Noetherianity and Combination Problems

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Abstract. In abstract algebra, a structure is said to be Noetherian if it does not admit infinite strictly ascending chains of congruences. In this paper, we adapt this notion to first-order logic by defining the class of Noetherian theories. Examples of theories in this class are Linear Arithmetics without ordering and the empty theory containing only a unary function symbol. Interestingly, it is possible to design a non-disjoint combination method for extensions of Noetherian theories. We investigate sufficient conditions for adding a temporal dimension to such theories in such a way that the decidability of the satisfiability problem for the quantifier-free fragment of the resulting temporal logic is guaranteed. This problem is firstly investigated for the case of Linear time Temporal Logic and then generalized to arbitrary modal/temporal logics whose propositional relativized satisfiability problem is decidable.

1 Introduction

Since full first-order temporal logics are known to be highly undecidable, researchers concentrated on finding fragments having good computational properties, such as the decidable monodic fragments investigated in, e.g., [17,9,12]. Although such fragments may also be used in verification, widely adopted formalisms for the specification of reactive or distributed systems (e.g., the one proposed by Manna and Pnueli [23] or the Temporal Logic of Actions by Lamport [19]) are such that the temporal part, used to describe the dynamic behavior of the systems, is parametric with respect to the underlying language of first-order logic, used to formalize the data structures manipulated by the systems. While the expressiveness of these formalisms helps in writing concise and abstract specifications, it is not clear how these can be amenable to automated analysis. The work presented in this paper contributes towards the solution of this problem, by analyzing what happens when we "add a temporal dimension" (in a sense similar to that investigated in [11]) to a decidable fragment of a *first-order theory* T with identity. By doing this, the hope is to transfer the decidability of the theory T to its "temporalized" version. This point of view has been pioneered by Plaisted in [29], where he further refined the semantics of the "temporalized T" by partitioning the symbols of the signature of T in *rigid* (whose interpretation is time-independent) and *flexible* (whose interpretation is time-dependent). This facilitates the expression of properties of both open and closed systems (see, e.g., [11] for more on this issue).

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In [14], we have presented a uniform framework where the approach in [29] has been clarified and extended. In particular, we have obtained undecidability and decidability results for quantifier-free satisfiability and model-checking problems in a temporal logic obtained by extending a decidable theory T with the operators of Linear time Temporal Logic (LTL). The key to obtain the results in [14] is a reduction of satisfiability and model-checking to the combination of (infinitely many) *partially renamed copies* of T (the symbols that are not renamed are those belonging to the rigid sub-signature Σ_r). The viewpoint of combination helps clarifying both decidability and undecidability issues. In fact, it is not always possible to transfer the decidability of the quantifier-free fragment of T to its "temporalized" version as shown by a simple reduction to known undecidable combination problems [5], even when the rigid subsignature Σ_r is empty. Fortunately, it is possible to use combination methods for non-disjoint theories in first-order logic [13] and find suitable requirements on the theory T to derive the decidability of both the satisfiability and the model-checking problem for the quantifier-free formulae of the "temporalized" version of T. The key ingredients are two. First (for correctness), it is assumed that T has a decidable universal fragment and is T_r-compatible [13], where T_r is the Σ_r-reduct of the universal fragment of T. Second (for termination), T_r is assumed to be locally finite [13]. Under these hypotheses, a (non-deterministic) combination schema can be obtained by using guessings over the finitely many (because of local finiteness) literals in the shared theory. This also simplifies the proof of correctness.

In this paper, we weaken the requirement of local finiteness to that of Noetherianity (cf. Section 3), and we focus our attention to the satisfiability problem, since model-checking is easily shown to be undecidable when considering Noetherian theories [15]. The *first contribution* of this paper is to show that our combinability requirements related to Noetherianity are met by any extension with a free unary function symbol of a stably infinite theory (cf. Section 3.2). The *second contribution* is to derive an amalgamation lemma (cf. Lemma 3.7) for combinations of (infinitely many) theories sharing a Noetherian theory (cf. Section 3.1). The combination procedure is more complex than in the locally finite case, since the exhaustive enumeration of guessings can no more be used to abstract away the exchange of now (possibly) infinitely many literals between the component theories and the combination results in [13,14] do not apply. The exchange mechanism is formalized by *residue enumerators*, i.e. computable functions returning entailed positive clauses in the shared theory. The *third contribution* of the paper is the application of the amalgamation lemma to show the decidability of the satisfiability problem for quantifier-free LTL formulae modulo a first order theory T, when T is an effectively Noetherian and T_r-compatible extension of T_r (cf. Section 4). Finally, the decidability result is extended to any modal/temporal logic whose propositional relativized satisfiability problem is decidable (cf. Section 5). Full proofs and all the technical details can be found in the extended version of this paper available online at the address `http://homes.dsi.unimi.it/~zucchell`

2 Formal Preliminaries

We adopt the usual first-order syntactic notions of signature, term, position, atom, (ground) formula, sentence, and so on. Let Σ be a first-order signature; we assume the binary equality predicate symbol '=' to be in any signature (so, if $\Sigma = \emptyset$, then Σ does not contain other symbols than equality). The signature obtained from Σ by adding it a set $\underline{a}$ of new constants (i.e., 0-ary function symbols) is denoted by $\Sigma^{\underline{a}}$. A *positive clause* is a disjunction of atoms. A *constraint* is a conjunctions of literals. A Σ*-theory* T is a set of sentences (called the axioms of T) in the signature Σ and it is *universal* iff it has universal closures of open formulae as axioms.

We also assume the usual first-order notion of interpretation and truth of a formula, with the proviso that the equality predicate = is always interpreted as the identity relation. We let $\bot$ denote an arbitrary formula which is true in no structure. A formula φ is *satisfiable* in $\mathcal{M}$ iff its *existential* closure is true in $\mathcal{M}$. A Σ-structure $\mathcal{M}$ is a *model* of a Σ-theory T (in symbols $\mathcal{M} \models T$) iff all the sentences of T are true in $\mathcal{M}$. If φ is a formula, $T \models \varphi$ ('φ *is a logical consequence of* T') means that the universal closure of φ is true in all the models of T. A Σ-theory T is *complete* iff for every Σ-sentence φ, either φ or $\neg\varphi$ is a logical consequence of T. T admits *quantifier elimination* iff for every formula $\varphi(\underline{x})$ there is a quantifier-free formula $\varphi'(\underline{x})$ such that $T \models \varphi(\underline{x}) \leftrightarrow \varphi'(\underline{x})$ (notations like $\varphi(\underline{x})$ mean that φ contains free variables only among the tuple $\underline{x}$). T is *consistent* iff it has a model, i.e., if $T \not\models \bot$. A sentence φ is T-consistent iff $T \cup \{\varphi\}$ is consistent.

The *constraint satisfiability problem* for the constraint theory T is the problem of deciding whether a Σ-constraint is satisfiable in a model of T (or, equivalently, T-satisfiable). In the following, we use free constants instead of variables in constraint satisfiability problems, so that we (equivalently) redefine a constraint satisfiability problem for the theory T as the problem of *establishing the consistency of* $T \cup \Gamma$ *for a finite set* Γ *of ground* $\Sigma^{\underline{a}}$*-literals* (where $\underline{a}$ is a finite set of new constants). For the same reason, we abbreviate 'ground $\Sigma^{\underline{a}}$-constraint' with 'Σ-constraint,' when $\underline{a}$ is clear from the context.

If $\Sigma_0 \subseteq \Sigma$ is a subsignature of Σ and if $\mathcal{M}$ is a Σ-structure, the Σ_0*-reduct* of $\mathcal{M}$ is the Σ_0-structure $\mathcal{M}_{|\Sigma_0}$ obtained from $\mathcal{M}$ by forgetting the interpretation of function and predicate symbols from $\Sigma \setminus \Sigma_0$. A Σ*-embedding* (or, simply, an embedding) between two Σ-structures $\mathcal{M} = (M, \mathcal{I})$ and $\mathcal{N} = (N, \mathcal{J})$ is any mapping $\mu : M \longrightarrow N$ among the corresponding support sets satisfying the condition

$$\mathcal{M} \models \varphi \quad \text{iff} \quad \mathcal{N} \models \varphi \tag{1}$$

for all Σ^M-atoms φ (here $\mathcal{M}$ is regarded as a Σ^M-structure, by interpreting each additional constant $a \in M$ into itself and $\mathcal{N}$ is regarded as a Σ^M-structure by interpreting each additional constant $a \in M$ into $\mu(a)$). If $M \subseteq N$ and if the embedding $\mu : \mathcal{M} \longrightarrow \mathcal{N}$ is just the identity inclusion $M \subseteq N$, we say that $\mathcal{M}$ is a *substructure* of $\mathcal{N}$ or that $\mathcal{N}$ is an *extension* of $\mathcal{M}$. In case condition (1) holds for all first order formulae, the embedding μ is said to be *elementary.*

3 Noetherian Theories

In abstract algebra, the adjective Noetherian is used to describe structures that satisfy an ascending chain condition on congruences (see, e.g., [22]): since congruences can have special representations, Noetherianity concerns, e.g., chains of ideals in the case of rings and chains of submodules in the case of modules. Although this is somewhat non-standard, we may take a more abstract view and say that a *variety* (i.e. an equational class of structures) is Noetherian iff finitely generated free algebras satisfy the ascending chain condition for congruences or, equivalently, iff finitely generated algebras are finitely presented. Now, congruences over finitely generated free algebras may be represented as sets of equations among terms. This allows us to equivalently re-state the Noetherianity of varieties as "there are no infinite ascending chains of sets of equations modulo logical consequence". This observation was the basis for the abstract notion of Noetherian Fragment introduced in [16], here adapted for an arbitrary first-order theory.

Definition 3.1 (Noetherian Theory). *A Σ_0-theory T_0 is* Noetherian *if and only if for every* finite *set of free constants $\underline{a}$, every infinite ascending chain*

$$\Theta_1 \subseteq \Theta_2 \subseteq \cdots \subseteq \Theta_n \subseteq \cdots$$

of sets of ground $\Sigma_0^{\underline{a}}$-atoms is eventually constant modulo T_0, i.e. there is an n such that $T_0 \cup \Theta_n \models A$, for every natural number m and atom $A \in \Theta_m$.

Natural examples of Noetherian theories are the first-order axiomatization (in equational logic) of varieties like K-algebras, K-vector spaces, and R-modules, where K is a field and R is a Noetherian ring (see [22] for further details). *Abelian semigroups* are also Noetherian (cf. Theorem 3.11 in [8]). Notice that, since any extension (in the same signature) of a Noetherian theory is also Noetherian, any theory extending the theory of a single Associative-Commutative symbol is Noetherian. This shows that the family of Noetherian theories is important for verification because theories axiomatizing *integer addition* or *multiset union* formalize crucial aspects of a system to be verified (e.g., multisets may be used to check that the result of some operations like sorting on a collection of objects yields a permutation of the initial collection). More examples will be considered below.

Before being able to describe our new combination method, we need to introduce some preliminary notions. In the remaining of this section, *we fix two theories $T_0 \subseteq T$ in their respective signatures $\Sigma_0 \subseteq \Sigma$.*

Definition 3.2 (T_0-basis). *Given a finite set Θ of ground clauses (built out of symbols from Σ and possibly further free constants) and a finite set of free constants $\underline{a}$, a T_0*-basis *for Θ w.r.t. $\underline{a}$ is a set Δ of* positive *ground $\Sigma_0^{\underline{a}}$-clauses such that*

(i) *$T \cup \Theta \models C$, for all $C \in \Delta$ and*
(ii) *if $T \cup \Theta \models C$ then $T_0 \cup \Delta \models C$, for every positive ground $\Sigma_0^{\underline{a}}$-clause C.*

Notice that only constants in $\underline{a}$ may occur in a T_0-basis for Θ w.r.t. $\underline{a}$, although Θ may contain constants not in $\underline{a}$.

Definition 3.3 (Residue Enumerator). *Given a finite set $\underline{a}$ of free constants, a T-*residue enumerator for T_0 w.r.t. $\underline{a}$ *is a computable function $Res^{\underline{a}}_T(\Gamma)$ mapping a Σ-constraint Γ to a finite T_0-basis of Γ w.r.t. $\underline{a}$.*

If Γ is T-unsatisfiable, then a residue enumerator can always return the singleton set containing the empty clause. The concept of (Noetherian) residue enumerator is inspired by the work on partial theory reasoning (see, e.g., [3]) and generalizes the notion of deduction complete procedure of [18]. Given a residue enumerator for constraints (cf. Definition 3.3), it is always possible to build one for clauses (this will be useful for the combination method, see below).

Lemma 3.4. *Given a finite set $\underline{a}$ of free constants and a T-residue enumerator for T_0 w.r.t. $\underline{a}$, there exists a computable function $Res^{\underline{a}}_T(\Theta)$ mapping a finite set of ground clauses Θ to a finite T_0-basis of Θ w.r.t. $\underline{a}$.*

If T_0 is Noetherian, then it is possible to show that a finite T_0-basis for Γ w.r.t. $\underline{a}$ always exists, for every Σ-constraint Γ and for every set $\underline{a}$ of constants, by using König lemma. Unfortunately, such a basis is not always computable; this motivates the following notion.

Definition 3.5. *The theory T is an* effectively Noetherian extension of T_0 *if and only if T_0 is Noetherian and there exists a T-residue enumerator for T_0 w.r.t. every finite set $\underline{a}$ of free constants.*

For example, the theory of commutative K-algebras is an effectively Noetherian extension of the theory of K-vector spaces, where K is a field (see [16,28] for details). Locally finite theories and Linear Real Arithmetic are further examples taken from the literature about automated theorem proving.

A Σ_0-theory T_0 is *locally finite* iff Σ_0 is finite and, for every finite set of free constants $\underline{a}$, there are finitely many ground $\Sigma_0^{\underline{a}}$-terms $t_1, \ldots, t_{k_{\underline{a}}}$ such that for every ground $\Sigma_0^{\underline{a}}$-term u, $T_0 \models u = t_i$ (for some $i \in \{1, \ldots, k_{\underline{a}}\}$). If such $t_1, \ldots, t_{k_{\underline{a}}}$ are effectively computable from $\underline{a}$, then T_0 is *effectively locally finite* and there are finitely many (*representative*) $\Sigma_0^{\underline{a}}$-atoms $\psi_1(\underline{a}), \ldots, \psi_m(\underline{a})$ such that for any $\Sigma_0^{\underline{a}}$-atom $\psi(\underline{a})$, there is some i such that $T_0 \models \psi_i(\underline{a}) \leftrightarrow \psi(\underline{a})$. Examples of effectively locally finite theories are Boolean algebras, Linear Integer Arithmetic modulo a given integer, and any theory over a finite purely relational signature. Also, theories consisting of sentences which are true in a fixed finite Σ_0-structure $\mathcal{M} = (M, \mathcal{I})$ are locally finite. Enumerated datatypes can be formalized by theories in this class. The class of locally finite theories is (strictly) contained in that of Noetherian theories: to see this, it is sufficient to notice that, once fixed a finite set of free constants $\underline{a}$, there are only finitely many $\Sigma_0^{\underline{a}}$-atoms which are not equivalent to each other modulo the locally finite theory. From this, it is obvious that any infinite ascending chain of sets of such atoms must be eventually constant. Under the hypotheses that T_0 is effectively locally finite and its extension T has decidable constraint satisfiability problem, it is straightforward to build a T-residue enumerator for T_0.

Example. Let us consider the signature $\Sigma = \{0, +, -, \{f_r\}_{r\in\mathbb{R}}, \leq\}$ where 0 is a constant, $-$ and f_r are unary function symbols, $+$ is a binary function symbol, $\leq$ is a binary predicate symbol, and $\Sigma_0 = \Sigma \setminus \{\leq\}$. We consider the theory $T_{\mathbb{R}}^{\leq} = Th_{\Sigma}(\mathbb{R})$, i.e. the set of all Σ-sentences true in $\mathbb{R}$, which is seen as an $\mathbb{R}$-vector space equipped with a linear ordering, where the f_r's represent the external product so that terms are all equivalent to homogeneous linear polynomials. Finally, let $T_{\mathbb{R}}$ be $Th_{\Sigma_0}(\mathbb{R})$, i.e. the set of all Σ_0-sentences true in $\mathbb{R}$, which is seen as an $\mathbb{R}$-vector space without the ordering (so $T_{\mathbb{R}}$ is the theory of the $\mathbb{R}$-vector spaces, not reduced to $\{0\}$). The Noetherianity of $T_{\mathbb{R}}$ follows from general algebraic properties (see, e.g., [22]). A $T_{\mathbb{R}}^{\leq}$-residue enumerator for $T_{\mathbb{R}}$ can be obtained as follows. Let $\Gamma = \{C_1, \ldots, C_m\}$ be a set of inequalities, i.e. Σ-atoms whose main predicate symbol is $\leq$. By Definition 3.2, a Σ_0-basis for Γ is the set of all the disjunctions of equalities implied by Γ. Actually, to compute a basis, it is sufficient to identify the set of *implicit* equalities in Γ, i.e. the equalities $C_i^{=}$ such that $T_{\mathbb{R}}^{\leq} \models \Gamma \to C_i^{=}$ (here $C_i^{=}$ is obtained from C_i by substituting $\leq$ with $=$). This is so because (i) $T_{\mathbb{R}}^{\leq}$ is Σ_0-*convex* (i.e. if $T_{\mathbb{R}}^{\leq} \models \Gamma \to (e_1 \vee \cdots \vee e_n)$, then there exists $i \in \{1, \ldots, n\}$ such that $T_{\mathbb{R}}^{\leq} \models \Gamma \to e_i$, for $n \geq 1$ and equalities $e_1, \ldots, e_n$) and (ii) given a system of inequalities Γ, if Δ is the collection of all the implicit equalities of Γ and e is an equality such that $T_{\mathbb{R}}^{\leq} \models \Gamma \to e$, then $T_{\mathbb{R}} \models \Delta \to e$ (see [21] for full details, [28] for the adaptation to our context). The interest of implicit equalities is that they can be easily identified by using the Fourier-Motzkin variable elimination method (see [20] for details on how to do this).

3.1 Combination over Noetherian Theories

Preliminarily, we recall the notion of T_0-compatibility [13] which is crucial for the completeness of our combination technique.

Definition 3.6 (T_0-compatibility [13]). *Let T be a theory in the signature Σ and let T_0 be a universal theory in a subsignature $\Sigma_0 \subseteq \Sigma$. We say that T is* T_0-compatible *iff $T_0 \subseteq T$ and there is a Σ_0-theory $T_0^{\star}$ such that (i) $T_0 \subseteq T_0^{\star}$; (ii) $T_0^{\star}$ has quantifier elimination; (iii) every model of T_0 can be embedded into a model of $T_0^{\star}$; and (iv) every model of T can be embedded into a model of $T \cup T_0^{\star}$.*

The requirements (i)-(iii) guarantee the uniqueness of the theory $T_0^{\star}$, provided it exists ($T_0^{\star}$ is the *model completion* of T_0, see e.g. [7]). Notice that if T_0 is the empty theory over the empty signature, then T_0^{*} is the theory axiomatizing an infinite domain, (i)-(iii) hold trivially, and (iv) can be shown equivalent to the stably infinite requirement of the Nelson-Oppen schema [27,31]. Examples of theories satisfying the compatibility condition are the following: (a) the theory of K-algebras is compatible with the theory of K-vector spaces, where K is a field (see [16,28]), (b) $T_{\mathbb{R}}^{\leq}$ is compatible with the universal fragment of $T_{\mathbb{R}}$ (this is so for $T_{\mathbb{R}}^{\leq} \supseteq T_{\mathbb{R}}$ and $T_{\mathbb{R}}$ eliminates quantifiers), (c) any equational extension over a larger signature of the theory BA of Boolean algebras is BA-compatible [13], and (d) any extension of T_0 whatsoever is T_0-compatible whenever T_0 eliminates quantifiers.

The following lemma is *our main technical tool* allowing us to reduce satisfiability in a "temporalized" extension of a (Noetherian) theory to satisfiability in first-order logic.

Lemma 3.7 (Amalgamation). *Let I be a (possibly infinite) set of indexes; $\Sigma_i^{\underline{c},\underline{a}_i}$ (for $i \in I$) be signatures (expanded with free constants $\underline{c}, \underline{a}_i$), whose pairwise intersections are all equal to a certain signature $\Sigma_r^{\underline{c}}$ (i.e. $\Sigma_i^{\underline{c},\underline{a}_i} \cap \Sigma_j^{\underline{c},\underline{a}_j} = \Sigma_r^{\underline{c}}$, for all distinct $i, j \in I$); T_i be Σ_i-theories (for $i \in I$) which are all T_r-compatible, where $T_r \subseteq \bigcap_i T_i$ is a universal Σ_r-theory; $\{\Gamma_i\}_{i \in I}$ be sets of ground $\Sigma_i^{\underline{c},\underline{a}_i}$-clauses; and $\mathcal{B}^\star$ be a set of positive ground $\Sigma_r^{\underline{c}}$-clauses not containing the empty clause and satisfying the following condition:*

$$\textit{if } T_i \cup \Gamma_i \cup \mathcal{B}^\star \models C, \textit{ then } C \in \mathcal{B}^\star,$$

for $i \in I$ and every positive ground $\Sigma_r^{\underline{c}}$-clause C. Then, there exists a $\bigcup_i(\Sigma_i^{\underline{c},\underline{a}_i})$-structure $\mathcal{M}$ such that $\mathcal{M} \models \bigcup_i(T_i \cup \Gamma_i)$ or, equivalently, there exist $\Sigma_i^{\underline{c},\underline{a}_i}$-structures $\mathcal{M}_i$ ($i \in I$) satisfying $T_i \cup \Gamma_i$, whose $\Sigma_r^{\underline{c}}$-reducts coincide.

This lemma can also be used to prove the "first-order version" of the combination result in [16], where residue enumerators permit the exchange of positive clauses between theories.

3.2 The Theory of a Free Unary Function Symbol

By collecting the observations above, it is easy to identify pairs of theories (T, T_0) such that T satisfies our relevant requirements to be 'combined over T_0' (i.e. T is such that $T_0 \subseteq T$ and T is a T_0-compatible effectively Noetherian extension of T_0). Here, we consider an entirely new (and somewhat remarkable) class of examples of such pairs (T, T_0) of theories.

Let f be a unary function symbol. If T is a theory, then T_f is the theory obtained from T by adding f to its signature (as a new free function symbol). So, e.g., if E the empty theory over the empty signature, E_f denotes the empty theory over the signature $\{f\}$.

Proposition 3.8. *E_f is Noetherian.*

A theory T is stably infinite (see, e.g., [27,31]) iff it is E-compatible, or, equivalently, iff any T-satisfiable constraint is satisfiable in a model of T whose domain is infinite.

Proposition 3.9. *If T is stably infinite and has decidable constraint satisfiability problem, then T_f is an effectively Noetherian extension of E_f.*

Proposition 3.10. *If T is stably infinite, then T_f is E_f-compatible.*

We are now ready to characterize our new class of theories.

Theorem 3.11. *Let T be a theory with decidable constraint satisfiability problem. If T is stably infinite, then T_f is an effectively Noetherian extension of E_f, which is also E_f-compatible.*

This result is a first step towards the integration in our framework of some theories that are useful for verification. For example, the theory of integer offsets can be seen as an extension of the theory of a loop-free unary function symbol (see, e.g., [1]). Properties of hardware systems can be expressed in a mixture of temporal logic – e.g., LTL or Computation Tree Logic (CTL) – and the theory of integer offsets [6]. Our decidability results on "temporalized" first-order theories below (see Theorems 4.11 and 5.4) can then be used to augment the degree of automation of tools attempting to solve this kind of verification problems.

4 Temporalizing a First-Order Theory

We introduce "temporalized" first-order theories, by using LTL to describe the temporal dimension. We use the formal framework introduced in [14] where formulae are obtained by applying temporal and Boolean operators (but no quantifiers) to first-order formulae over a given signature.

Definition 4.1 (LTL($\Sigma^{\underline{a}}$)-Sentences [14]). *Given a signature Σ and a (finite or infinite) set of free constants $\underline{a}$, the set of LTL($\Sigma^{\underline{a}}$)-sentences is inductively defined as follows: (a) if φ is a first-order $\Sigma^{\underline{a}}$-sentence, then φ is an LTL($\Sigma^{\underline{a}}$)-sentence and (b) if ψ_1, ψ_2 are LTL($\Sigma^{\underline{a}}$)-sentence, so are $\psi_1 \wedge \psi_2$, $\psi_1 \vee \psi_2$, $\neg\psi_1$, $X\psi_1$, $\Box\psi_1$, $\Diamond\psi_1$, $\psi_1 U \psi_2$.*

The free constants $\underline{a}$ allowed in LTL($\Sigma^{\underline{a}}$)-sentences will be used to model the variables and the parameters of (reactive) systems.

Definition 4.2 ([14]). *Given a signature Σ and a set $\underline{a}$ of free constants, an* LTL($\Sigma^{\underline{a}}$)-structure *(or simply a* structure*) is a sequence $\mathcal{M} = \{\mathcal{M}_n = (M, \mathcal{I}_n)\}_{n\in\mathbb{N}}$ of $\Sigma^{\underline{a}}$-structures. The set M is called the* domain *(or the* universe*) and $\mathcal{I}_n$ is called the n*-th level interpretation function *of the LTL($\Sigma^{\underline{a}}$)-structure.*[1]

When considering a background Σ-theory T, the structures $\mathcal{M}_n = (M_n, \mathcal{I}_n)$ will be taken to be models of T (further requirements will be analyzed later on).

Definition 4.3 ([14]). *Given an LTL($\Sigma^{\underline{a}}$)-sentence φ and $t \in \mathbb{N}$, the notion of "φ being true in the LTL($\Sigma^{\underline{a}}$)-structure $\mathcal{M} = \{\mathcal{M}_n = (M, \mathcal{I}_n)\}_{n\in\mathbb{N}}$ at the instant t" (in symbols $\mathcal{M} \models_t \varphi$) is inductively defined as follows:*

- *if φ is an first-order sentence, $\mathcal{M} \models_t \varphi$ iff $\mathcal{M}_t \models \varphi$;*
- *$\mathcal{M} \models_t \neg\varphi$ iff $\mathcal{M} \not\models_t \varphi$;*
- *$\mathcal{M} \models_t \varphi \wedge \psi$ iff $\mathcal{M} \models_t \varphi$ and $\mathcal{M} \models_t \psi$;*
- *$\mathcal{M} \models_t \varphi \vee \psi$ iff $\mathcal{M} \models_t \varphi$ or $\mathcal{M} \models_t \psi$;*
- *$\mathcal{M} \models_t X\varphi$ iff $\mathcal{M} \models_{t+1} \varphi$;*
- *$\mathcal{M} \models_t \Box\varphi$ iff for each $t' \geq t$, $\mathcal{M} \models_{t'} \varphi$;*
- *$\mathcal{M} \models_t \Diamond\varphi$ iff for some $t' \geq t$, $\mathcal{M} \models_{t'} \varphi$;*
- *$\mathcal{M} \models_t \varphi U \psi$ iff there exists $t' \geq t$ such that $\mathcal{M} \models_{t'} \psi$ and for each t'', $t \leq t'' < t' \Rightarrow \mathcal{M} \models_{t''} \varphi$.*

[1] In more detail, $\mathcal{I}_n$ is such that $\mathcal{I}_n(P) \subseteq M^k$ for every predicate symbols $P \in \Sigma$ of arity k, and $\mathcal{I}_n(f) : M^k \longrightarrow M$ for each function symbol $f \in \Sigma$ of arity k.

Let φ be an LTL($\Sigma^{\underline{a}}$)-sentence; we say that φ *is true in* $\mathcal{M}$ or, equivalently, that $\mathcal{M}$ *satisfies* φ (in symbols $\mathcal{M} \models \varphi$) iff $\mathcal{M} \models_0 \varphi$.

Since we distinguish between rigid (i.e. time-independent) and flexible (i.e. time-dependent) symbols of the signature, we need to introduce a notion of first-order theory that fixes a sub-signature and distinguish between two kinds of free constants.

Definition 4.4. *A* data-flow theory *is a 5-tuple* $\mathcal{T} = \langle \Sigma, T, \Sigma_r, \underline{a}, \underline{c} \rangle$ *where* Σ *is a signature,* T *is a* Σ*-theory (called the underlying theory of* $\mathcal{T}$*),* Σ_r *is the* rigid subsignature *of* Σ*,* $\underline{a}$ *is a set of free constants (called* system variables*), and* $\underline{c}$ *is a set of free constants (called* system parameters*).*

A data-flow theory $\mathcal{T} = \langle \Sigma, T, \Sigma_r, \underline{a}, \underline{c} \rangle$ is *totally flexible* iff Σ_r is empty and is *totally rigid* iff $\Sigma_r = \Sigma$. In [14], data-flow theories are called LTL-theories. Here, we prefer to use the more abstract term of data-flow theory in order to prepare for the generalization of the decidability result in the next section.

Definition 4.5 ([14]). *An LTL(*$\Sigma^{\underline{a},\underline{c}}$*)-structure* $\mathcal{M} = \{\mathcal{M}_n = (M, \mathcal{I}_n)\}_{n \in \mathbb{N}}$ *is* appropriate *for a data-flow theory* $\mathcal{T} = \langle \Sigma, T, \Sigma_r, \underline{a}, \underline{c} \rangle$ *iff for all* $m, n \in \mathbb{N}$*, for all function symbol* $f \in \Sigma_r$*, for all relational symbol* $P \in \Sigma_r$*, and for all constant* $c \in \underline{c}$*, we have*

$$\mathcal{M}_n \models T, \quad \mathcal{I}_n(f) = \mathcal{I}_m(f), \quad \mathcal{I}_n(P) = \mathcal{I}_m(P), \quad \mathcal{I}_n(c) = \mathcal{I}_m(c).$$

The satisfiability problem for $\mathcal{T}$ *is the following: given an LTL(*$\Sigma^{\underline{a},\underline{c}}$*)-sentence* φ*, decide whether there is an LTL(*$\Sigma^{\underline{a},\underline{c}}$*)-structure* $\mathcal{M}$ *appropriate for* $\mathcal{T}$ *such that* $\mathcal{M} \models \varphi$*. The* ground satisfiability problem for $\mathcal{T}$ *is similarly introduced, but* φ *is assumed to be ground.*

Notice that appropriate structures are such that the equality symbol is always interpreted as the identity relation, since the equality is included in every signature (hence also in the rigid signature Σ_r).

In the sequel, we shall concentrate on the ground satisfiability problem for data-flow theories; for this reason, we shall assume from now on that **the underlying theory** T **of any data-flow theory** $\mathcal{T} = \langle \Sigma, T, \Sigma_r, \underline{a}, \underline{c} \rangle$ **has decidable constraint satisfiability problem**. Unfortunately, this assumption is insufficient to guarantee decidability.

Theorem 4.6 ([14]). *There exists a totally flexible data-flow theory* $\mathcal{T}$ *whose ground satisfiability problem is undecidable.*

Notwithstanding the undecidability of the ground satisfiability problem, the following compatibility requirement can be used to re-gain decidability.

Definition 4.7. *A data-flow theory* $\mathcal{T} = \langle \Sigma, T, \Sigma_r, \underline{a}, \underline{c} \rangle$ *is said to be* Noetherian compatible *iff there is a* Σ_r*-universal theory* T_r *such that* T *is an effectively Noetherian and* T_r*-compatible extension of* T_r*.*

The definition above refers to a Σ_r-theory T_r such that T is T_r-compatible. Although not relevant for the results in this paper, we notice that if such a theory T_r exists, then one can always take T_r to be the theory axiomatized by the universal Σ_r-sentences which are logical consequences of T.

4.1 A Decision Procedure for the Noetherian Compatible Case

Preliminarily, we recall that it is possible to define the notion of ground model-checking problem in our framework [14] and to show its undecidability when the underlying theory is Noetherian. The argument of the proof is a simple reduction to the (undecidable) reachability problem of Minsky machines [26,10] by using the reduct of Presburger Arithmetic obtained by forgetting addition and ordering, which is capable of encoding counters (see [15] for details). This is why here we focus on the ground satisfiability problem in the Noetherian compatible case.

Before developing our decision procedure, some preliminary notions are required.

Definition 4.8 (PLTL-Abstraction [14]). *Given a signature $\Sigma^{\underline{a}}$ and a set of propositional letters $\mathcal{L}$ of the appropriate cardinality, let $[\![\cdot]\!]$ be a bijection from the set of ground $\Sigma^{\underline{a}}$-atoms into $\mathcal{L}$. By translating identically Boolean and temporal connectives, the map is inductively extended to a bijective map (also called $[\![\cdot]\!]$) from the set of LTL($\Sigma^{\underline{a}}$)-sentences onto the set of propositional $\mathcal{L}$-formulae.*

Given a ground LTL($\Sigma^{\underline{a}}$)-sentence φ, we call $[\![\varphi]\!]$ the *PLTL-abstraction* of φ; moreover, if Θ is a set of ground LTL($\Sigma^{\underline{a}}$)-sentences, the PLTL-abstraction $[\![\Theta]\!]$ of Θ denotes the set $\{[\![\varphi]\!] \mid \varphi \in \Theta\}$.

Definition 4.9 (φ-Guessing). *Let φ be a ground LTL($\Sigma^{\underline{a},\underline{c}}$)-sentence. A φ-*guessing *is a Boolean assignment to literals of φ (we view a guessing as the set $\{\ell \mid \ell$ is an atom occurring in φ and ℓ is assigned to true$\} \cup \{\neg\ell \mid \ell$ is an atom occurring in φ and ℓ is assigned to false$\}$).*

We say that a (non-empty) set of φ-guessings $\mathcal{G}_{(\varphi)} := \{G_1, \ldots, G_k\}$ is *φ-compatible* if and only if $[\![\varphi \wedge \Box \bigvee_{i=1}^{k} G_i]\!]$ is PLTL-satisfiable.

Let $\mathcal{T} = \langle \Sigma, T, \Sigma_r, \underline{a}, \underline{c} \rangle$ be a Noetherian compatible data-flow theory. The procedure NSat (see Algorithm 1) takes a ground LTL($\Sigma^{\underline{a},\underline{c}}$)-sentence φ as input and returns *"satisfiable"* if there is an appropriate LTL($\Sigma^{\underline{a},\underline{c}}$)-structure $\mathcal{M}$ for $\mathcal{T}$ such that $\mathcal{M} \models \varphi$; otherwise, it returns *"unsatisfiable"*. The procedure relies on a decision procedure for the PLTL-satisfiability problem in order to recognize the φ-compatible sets of φ-guessings (cf. the outer loop of NSat). Moreover, Dp-t is a decision procedure for the satisfiability problem of arbitrary Boolean combinations of atoms of the theory T (i.e., it is capable of checking the T-satisfiability of sets of ground $\Sigma^{\underline{a},\underline{c}}$-clauses and not only of ground $\Sigma^{\underline{a},\underline{c}}$-literals). Notice that Dp-t can be implemented by Satisfiability Modulo Theories solvers (see, e.g., [30]). Finally, $Res^{\underline{c}}_{T}$ is the T-residue enumerator for T_r w.r.t. $\underline{c}$.

In the outer loop of NSat, all possible φ-compatible sets of φ-guessings are enumerated. Let $\mathcal{G}_{(\varphi)} := \{G_1, \ldots, G_n\}$ be the current φ-guessing. The local variable $\mathcal{B}$ is initialized to the empty set (line 3) and then updated in the inner loop (lines 4-10) as follows: the T_r-bases $\mathcal{B}_i$ for $G_i \cup \mathcal{B}$ w.r.t. $\underline{c}$ are computed (for $i = 1, \ldots, n$), and the new value of $\mathcal{B}$ is set to $\bigcup_i \mathcal{B}_i$ (line 5 saves in $\mathcal{B}'$ the old value of $\mathcal{B}$). The inner loop is iterated until $\mathcal{B}$ is logically equivalent to $\mathcal{B}'$

Algorithm 1. The satisfiability procedure for the Noetherian compatible case

Require: φ ground LTL($\Sigma^{\underline{a},\underline{c}}$)-sentence
procedure NSAT(φ)
  for all φ-compatible set of φ-guessing $\mathcal{G}_{(\varphi)}$ **do**
    $\mathcal{B} \leftarrow \emptyset$
    repeat
      $\mathcal{B}' \leftarrow \mathcal{B}$
      for all $G_i \in \mathcal{G}_{(\varphi)}$ **do**
        $\mathcal{B}_i \leftarrow Res_T^{\underline{c}}(G_i \cup \mathcal{B})$
      end for
      $\mathcal{B} \leftarrow \bigcup_i \mathcal{B}_i$
    until DP-T($\mathcal{B}' \wedge \neg\mathcal{B}$) = *"unsatisfiable"*
    if DP-T($\mathcal{B}$) = *"satisfiable"* **then**
      return *"satisfiable"*
    end if
  end for
  return *"unsatisfiable"*
end procedure

modulo T. At this point, if $\mathcal{B}$ is T-consistent, the procedure stops and returns *"satisfiable"*; otherwise it tries another φ-compatible set of φ-guessings. If for all φ-compatible sets of φ-guessings the $\mathcal{B}$'s returned after the execution of the inner loop are T-inconsistent, the procedure returns *"unsatisfiable"*.

Proposition 4.10 (Correctness of NSat). *Let $\mathcal{T} = \langle \Sigma, T, \Sigma_r, \underline{a}, \underline{c} \rangle$ be a Noetherian compatible data-flow theory and φ be a ground LTL($\Sigma^{\underline{a},\underline{c}}$)-sentence. Then,* NSAT*(φ) returns "satisfiable" iff there exists an LTL($\Sigma^{\underline{a},\underline{c}}$)-structure $\mathcal{M}$ appropriate for $\mathcal{T}$ such that $\mathcal{M} \models \varphi$.*

Indeed, the termination of NSAT is a consequence of the Noetherianity of the underlying theory of $\mathcal{T}$ by using the fact that every infinite ascending chain of sets of positive ground $\Sigma_r^{\underline{c}}$-clauses is eventually constant for logical consequence. The correctness and termination of NSAT yield our main decidability result.

Theorem 4.11. *The ground satisfiability problem for Noetherian compatible data-flow theories is decidable.*

The theories considered in the previous section (especially, those in Section 3.2) satisfy the hypothesis of the theorem above.

5 Extensions to Abstract Temporal Logics

By considering the proof of the correctness of NSAT, it becomes evident that only very few of the characteristic properties of LTL are used. It turns out that a simple generalization of NSAT can be used to decide satisfiability problems of "temporalized" extensions of Noetherian theories whose flow of time is not linear, e.g., branching as in CTL.

In order to formalize the observation above, we regard modal/temporal operators as functions operating on powerset Boolean algebras. In this way, logics for various flows of time, as well as CTL, Propositional Dynamic Logic (PDL), and the μ-calculus fall within the scope of our result (see [2] for a similar approach).

Definition 5.1. *An* abstract temporal signature[2] *I is a purely functional signature extending the signature BA of Boolean algebras.*[3] *An* abstract temporal logic *L is a class of I-structures, whose Boolean reducts are powerset Boolean algebras. Given an I-term t, deciding whether $t \neq 0$ is satisfied in some member of L is the* satisfiability *problem for L. Given I-terms t, u, deciding whether $u = 1$ & $t \neq 0$ is satisfied in some member of L is the* relativized satisfiability *problem for L.*

In many cases (e.g., LTL, CTL, PDL, and the μ-calculus), it is possible to reduce the relativized satisfiability problem to that of satisfiability (by using the so-called "master modality"); however, there are logics for which the latter is decidable whereas the former is undecidable (see [12]).

Definition 5.2 ($I(\Sigma^{\underline{a}})$-sentence). *Given a signature Σ, a (finite or infinite) set of free constants $\underline{a}$, and an abstract temporal signature I, the set of $I(\Sigma^{\underline{a}})$-sentences is inductively defined as follows: (a) if φ is a first-order $\Sigma^{\underline{a}}$-sentence, then φ is an $I(\Sigma^{\underline{a}})$-sentence, (b) if φ_1, φ_2 are $I(\Sigma^{\underline{a}})$-sentences, so are $\varphi_1 \wedge \varphi_2, \varphi_1 \vee \varphi_2, \neg\varphi_1$, and (c) if $\psi_1, \ldots, \psi_n$ are $I(\Sigma^{\underline{a}})$-sentences and $O \in I \setminus BA$ has arity n, then $O(\psi_1, \ldots, \psi_n)$ is a $I(\Sigma^{\underline{a}})$-sentence.*

When I is LTL, $I(\Sigma^{\underline{a},\underline{c}})$-sentences coincide with LTL$(\Sigma^{\underline{a},\underline{c}})$-sentences (cf. Definition 4.1). We defined an abstract temporal logic L (based on I) as a class of I-structures based on powerset Boolean algebras: such structures (also called I-frames) will be denoted with $\mathcal{F} = (\wp(F), \{O^{\mathcal{F}}\}_{O \in I \setminus BA})$.

Definition 5.3. *Let a signature Σ, a set $\underline{a}$ of free constants, and an abstract temporal signature I be given; an $I(\Sigma^{\underline{a}})$-*structure *(or simply a* structure*) is a pair formed by an I-frame $\mathcal{F} = (\wp(F), \{O^{\mathcal{F}}\}_{O \in I \setminus BA})$ and a collection $\mathcal{M} = \{\mathcal{M}_n = (M, \mathcal{I}_n)\}_{n \in F}$ of $\Sigma^{\underline{a}}$-structures (all based on the same domain).*

An $I(\Sigma^{\underline{a}})$-sentence φ is true in the $I(\Sigma^{\underline{a}})$-structure $(\mathcal{F}, \mathcal{M})$ at $t \in F$ (noted $\mathcal{F}, \mathcal{M} \models_t \varphi$) iff the following holds: (a) if φ is a first-order sentence, then $\mathcal{F}, \mathcal{M} \models_t \varphi$ holds iff $\mathcal{M}_t \models \varphi$ and (b) if the main operator of φ is a Boolean connective, truth of φ is defined in a truth-table manner; (c) if φ is of the kind $O(\psi_1, \ldots, \psi_n)$, then $\mathcal{F}, \mathcal{M} \models_t \varphi$ holds iff $t \in O^{\mathcal{F}}(\{u \mid \mathcal{F}, \mathcal{M} \models_u \psi_1\}, \ldots, \{u \mid \mathcal{F}, \mathcal{M} \models_u \psi_n\})$.

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[2] From the modal/temporal literature viewpoint, the adjective "intensional" might be preferable to "abstract temporal". We have chosen the latter, in order to emphasize that our results are deemed as significant for a class of logics whose modalities concern flows of time.

[3] This signature contains two binary function symbols for meet and join, a unary function symbol for complement, and two constants for zero and one (the latter are denoted with 0 and 1, respectively).

If a data-flow theory $\mathcal{T}$ is given, we say that an $I(\Sigma^{\underline{a}})$-structure is appropriate for $\mathcal{T}$ iff it satisfies the requirements of Definition 4.5. The (ground) satisfiability problem for an abstract temporal logic L (based on I) and for a data-flow theory $\mathcal{T}$ is now the following: given a (ground) $I(\Sigma^{\underline{a}})$-sentence φ, decide whether there is a $I(\Sigma^{\underline{a}})$-structure $(\mathcal{F}, \mathcal{M})$ appropriate for $\mathcal{T}$, such that $\mathcal{F} \in L$ and such that $\mathcal{F}, \mathcal{M} \models_t \varphi$ holds for some t.

Theorem 5.4. *The ground satisfiability problem for $\mathcal{T}$ and L is decidable if (i) $\mathcal{T}$ is Noetherian compatible and (ii) the relativized satisfiability problem for L is decidable.*

When I is LTL, this result simplifies to Theorem 4.11. To prove Theorem 5.4, it is possible to re-use NSat (cf. Algorithm 1) almost 'off-the-shelf', by preliminarily adapting the definition of PLTL-abstraction function $[\![\cdot]\!]$ (cf. Definition 4.8) to L in the obvious way. It turns out that only the compatibility of guessings should be changed: a finite set of φ-guessings $\mathcal{G}_{(\varphi)} := \{G_1, \ldots, G_k\}$ is φ-compatible if and only if the relativized satisfiability problem

$$[\![\varphi]\!] \neq 0 \quad \& \quad [\![\bigvee_{i=1}^{k} G_i]\!] = 1$$

is satisfiable in L (this is the only modification required to the definitions and proofs from Section 4.1).

While Theorem 4.11 is relevant to augment the degree of mechanization of deductive approaches for the verification of reactive systems based on LTL (e.g., the one put-forward by Manna and Pnueli [23]), one may wonder about the relevance of its generalization, i.e. Theorem 5.4. To see its usefulness, consider TLA [19]. For such a specification formalism, it is difficult to reuse techniques and tools for (classic) temporal/modal logic since TLA features some non-standard characteristics which are quite useful for practitioners (see [25] for an extensive discussion on this and related issues). On the other hand, deductive verification of TLA specifications can be supported by proof assistants (e.g., [24]). While applying the inference rules of TLA [19], it has been observed [25] that some of the resulting sub-goals may belong to a fragment of TLA which is equivalent to the modal logic S4.2 [4]. Now, the relativized satisfiability problem for this logic is decidable (see again [4]) so that NSat can be used to automatically discharge some of the sub-goals, whenever the data-flow theory formalizing the data structure manipulated by the system modelled in TLA is Noetherian compatible.

6 Conclusions

We have investigated the role of Noetherianity for the decidability of the satisfiability problem for "temporalized" first-order theories (cf. Sections 4 and 5). The key technical contribution is Lemma 3.7, which allows us to obtain amalgamations of (possibly infinite) sequences of first-order structures corresponding to temporal structures. This lemma is the basis of a method for combinations of first-order theories over Noetherian theories. An important class of stably infinite theories extending the empty theory over a single unary function symbol has been shown to satisfy the hypotheses for the decidability of both the combination schema and the satisfiability of "temporalized" first-order theories (cf. Section 3.2).

The results in this paper extends those of [14] in two ways. First, the requirement of local finiteness of the (rigid) sub-theory is weakened to that of Noetherianity. Second, decidability is parametric w.r.t. a modal/temporal logic, provided that relativized satisfiability problem is decidable in the latter.

References

1. Armando, A., Bonacina, M.P., Ranise, S., Schulz, S.: On a rewriting approach to satisfiability procedures: extension, combination of theories and an experimental appraisal. In: Gramlich, B. (ed.) FroCoS 2005. LNCS (LNAI), vol. 3717, pp. 65–80. Springer, Heidelberg (2005)
2. Baader, F., Lutz, C., Sturm, H., Wolter, F.: Fusions of description logics and abstract description systems. Journal of A.I. Research 16, 1–58 (2002)
3. Baumgartner, P., Furbach, U., Petermann, U.: A unified approach to theory reasoning. Research Report 15–92, Universität Koblenz-Landau (1992)
4. Blackburn, P., de Rijke, M., Venema, Y.: Modal Logic. Cambridge University Press, Cambridge (2002)
5. Bonacina, M.P., Ghilardi, S., Nicolini, E., Ranise, S., Zucchelli, D.: Decidability and undecidability results for Nelson-Oppen and rewrite-based decision procedures. In: Furbach, U., Shankar, N. (eds.) IJCAR 2006. LNCS (LNAI), vol. 4130, pp. 513–527. Springer, Heidelberg (2006)
6. Bryant, R.E., Lahiri, S.K., Seshia, S.A.: Modeling and verifying systems using a logic of counter arithmetic with lambda expressions and uninterpreted functions. In: Brinksma, E., Larsen, K.G. (eds.) CAV 2002. LNCS, vol. 2404, pp. 78–92. Springer, Heidelberg (2002)
7. Chang, C.-C., Keisler, J.H.: Model Theory, 3rd edn. North-Holland, Amsterdam-London (1990)
8. Le Chenadec, P.: Canonical Forms in Finitely Presented Algebras. Research Notes in Theoretical Computer Science. Pitman-Wiley (1986)
9. Degtyarev, A., Fisher, M., Konev, B.: Monodic temporal resolution. ACM Transaction on Computational Logic 7(1), 108–150 (2006)
10. Ebbinghaus, H.-D., Flum, J., Thomas, W.: Mathematical logic. In: Undergraduate Texts in Mathematics, 2nd edn., Springer, Heidelberg (1994)
11. Finger, M., Gabbay, D.M.: Adding a temporal dimension to a logic system. Journal of Logic, Language, and Information 1(3), 203–233 (1992)
12. Gabbay, D.M., Kurucz, A., Wolter, F., Zakharyaschev, M.: Many-Dimensional Modal Logics: Theory and Applications. Studies in Logic and the Foundations of Mathematics, vol. 148, North-Holland, Amsterdam (2003)
13. Ghilardi, S.: Model theoretic methods in combined constraint satisfiability. Journal of Automated Reasoning 33(3-4), 221–249 (2004)
14. Ghilardi, S., Nicolini, E., Ranise, S., Zucchelli, D.: Combination methods for satisfiability and model-checking of infinite-state systems. In: Pfenning, F. (ed.) CADE 2007. LNCS, vol. 4603, pp. 362–378. Springer, Heidelberg (2007)

15. Ghilardi, S., Nicolini, E., Ranise, S., Zucchelli, D.: Combination methods for satisfiability and model-checking of infinite-state systems. Technical Report RI313-07, Università degli Studi di Milano (2007) Available at http://homes.dsi.unimi.it/ zucchell/publications/techreport/GhiNiRaZu-RI313-07.pdf
16. Ghilardi, S., Nicolini, E., Zucchelli, D.: A comprehensive framework for combined decision procedures. ACM Transactions on Computational Logic (to appear)
17. Hodkinson, I.M., Wolter, F., Zakharyaschev, M.: Decidable fragment of first-order temporal logics. Annals of Pure and Applied Logic 106(1–3), 85–134 (2000)
18. Kirchner, H., Ranise, S., Ringeissen, C., Tran, D.-K.: On superposition-based satisfiability procedures and their combination. In: Van Hung, D., Wirsing, M. (eds.) ICTAC 2005. LNCS, vol. 3722, pp. 594–608. Springer, Heidelberg (2005)
19. Lamport, L.: The temporal logic of actions. ACM Transactions on Programming Languages and Systems 16(3), 872–923 (1994)
20. Lassez, J.-L., Maher, M.J.: On Fourier's algorithm for linear arithmetic constraints. Journal of Automated Reasoning 9(3), 373–379 (1992)
21. Lassez, J.-L., McAloon, K.: A canonical form for generalized linear constraints. Journal of Symbolic Computation 13(1), 1–24 (1992)
22. MacLane, S., Birkhoff, G.: Algebra, 3rd edn. Chelsea Publishing, New York (USA) (1988)
23. Manna, Z., Pnueli, A.: Temporal Verification of Reactive Systems: Safety. Springer, Heidelberg (1995)
24. Merz, S.: Isabelle/TLA (1999), Available at http://isabelle.in.tum.de/library/HOL/TLA.
25. Merz, S.: On the logic of TLA. Computing and Informatics 22, 351–379 (2003)
26. Minsky, M.L.: Recursive unsolvability of Post's problem of "tag" and other topics in the theory of Turing machines. Annals of Mathematics 74(3), 437–455 (1961)
27. Nelson, G., Oppen, D.C.: Simplification by cooperating decision procedures. ACM Transaction on Programming Languages and Systems 1(2), 245–257 (1979)
28. Nicolini, E.: Combined decision procedures for constraint satisfiability. PhD thesis, Dipartimento di Matematica, Università degli Studi di Milano (2007)
29. Plaisted, D.A.: A decision procedure for combination of propositional temporal logic and other specialized theories. Journal of Automated Reasoning 2(2), 171–190 (1986)
30. Ranise, S., Tinelli, C.: Satisfiability modulo theories. IEEE Magazine on Intelligent Systems 21(6), 71–81 (2006)
31. Tinelli, C., Harandi, M.T.: A new correctness proof of the Nelson-Oppen combination procedure. In: Proc. of FroCoS 1996, Applied Logic, pp. 103–120. Kluwer Academic Publishers, Dordrecht (1996)