:

- the empty bottle, m_1
- the bottle containing the powder sample only, m_2
- the bottle containing the powder sample filled with the liquid, m_3
- the bottle filled with the liquid only, m_4

The mass of the powder sample is given by $(m_2 - m_1)$, while its volume is given by $[(m_4 - m_3) - (m_2 - m_1)]/\rho_l$, where ρ_l is the density of the pycnometric liquid. The density can then be found by dividing the mass of the sample with its volume.

If the pycnometric fluid penetrates into the pores of the particles, the density determined will be the skeletal density. In order to find the envelope

density, some pycnometric fluid has to be found that will penetrate the interstitial space between the particles, but not the pores within the particles. Mercury can be used for this, whereby the mercury is added under vacuum, so that it will penetrate the interstices between the particles properly. If one is interested in the skeletal density, and the pores in the particles are fine, air can be used as the pycnometric fluid in specially designed equipment.

We note that if, during particle sizing, the dynamically equivalent particle size is determined, the problem of determining the appropriate particle density is solved already.

We have now reviewed the most essential topics necessary for an appreciation of the basic working of gas cyclones. We shall make frequent use of these developments in the subsequent chapters, and we shall look at models for cyclone performance based on the above-mentioned basic principles. We hasten to add that a cyclone design based on these principles is only a starting point. Many key issues for practical cyclone design and operation are of a highly complex nature and cannot be described using the basic ideas of single particles in swirling flows alone. Examples are the effect of inlet solids loading on cyclone separation efficiency, the 'natural turning point' of the vortex, the phenomenon of 'hopper crossflow', hopper venting, and the issue of cyclone erosion. We will be discussing these and some related issues later on.

2.A Ideal Vortex Laws from the Navier-Stokes Equations

In this appendix, we will derive the essential equations for swirling flow from the basic equations of fluid mechanics: the Navier-Stokes equations. The Navier-Stokes equations are derived in most textbooks on fluid mechanics, for instance Bird et al. (2002).

The 'equation of continuity' states that material can neither be generated nor destroyed. For an incompressible fluid the equation is:

$$\nabla \cdot \mathbf{v} = 0 \tag{2.A.1}$$

with $\mathbf{v}$ the fluid velocity vector.

The 'momentum conservation' equation states Newton's law for a fluid element: its mass times acceleration equals the sum of the forces acting on it:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g} \tag{2.A.2}$$

where ρ is the density, p the pressure and $\mathbf{g}$ the gravitational acceleration. $\boldsymbol{\tau}$ is the deviatoric stress tensor (see below). The terms in (2.A.2) represent from left to right:

- The mass times acceleration per unit volume. This is the density multiplied by the absolute (or 'material') derivative of the velocity. The material

derivative D/Dt gives the acceleration of a fluid element in a Eulerian[3] frame of reference.

- The net force due to normal stresses per unit volume.
- The net force due to shear stresses per unit volume. $\boldsymbol{\tau}$ is the 'deviatoric' stress tensor, meaning that the pressure has been subtracted from the total stress tensor, so that the sum of the three diagonal elements is zero. This essentially leaves us with the shear stresses.
- The gravitational force per unit volume.

Equation (2.A.2) can be expressed in terms of its coordinate components. For cyclones, it is convenient to use a cylindrical coordinate system (r, θ, z), with the z-axis along the cyclone axis. Writing out the θ-component of Eq. (2.A.2) gives (Bird et al., 2002):

$$\begin{aligned} &\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)= \\ &\quad \text{I} \qquad \text{II} \qquad \text{III} \qquad \text{IV} \qquad \text{V} \\ &-\frac{1}{r}\frac{\partial p}{\partial \theta}-\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right)+\frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta}+\frac{\partial \tau_{\theta z}}{\partial z}\right)+\rho g_z \\ &\quad \text{VI} \qquad\qquad \text{VII} \qquad\qquad \text{VIII} \qquad \text{IX} \qquad \text{X} \end{aligned} \tag{2.A.3}$$

In the shear stress components τ, the first index indicates the plane on which the stress acts, and the second its direction.

This complicated looking equation can be simplified to give useful information about swirling flow. In steady, axisymmetrical vortex flow with negligible velocity in the r and the z-directions, the terms listed in Table 2.A.1 can be eliminated[4].

This leaves only term VII, and, since $1/r^2 \neq 0$:

$$\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right)=0. \tag{2.A.4}$$

And, because the derivative of the quantity in parenthesis is equal to zero, the quantity itself is a constant, which we call C_1:

[3] In a 'Eulerian' frame of reference time derivatives are stated in a stationary frame. Therefore, in order to write the time derivative of a property φ for a fluid element, we have to include both the *local* derivative (the rate of change of φ at the stationary point) and the *convective* derivative (the rate of change of φ in the direction in which the fluid element is moving). The Eulerian frame is in contrast to a 'Lagrangian' frame where we state time derivatives following a fluid element (or a particle).

[4] We note that this is an idealized flow pattern, if the radial velocity in a cyclone were zero, the cyclone would not function since, then none of the entering fluid could make its way to the inner core, and, hence, out the vortex finder.

Table 2.A.1. Eliminating terms in Eq. (2.A.3)

Term eliminated	Reason
I	steady flow, no change with time
II and IV	no radial velocity
III and VI and VIII	no gradients in the θ-direction (axisymmetric flow)
V and IX	no gradients in the z-direction

$$\tau_{r\theta} = \frac{C_1}{r^2}. \tag{2.A.5}$$

The next step is to relate the shear stress $\tau_{r\theta}$ to the velocity field. The simplest way of doing this is to assume a constant Newtonian viscosity, μ. Then the expression for $\tau_{r\theta}$ becomes (Bird et al., 2002):

$$\tau_{r\theta} = -\mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]. \tag{2.A.6}$$

Here the second term on the right-hand side can be eliminated if there are no gradients in the θ direction. Doing this and substituting in the first part of (2.A.4) gives:

$$\frac{\partial}{\partial r} \left[r^3 \left(-\mu \right) \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right] = 0 \;\Rightarrow\; \frac{\partial}{\partial r} \left[r^3 \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right] = 0 \tag{2.A.7}$$

- since μ is independent of r. The first equation above only implies the second if $\mu \neq 0$. If $\mu = 0$, *any* v_θ profile will satisfy the first part.

The solution to (2.A.7) is:

$$v_\theta = C_1 r + \frac{C_2}{r}. \tag{2.A.8}$$

If we require that v_θ does not become infinite at $r = 0$, C_2 has to be zero, giving the well known equation for a 'forced vortex' or 'solid-body rotation':

$$v_\theta = C_1 r = \Omega r. \tag{2.A.9}$$

This is one ideal vortex motion, where, as mentioned in the main text, the angular velocity Ω is constant.

Another is the 'loss-free' vortex, which is a vortex motion in a fluid with a viscosity of zero. We saw that if $\mu = 0$, any radial v_θ profile would satisfy the first part of (2.A.7) under the assumptions. If we allow a radial velocity, so that fluid elements can move radially in the vortex, this is no longer so. Doing this and setting the viscosity and, therefore, the shear stress $\tau_{r\theta}$ equal to zero, we see that terms II and IV re-emerge, and that term VII of Eq. (2.A.3) is eliminated. This leads to:

$$v_r \left(\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \right) = 0, \tag{2.A.10}$$

which has the solution:

$$v_\theta = \frac{C}{r} \tag{2.A.11}$$

with C an integration constant. This is the familiar equation for the tangential velocity distribution in a loss-free vortex. In this type of flow the moment-of-momentum of fluid elements is constant in the radial direction.

Note that we could not have derived (2.1.2) by letting C_1 equal zero in (2.A.8) since the viscosity μ was assumed to be nonzero to arrive at (2.A.8) in the first place.

In the same way that (2.A.7) was derived from the θ-component equation of (2.A.2), two other differential equations for the flow field in a vortex motion can be derived from the r- and the z-component equations. They are, respectively:

$$\frac{\partial p}{\partial r} = \rho \frac{v_\theta^2}{r} \tag{2.A.12}$$

and:

$$\frac{\partial p}{\partial z} = \rho g_z. \tag{2.A.13}$$

Equation (2.A.12) is the balance between the centrifugal force (or the mass times the centripetal acceleration) and the pressure force, all on a per unit volume basis. It shows, as we also saw on basis of heuristic arguments in the main text, that the pressure in a vortex flow increases towards the periphery and more so the stronger the tangential velocity. The radial pressure distribution can be obtained by integrating the right-hand side over r.

Equation (2.A.13) simply says that the axial pressure distribution is the hydrostatic pressure, which in gas cyclones is not very interesting, since the fluid density is low.

This completes the derivation of the basic equations for swirling flows from the Navier-Stokes equations. When deriving flow equations, particularly in cylindrical coordinates, this method is safer than using heuristic arguments.

2.B Common Model Functions for Particle Size Distributions

In this appendix, the most frequently used particle size distributions are given for the reader's reference. If you are a researcher interested in these distributions, it is very instructive to program and graph these models using a mathematics package (for instance Mathematica, or a freeware program called MathGV), and have a look at how the shapes of the distributions change with the parameters.

When studying cyclone performance, or any issue in powder technology, it can be advantageous to fit models to the experimentally determined particle size distributions obtained from laboratory measurements. In this way the particle size distribution can be characterized using only two parameters: the

mean size and the spread. Model functions may also make it possible to make up for incomplete information about size distributions, as long as one is aware of the dangers of doing so pointed out in Appendix 3.A.

2.B.1 The Normal Distribution

The density function for the normal distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \langle x\rangle)^2}{2\sigma^2}\right). \tag{2.B.1}$$

To obtain the cumulative undersize function, we must integrate this:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(z - \langle z\rangle)^2}{2\sigma^2}\right) dz. \tag{2.B.2}$$

For this distribution, the mean particle size $\langle x\rangle$, the median, and the mode are all equal, and the spread is σ. In this purely mathematical distribution, x can take on negative values, which is not physically meaningful.

2.B.2 The Log-Normal Distribution

The log-normal distribution is defined as: *the distribution of a variable, the natural log of which is normally distributed*. Thus, for the distribution of the natural log of particle diameters we get:

$$f(\ln x) = \frac{dF'(\ln x)}{d\ln x} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \langle \ln x\rangle)^2}{2\sigma^2}\right). \tag{2.B.3}$$

To obtain the distribution of the particle diameter itself rather than that of its logarithm, we note that $F'(\ln x)$, the fraction of particles with the logarithm of their diameter less than $\ln x$, is the same as $F(x)$, the fraction of particles with diameter less than x. Thus:

$$F'(\ln x) = \int_{-\infty}^{\ln x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln z - \langle \ln z\rangle)^2}{2\sigma^2}\right) d\ln z = F(x). \tag{2.B.4}$$

In order to write $F(x)$ in terms of x rather than $\ln x$, we change the variable of integration:

$$F(x) = \int_{0}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln z - \langle \ln z\rangle)^2}{2\sigma^2}\right) \frac{1}{z} dz, \tag{2.B.5}$$

which shows that the density function of x for the log-normal distribution is:

$$f(x) = \frac{1}{x}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln x - \langle \ln x \rangle)^2}{2\sigma^2}\right). \tag{2.B.6}$$

One should be most careful when converting between cumulative and density distributions for the log-normal distribution.

The log-normal distribution is skewed with a long tail at large particle sizes. It fits the volume distributions of many powders very well. Because it is skewed, the mode, the median and the mean particle sizes are all different.

2.B.3 The Rosin-Rammler Distribution

The Rosin-Rammler distribution is one that applies specifically to dusts generated by crushing. The density function is:

$$f(x) = nkx^{n-1}\exp(-kx^n). \tag{2.B.7}$$

The shape of $f(x)$ depends on the constants n and k. Integrating to find $F(x)$ and adding a constant to make it start at the origin gives:

$$F(x) = 1 - \exp(-kx^n). \tag{2.B.8}$$

For this distribution the mode, median and mean sizes are different. Using Eqs. (2.3.4) and (2.B.7) the mean particle size becomes:

$$\langle x \rangle = k^{-\frac{1}{n}}\Gamma\left(\frac{1}{n} + 1\right) \tag{2.B.9}$$

where Γ is the Gamma function.

Mathematics packages, such as Mathematica, Matlab or Mathcad and MathGV make it easy to fit these model distributions to sets of experimental data. It is often helpful to do so, since this allows the particle size distribution to be described by only two parameters. This also has its limitations, though. We shall come across one in Appendix 3.A.