Color Image Segmentation Based on Type-2 Fuzzy Sets and Region Merging

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Abstract. This paper focuses on application of fuzzy sets of type 2 (FS2) in color images segmentation. The proposed approach is based on FS2 entropy application and region merging. Both local and global information of the image are employed and FS2 makes it possible to take into account the total uncertainty inherent to the segmentation operation. Fuzzy entropy is utilized as a tool to perform histogram analysis to find all major homogeneous regions at the first stage. Then a basic and fast region merging process, based on color similarity and reduction of small clusters, is carried out to avoid oversegmentation. The experimental results demonstrate that this method is suitable to find homogeneous regions for natural images, even for noisy images.

1 Introduction

Segmentation remains one of the most important problems in color image analysis nowadays. The two main techniques described in the literature are **region reconstruction** (image plane analysis using region growing process, [6], [1]) and **color space analysis** (the color of each pixel is represented in a color space). Many authors have tried to determine the best color space for some specific color image segmentation problems ([22]), but, unfortunately, there does not exist an unique color space for all the segmentation problems. Computational complexity may increase significantly (in comparison with gray scale image segmentation), so we have classically chosen to work in the (R, G, B) color space, where a color point is defined by the color component levels of the corresponding pixel, red (R), green (G) and blue (B). These two techniques have considerable drawbacks. The region-oriented techniques tend to over-segment images, and the second techniques are not robust to significance appearance changes because they do not include any spatial information. Fuzzy logic is considered to be an appropriate tool for image analysis, and particularly for gray scale segmentation ([2], [14], [30]). These techniques have been tested with success for color image analysis. Recently, fuzzy region oriented techniques for color image segmentation have been presented ([15], [3]), defining a region as a fuzzy subset of pixels, where each pixel in the image has a membership degree to each region. These techniques are based on fuzzy logic with type-1 fuzzy sets. Other techniques

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have been presented to perform color clustering in a color space ([5], [4]), based on type-1 fuzzy sets and a homogeneity measure (homogeneity of the "paths" connecting the pixels, [9], or fuzzy homogeneity calculated by fuzzy entropy, [4]). The major concern of these techniques is that spatial ambiguity among pixels has inherent vagueness rather than randomness. However, there remain some sources of uncertainties in type-1 fuzzy sets (see: [18]): the meanings of the words that are used, measurements may be noisy, the data used to tune the parameters of type-1 fuzzy sets may also be noisy. Imprecision and uncertainty are naturally present in image processing ([23]), and particularly these three kinds of uncertainty. Techniques that are not much used for the moment in color images analysis are type-2 fuzzy sets. Mendel ([8], [11], [18]) shows that **type-2 fuzzy sets** (or FS2) may be applied to take into account these three kinds of uncertainty (measurement noise, data-generating mechanism, and description of features that are all nonstationary, when the nature of the nonstationarities cannot be expressed mathematically), and we have investigated this new scheme in this paper. The concept of a type-2 fuzzy set was introduced first by Zadeh ([24]) as an extension of the concept of an ordinary fuzzy set (type-1 fuzzy set). Type-2 fuzzy sets have grades of membership that are themselves fuzzy. At each value of the primary variable (discourse universe X), the membership is a function (and not just a point value) - the secondary membership function - whose domain (the primary membership) is in the interval [0,1] and whose range (the secondary grades) may also be in [0,1]. Hence, the membership function of a type-2 fuzzy set is three dimensional, and it is the new third dimension that provides new design degrees of freedom for handling uncertainty. In this paper we propose to use FS2 for the segmentation of color images in the color (RGB) space. The paper is organized as follows:

- Section 2 briefly describes the type-2 fuzzy sets
- Section 3 introduces image segmentation using type-2 fuzzy sets
- In section 4 we present some results
- Finally, the paper is summarized with some conclusions in section 5.

2 Type-2 Fuzzy Sets

2.1 Definition

Type-1 fuzzy sets that are used in image processing are often fuzzy numbers. However, it is not possible to say which membership function is the best one. This is the major motivation of this work to remove the uncertainty of membership values by using type-2 fuzzy sets. For example, the type-1 fuzzy sets may be interesting to modelize the imprecision of patients in telemedicine (visual acuity of vision tested by fuzzy logic with an application in ophthalmology, [13]). This imprecise value allows the modelization of the visual acuity (from 0 to 10), but it is not possible to take into account the bounds of the intervals of modelization. One possible approach consists in calculating an average value of the bounds observed with n patients. Another possible approach consists in

making use of the average values and the standard deviation for the two end-points of the membership function (representing the type-1 fuzzy set), and leads to a continuum of fuzzy numbers. Let be X the discourse universe:
A type-2 fuzzy set (FS2) A, or $\tilde{A}$, is characterized by its membership function (MF) $\mu(x, u)$, where $x \in X$ and $u \in J_x \subseteq$ [0,1], with:
$\tilde{A} = \{((x, u),\ \mu(x, u)) \mid \forall\ x \in X, \forall\ u \in J_x \subseteq [0,1]\}$, where $0 \leq \mu(x, u) \leq 1$. $\tilde{A}$ may also be characterized as following:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} (\mu(x, u)/(x, u)) dx du \tag{1}$$

2.2 Uncertainty Representation

A way to visualize a type-2 fuzzy set $\tilde{A}$ is to sketch its two-dimensional domain, its "footprint of uncertainty" (FOU, [18]). The heights of a type-2 MF (its secondary grades) sit atop of its FOU. The FOU is the union of all primary MFs:

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x \tag{2}$$

The computation of the general type-2 fuzzy set is prohibitive because the general FS2 operations are complex. A special case of FS2 is the interval type-2 fuzzy set (ITFS2), where all the secondary grades equal to one so that the set operations can be simplified to interval calculations ([18]). The interval type-2 fuzzy sets are the most widely used type-2 fuzzy sets (because they are simple to use, and it is very difficult to justify the use of any other kind of type-2 fuzzy set to date). The ITFS2 is a special case of the general type-2 fuzzy set (FS2) A, or $\tilde{A}$, and can be expressed as following:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} (1/(x, u)) dx du \tag{3}$$

Mendel ([19]) has shown that the footprint of uncertainty (FOU) represents uncertainty in the primary memberships of an ITFS2. Figure 1 presents primary (gaussian) membership function of a ITFS2.
The upper membership function ($\mu_U(x)$) and the lower membership function ($\mu_L(x)$) of $\tilde{A}$ are two FS1 membership functions that bound the FOU (FOU is shaded in figure 1).

2.3 Type-2 Fuzzy Set Entropy

The process of selecting the necessary information to proceed segmentation must lead here to the correct estimate of the regions of the color image. The present work presents an application of the theory of fuzzy sets to evaluate these regions, with the best accuracy as possible. The terms *fuzziness index* ([26]) and *entropy* ([28]) provide the measurement of fuzziness in a set and are used to define the degree of uncertainty of the segmentation process (the total amount of uncertainty being difficult to calculate in this case). These data make it possible to define an index relevant for the process, being used as a criterion to find fuzzy region width and thresholds for segmentation automatically.

An ordinary fuzzy set A of a set X is classically defined by its membership function $\mu_A(x)$ written as:

$$\mu_A: X \to [0,1], \text{ with } x \in X$$

where the membership function denotes the degree in which an event x may be a member of A. A point x for which $\mu_A(x) = 0.5$ *is said to be a crossover point of the fuzzy set A.*. The uncertainty brought by the variable is represented by the " α-cut " of the fuzzy set A. Let X be a classical set and $A \subseteq X$ an ordinary fuzzy set characterized by its membership function $\mu_A(x)$. Considering a threshold $\alpha \in [0, 1]$, the membership function can be defined as μ_A^α (classical set A^α or α-cut of the fuzzy set A):

$$\mu_A^\alpha : X \to \{0,1\}$$
$$\mu_A^\alpha(x) = \begin{Bmatrix} 1 \ \forall x \geq \alpha \\ 0 \ \forall x < \alpha \end{Bmatrix} \forall x \in X$$

The **fuzziness index** γ of a fuzzy set A reflects the degree of ambiguity by measuring the distance between A and its nearest ordinary set $A^{0.5}$. It is defined as ([25]):

$$\gamma(A) = 2.d(A, A^{0.5})/n^{1/\mathrm{p}} \tag{4}$$

where $d(A, A^{0.5})$ denotes the distance between A and its nearest ordinary set $A^{0.5}$. A positive scalar p is introduced to make γ (A) lie between zero and one. Its value depends on the type of distance function used. For example, p=1 represents a generalized Hamming distance, whereas p= 2 represents an Euclidean distance. The term **entropy of an ordinary fuzzy set A** was first introduced by Deluca and Termini ([28]) as:

$$H(A) = (\Sigma S_\mathrm{n}(\mu_\mathrm{A}(x)))/n.ln2 \tag{5}$$

Where $S_n(\mu_A(x)) = -\mu_A(x)\ln(\mu_A(x)) - (1-\mu_A(x))\ln(1-\mu_A(x))$, (ln standing for natural logarithm). Yager ([25]) and Kaufmann ([26]) proposed other possible measures of the entropy, motivated by the classical Shannon entropy function, that we do not present here (Fan and Ma ([10]) proposed a complete analysis of fuzzy entropy formulas).

γ (A) and H(A) are such that :
$\gamma_{\min} = H_{\min} = 0$, for $\mu = 0$ or 1
$\gamma_{\max} = H_{\max} = 1$, for $\mu = 0.5$.

Therefore, γ and H are monotonic functions and increase in the interval [0, 0.5], and decrease in [0.5, 1] with a maximum of one at $\mu = 0.5$. So it is possible to use one or the other expression to define the degree of uncertainty. In this work, we use the extension of the "De Luca and Termini" measure ([28]) to discrete images, proposed by Pal ([27]) and which is well adapted to our problem. For an MxN image subset $A \subseteq X$ with L gray levels $g \in [0, L-1]$, the histogram h(g) and the membership function $\mu_X(g)$, the (linear) index of fuzzyness can now be defined as follows:

$$\gamma(A) = \frac{1}{MN}\sum_{g=0}^{L-1} h(g) * min[\mu_A(g), 1 - \mu_A(g)] \tag{6}$$

There have been numerous applications of fuzzy entropies in gray scale image segmentation ([13], [14], [30]). The entropy of FS2 has not yet been studied in the literature. However, for type-2 fuzzy sets, it is very easy to extend the previous concepts of FS1 for ITFS2, as proposed by ([30]), and to define the (linear) index of fuzziness as follows:

$$\gamma(\tilde{A}) = \frac{1}{MN}\sum_{g=0}^{L-1} h(g) * [\mu_U(g) - \mu_L(g)] \tag{7}$$

In this last formula, $\mu_U(g)$ and $\mu_L(g)$ are defined in the following paragraph. This basic definition verify the four conditions proposed by Kaufman ([26]) for the measure of uncertainty of a fuzzy set, and among the numerous frames of uncertainty modelization, this last equation seems to be an interesting way in image processing.

2.4 Signal Processing Applications of Type-2 Fuzzy Sets

Recently, some applications of type-2 fuzzy set have been presented in the literature. Gader and al. ([12]) presented land mines detection with very good results. Hagras ([7]) proposed a hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots, and [8] and [11] proposed applications for the design of fuzzy logic systems (used for the control of plants). Rhee and Wang studied uncertainty associated with the parameters used in fuzzy clustering algorithms and showed that interval type-2 fuzzy approach aids cluster prototype to converge to a more desirable location than a type-1 fuzzy approach ([16], [17]).

Tizhoosh ([30]) applied type-2 fuzzy sets to gray scale images thresholding. He obtained good results with very noisy images. As proposed in [18], he used interval type-2 fuzzy sets, with the following "FOU" (figure 1):

- Upper limit: $\mu_U(x)$: $\mu_U(x) = [\mu(x)]^{0.5}$
- Lower limit: $\mu_L(x)$: $\mu_L(x) = [\mu(x)]^2$

The study he made about these functions showed that they are well adapted in image processing. So we shall use the same functions in color images segmentation. We now present the application of type-2 fuzzy sets to color images segmentation.

3 Color Image Segmentation with Type-2 Fuzzy Sets

In this paper, we consider color uniformity as a relevant criterion to partition an image into significant regions. We propose a fuzzy entropy approach to take into account simultaneously the color and spatial properties of the pixels.

3.1 Proposed Scheme

The segmentation scheme is divided into two steps. In the first one, the color image is considered as a combination of three color component images. A color component image is a monochromatic image where each pixel is characterized by the level of one color component. Each of these component images is analyzed using type-2 fuzzy set (both the occurence of the gray levels and the neighboring homogeneity value among pixels is considered) and then fuzzy entropy. So local and global information is employed in the algorithm. In the second step, the entropy is utilized as a tool to perform histograms analysis for finding all major homogeneous regions at the first stage. The classes built by the analyses of the three color component images are combined to form the classes of pixels of the color image (merging stage).

3.2 Type-2 Fuzzy Set Entropy

The membership function of the type-2 fuzzy set is shifted over the gray-level range (corresponding to one color component, R, G or B) and the amount of fuzziness is calculated (using equation 7). So we are able to transform an image into fuzzy domains with maximum fuzzy entropy. The proposed color image segmentation method could be described as a system whose inputs are a color image and the entropy threshold value. The output of the system is the segmented image (the threshold value is applied for each color component independently).

3.3 Algorithm

The general algorithm for color image segmentation based on type-2 fuzzy sets and measure of fuzzyness γ can be formulated as follows:

- Select the shape of MF (here interval-based type-2 fuzzy set)
- Calculate the image histogram for each color component of the color space (R, G, B)
- Initialize the position of the membership function
- Shift the MF along the gray-level ranges (R, G, B) (as illustrated figure 2)
- Calculate in each position (g) the MF values $\mu_U(g)$ and $\mu_L(g)$
- Calculate in each position (g) the amount of uncertainty γ
- Find the maximum values of γ
- Threshold the image with γ_{max}
- Region merging process from the obtained classes of pixels

3.4 Color Region Merging

At the previous stage, a coarse segmentation of the image is obtained. Color region merging technique is needed in order to refine the segmentation results. In fact, regions with small numbers of pixels should be merged and homogeneous regions with narrow color transition might be split as separate regions having small color difference. These cases often appear in natural images characterized by imprecise regions such as shadows, brights and color gradients.

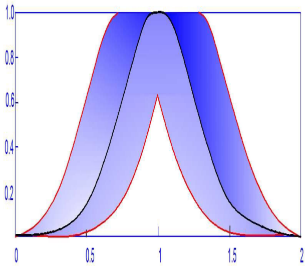

Fig. 1. FOU of a ITFS2

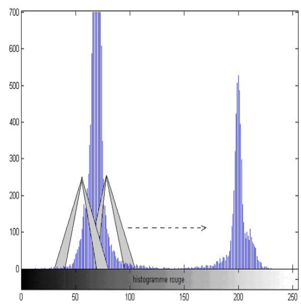

Fig. 2. Shifting of the membership function

The region merging criterion. Classical problem with region merging is how to define merging criteria. Incorporating specific knowledge of psychophysical perception is an ideal way, but it is not practical for application. In this paper, the definition of a region is based on similar colour (homogeneity), so we take into account color similarity to decide if two regions are to be merged. We adopt an approach similar to [4]. In the RGB color space, we use the distance between two clusters C_1 and C_2:

$$\text{dist}(C_1, C_2) = \max(|R_1 - R_2|, |G_1 - G_2|, |B_1 - B_2|)$$

where (R_1, G_1, B_1) and (R_2, G_2, B_2) are the average color values of clusters C_1 and C_2.

Region merging algorithm. The strategy we follow in this first work is the following:

– From the segmented image obtained with the application of FS2, we merge clusters whose number of pixels is less than a predefined threshold into its closest cluster (first stage of merging)
– Then region merging is performed iteratively by combining the two closest regions each time until the distances of all pairs of regions are greater than a specified global threshold.

4 Experimental Results

In order to test the performance of the proposed technique, a classical synthetic image (named "Savoyse", and composed of five areas on an uniform background, with an additive gaussian noise) is first tested. Other images are natural well-known scene images (named "House" and Lena), have been also tested. These images are presented respectively figures 3, 4 and 5, (the intensity value for each color component of the test images is from 0 to 255). The algorithm has been implemented on the well-known software "Matlab" on PC (it is important to note that the software has not been optimized). We can remark that the most time consuming parts of our Matlab implementation is the region merging procedure (due to the non-optimzed data structure used). Without the region merging procedure, running time is only some seconds. The characteristics of the images, the number of colors, the CPU time, etc... are listed table 1.

4.1 Type-2 Fuzzy Sets Entropy Approach

We have applied the unsupervised segmentation algorithm we propose on these images, presented respectively figure 3, 4 and 5: the first image (left) is the original image, the second image is the image obtained with the FS2 segmentation method, the third image is the result after the first merging stage and the fourth image (right) is the result obtained after the second merging stage.

It is interesting to analyse these results. First, we can easily remark that the number of colors is drastically low after the segmentation process using FS2: thresholding using FS2 entropy is very effective. Mendel has shown that the amount of uncertainty associated to a FS2 is characterized by its lower and upper membership functions. So we are intuitively able to explain these results (compared with a FS1 for example). On the synthetic image (Savoyse), the two regions corresponding to the two green concentric discs are correctly extracted. This result shows that the method is able to handle unequiprobable and overlapping classes of pixels. The segmentation of the other (natural) images is challenging because of the presence of shadows and highlights effects. Segmentation results for the house image show that low-contrast regions are merged, and the number of segmented regions dropped from 17 to 4 colors. For ending, consider the well-known benchmark "Lena" image. It is well known that segmentation techniques based solely on low-level cues (such as colors) are very difficult to apply, due to the distribution of colors. Nevertheless, our method provides good results: the hat of the girl and her face remain cleanly separated from the background.

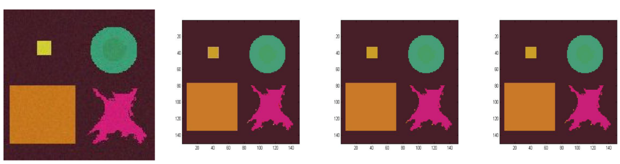

Fig. 3. Original and segmented images, Savoyse

So we are able to sum up some important results:

- The image is able to be transformed into fuzzy domains using type-2 fuzzy membership functions
- These fuzzy domains consider both the occurence of the gray levels (of each color component) and the neighboring homogeneity among pixels (spatial information)
- The analysis of the entropy function (of each color component) performs image segmentation (regions and contours)
- The segmentation process is unsupervised (we do not need to know the number of clusters of pixels), and apparently the results we obtained seem

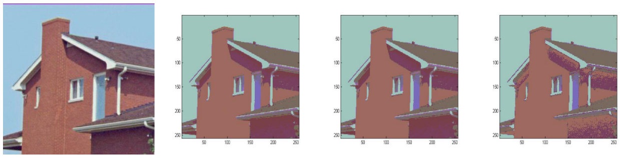

Fig. 4. Original and segmented images, House

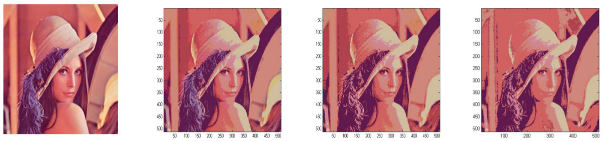

Fig. 5. Original and segmented images, Lena

robust to noise and to membership function shapes (we obtain the same results with different kinds of membership functions).

4.2 Type-2 Fuzzy Sets Entropy vs. Type-1 Fuzzy Sets Entropy

Then, we have compared type-1 fuzzy sets entropy approach (using the equation 6) to its counterpart with type-2 fuzzy sets. In particular, we are able to remark that the peak values of the entropy using type-2 fuzzy sets are more important that their counterparts using type-1 fuzzy sets (figure 6, middle), so that regions will be easier to extract in a noisy image, and this proves qualitatively the advantage of this approach (more uncertainty is taken into account using type-2 fuzzy sets, as it is suggested previously). This result is well illustrated on the results obtained with image "Lena". On the figure 6, we present the segmented image using FS2 on the left, and the segmented image using FS1 on the right (these results are obtained without region merging to make interpretation easier). It is clear that type-2 fuzzy sets aids to obtain better results. Type-2 fuzzy sets are able to model imprecision and uncertainty which type-1 fuzzy sets find difficult or impossible to handle. Local entropy in information theory represents the variance of local region and catches the natural properties of transition region. So FS2 being able to deal with a greater amount of uncertainty than FS1, transition regions are more acute and homogeneous regions are better drawn. It is possible to illustrate this assertion using the results of table 1. For "Lena" image, 50 colors are obtained with FS2 instead of 17 with FS1.

It will be interesting in the future to use a measure of performance, to compare these two approaches (and with non-fuzzy references) on different sets of images. Computational complexity and calculus time are small, and should be also compared with other algorithms with interest. Particularly, a complete study about the application of our method on noisy images is on work, to establish a link between the "FOU" of FS2 and the level of noise, and will be presented in a future paper.

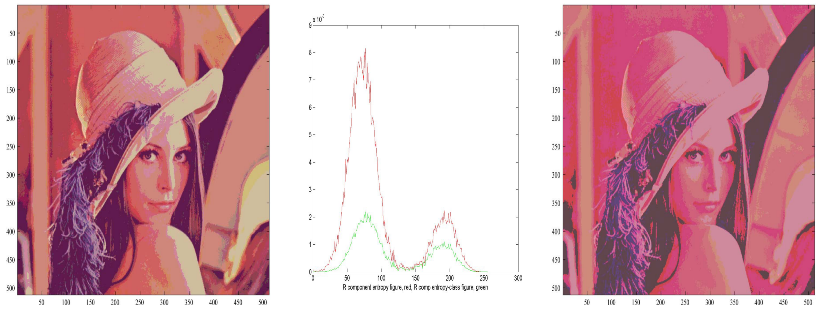

Fig. 6. Segmented (FS2, left and FS1, right) images, Lena, and fuzzy sets entropies, (middle)

Table 1. Results of the Proposed Approach in RGB Color Space

Image Name	Size	CPU Time	number of colors				
	pixels	sec	Initial	FS1	FS2	FS2 after 1st merging	FS2 after 2nd merging
SAVOYSE	150x150	0.5	5330	15	16	7	7
HOUSE	256x256	1 to 2	33925	12	17	5	4
LENA	512x512	9	67189	17	50	7	5

4.3 Color Spaces

The proposed approach operates in RGB color space, which is the most commonly used model in the literature. The major disadvantage of RGB for color scene segmentation is the high correlation among the R, G, and B components. The HSI system is another commonly used color space in image processing, which is more intuitive to the human vision. Anyway, the non-removable singularity of hue may create spurious modes in the distribution of values resulting from non-linear transformations, which makes the entropy calculus of hue value unreliable for segmentation. RGB color space does not have such a problem. But for color images with high saturation, segmentation using HSI can generate good results, and a comparison between RGB results and HSI results is under investigation.

5 Conclusion

Color image segmentation is a difficult task in image processing. A unique algorithm will certainly never be established to be applied to all kinds of images. We have tried to apply a new algorithm provided by fuzzy set theory. The central idea of this paper was to introduce the application of type-2 fuzzy sets, to take into account the total amount of uncertainty present at the segmentation stage, and this idea seems to be very promising. So a new segmentation algorithm has been presented and some examples have demonstrated the applicability of this algorithm. We have now to compare this algorithm with other ones (non-fuzzy and fuzzy algorithms) and to lead additional experiments with different test images to confirm the results we obtain (in a relevant benchmark) and to reinforce the potentiality of this new method. In particular, more extensive investigations on other measures of entropy and the effect of parameters influencing the width (length) of FOU are under investigation. We are also working about incorporating specific knowledge of psychophysical perception to obtain better results in the merging stage of our method. So this first study, with the good results we obtain, may lead to interesting studies in the future.

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