Structural Aspects of the Evaluation of Agent Organizations

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Abstract. A multi-agent system can be analyzed and specified as an organization consisting of roles and their relations. The performance of an organization depends on many factors among which the type of its organizational structure, i.e., the set of relations holding between its roles. This work focuses on the structure of organizations and addresses the issue of the analysis, evaluation, and comparison of organizational structures which can contribute to develop general methods for the assessment of multi-agent systems' performance. Specifically, quantitative concepts from graph theory are used to provide numerical analyses of organizational structures. It is argued that these analyzes can be used for evaluating to what extent an organizational structure exhibits some characteristic properties such as robustness, flexibility and efficiency.

1 Introduction

A great deal of ongoing research in the field of organization-based multi-agent systems (MAS) is devoted to comparing and evaluating different types of organizations and their performance. Work on these issues varies from surveys comparing organizational paradigms [6], to frameworks for representing and verifying organizational designs [7,19], to studies concerning properties and performance of specific types of organizations [13,17].

The present paper aims at contributing to the establishment of a number of techniques for evaluating MAS organizations and their performance. The notion of organization plays an important role in multi-agent systems, which is also reflected in many agent-oriented software methodologies (cf. GAIA, TROPOS). The performance of different organizations depends on organizations' characteristics such as robustness, flexibility, and efficiency. For example, hierarchies are known not to perform well in rapidly changing environments because of their poor flexibility. The paper is based on the intuition that a connection can be drawn between some of these characteristics and graph-theoretical properties of the structure of organizations. For example, flexibility depends on how strongly the roles in the organization are connected with one another. The notion of flexibility, though complex and multi-faceted, can definitely be correlated

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with structural aspects of the organization. Intuitively, the more are the connections between the roles in the organization, the more flexible is the organization. The point is to relate the notion of flexibility to precise properties of the organizational structure. Given an organization, can we say it is flexible? And how flexible? Is it more flexible than another one as far as structure is concerned? How can a designer foster flexibility in a MAS just working on its structure? These types of questions constitute, in a nutshell, the target of the present work.

We claim that an investigation of this connection is important for the development of appropriate methods for comparing and evaluating different types of organizations and their performances. In order to tackle the evaluation problem, "the space of organizational options must be mapped, and their relative benefits and costs understood" [6], and to provide such a "map" a rigorous analysis of organizational structure plays a crucial role. The perspective chosen consists thus in addressing the evaluation issue from a structural perspective, that is to say, analyzing the organizational structure of MAS and providing a way to rigorously describe the pros and cons of them which lie in their structures.

We will proceed as follows. Firstly (Section 3), building on the results presented in [3] (briefly recapitulated in Section 2) we investigate a number of simple equations which can provide ways of measuring to what extent a given organizational structure enjoys some specific graph-theoretical properties. For instance, to what degree is the structure connected? These measures already provide a way to evaluate, in an exact fashion, the adherence of organizational structures to structural constraints a designer might take into consideration. Secondly (Section 4), the proposed measures are linked to commonly used criteria for the classification and evaluation of organizations. The criteria on which we focus are robustness, flexibility and efficiency. We show then (Section 5) how these criteria can conflict with each other, and how to ground a structural analysis of these conflicts as well. Conclusions follow in Section 6.

2 Organizational Structure

2.1 Some Terminology

Before getting started it is worth recollecting some standard graph theoretical notions which will be used in the proceeding of the paper. An R_k-path (of length n) is a sequence $\langle x_1, ..., x_{n+1}\rangle$ of distinct elements of $Roles$ s.t. $\forall x_i\ 1 \leq i \leq n$, $(x_i, x_{i+1}) \in R_k$. A R_k-semipath (of length n) is a sequence $\langle x_1, ..., x_{n+1}\rangle$ of distinct elements of $Roles$ s.t. $\forall x_i\ 1 \leq i \leq n$, $(x_i, x_{i+1}) \in R_k$ or $(x_{i+1}, x_i) \in R_k$. A *source* in $Roles$ is an element s s.t. $\forall d \in Roles$ with $d \neq s$ there exists a R_k-path from s to d. The *indegree* $id_k(d)$ of a point d in structure k is the number of elements d_1 s.t. $(d_1, d) \in R_k$. The *outdegree* $od_k(d)$ of a point d in structure k is the number of elements d_1 s.t. $(d, d_1) \in R_k$. We say a point d to be incident w.r.t. a k link if $id_k(d) \leq 1$, and it is said to have emanating k links if $od_k(d) \leq 1$.

2.2 Representing Organizational Structures

In [3] a view on organizational structure has been proposed, inspired by foundational work on the theory of organizations [11,15], which is based on the claim that organizations do not exhibit only one structural dimension, but rather a multiplicity of interrelated dimensions, the dimensions of *power*, *coordination* and *control*. A natural way of modeling this notion of organizational structure is via directed graphs, which we represent here as systems of relations.

Definition 1. (Organizational structure)
An organizational structure OS is a tuple:

$$\langle Roles, R_{Pow}, R_{Coord}, R_{Contr} \rangle$$

where $Roles$ is the finite set of roles, and $R_{Pow}, R_{Coord}, R_{Contr}$ are three irreflexive binary relations on $Roles$ characterizing the Power, respectively, the Coordination and the Control structures.

For every R_k s.t. $k \in \{Pow, Coord, Contr\}$, we denote with $Roles_k$ the smallest subset of $Roles$ such that, if $(x, y) \in R_k$ then $x, y \in Roles_k$. In other words, sets $Roles_k$ denote the set of roles involved in the structural dimension k. Each digraph $\langle Roles_k, R_k \rangle$ in OS will be also referred to as the *structural dimension* k of OS.

Some observations are in order. First, it is worth noticing that in [3] the enactment relations between agents and roles are also included under the notion of organizational structure . In that work, it was necessary to include agents in the explicit representation of the structure in order to give an account of the effects that structural links bear on agents' performance. That study proposes also a formal analysis of the meaning of structural links in terms of the effects that they have on the activities of the agents playing roles in the organization. To briefly recapitulate it, the *power structure* defines the task delegation patterns possible within the organization. The *coordination structure* concerns the flow of knowledge within the organization, and the *control structure* has finally to do with the task recovery functions of the organization. In other words, the existence of a power link between role a and role b implies that every delegation of tasks from agent a (agent enacting role a) to agent b (agent enacting role b) ends up in the creation of an obligation directed to agent b. If a and b are connected via a coordination link, then every information act from a to b ends up in creating the corresponding knowledge in agent b. Finally, a control link between a and b implies that agent a has to monitor the activities of agent b, possibly taking over the tasks of agent b which have not been accomplished. In the present work however, such concern about the "semantics" of the structural links is left aside, and the main focus is settled only on the structural configurations linking the roles of the organization. This emphasizes also the generality of the method proposed here. In fact, the technical results that are going to be presented in Section 3 abstract from the meaning attached to the links, and can thus be applied to any kind of organizational structure representable in the fashion of Definition 1.

Second, we consider the roles on which the organizational structure ranges (i.e., the elements of set $Roles$) to be enacted by one and only one agent. The reason for this choice is illustrated by the following example. Suppose we need to model an organization for a soccer team implementing a 4-3-3 strategy. in such a way that the organizational structure inherent in the strategy is made explicit. Three roles can be defined in every team: 'attacker', 'defender' and 'midfielder', which are connected by appropriate power, control and coordination relations. An option would be to model the organization via imposing complex enactment constraints such as: "the role 'attacker' should be enacted by three agents such that the first agent should communicate with the third one, the second agent should monitor the first and third ones, etc.”. However, this would make implicit in the enactment constraints the power, coordination and control links that are present between all the various attackers in the 4-3-3 strategy. A better option would be to explicitly define three new roles, which can be seen as specializations of the ‘attacker’ role and which can be enacted by only one agent. The organizational links existing between these three new roles could thus be made explicit, and the resulting organizational structure satisfactorily modeled. This is the perspective we assume here. In practice, this boils down to a modeling issue: if two agents enacting a same role have to be connected by power coordination or control links, then two different roles have to be specified which substitute the first one and which are played by only one agent. This finer level of granularity is essential in order to suitably evaluate the adherence of the organizations to desired criteria, which constitute the primary target of the paper: for example, is the 4-3-3 organization flexible? An analysis at a level where roles do not specify the precise relative positions of all agents with respect to all the structural dimensions would fall short, missing many relevant structural links. It follows from this distinction that a study of the organizational structure ranging on role types would abstract from those power, coordination, and control links that might be present between role tokens specializing the same role type (for instance the three attackers in a 4-3-3 strategy). Here we are instead interested in the analysis of structure at the level of the actual agents’ positions within the organization, and thus at a finer level of granularity. The elements of the set $Roles$ in an OS are then to be considered role tokens. In the rest of the paper, if not stated otherwise, we use the word role intending role-token.

Finally, besides the analysis of the power, coordination and control dimensions, [3] proposes a number of ‘soundness’ properties of organizational structures, which concern the interplay between the different structural dimensions.

Definition 2. (Sound OS)
A sound organizational structure is a tuple: $\langle Roles, R_{Pow}, R_{Coord}, R_{Contr}\rangle$ where $Roles$ is the finite set of roles, and $R_{Pow}, R_{Coord}, R_{Contr}$ are three irreflexive binary relations on $Roles$ such that $\forall r, s \in Roles$:

$$(r, s) \in R_{Pow} \Rightarrow \text{there exists a } R_{Coord}\text{–path from } r \text{ to } s;$$
$$(r, s) \in R_{Pow} \Rightarrow \text{there exists a } t \in Roles \text{ s.t. } R_{Contr}(t, s).$$

The occurrence of a power relation between role r and role s requires: the existence of a (finite) coordination path from r to s so that effective informative actions can transmit the relevant knowledge of agents enacting role r to agents enacting role s; and the existence of at least one element t (which, notice, might be r itself) which is in a control relation with s.

3 Measuring Structure

This section presents some equations measuring specific graph-theoretical aspects of organizational structures[1].

[1] Equations 2, 3 and 4 below are an adaptation of equations presented in [8].

3.1 Completeness, Connectedness, Economy

Completeness and connectedness of an OS have to do with how strongly roles are linked with one another within one of the structural dimensions k. How much does the given structure approximate the structure where all directed links are present (*completeness*)? And how much is the given structure split in fragments (*connectedness*)?

$$\text{Completeness}_k(OS) = \frac{|R_k|}{|Roles_k| * (|Roles_k| - 1)} \tag{1}$$

$$\text{Connectedness}_k(OS) = 1 - \frac{|\text{DISCON}_k|}{|Roles_k| * (|Roles_k| - 1)} \tag{2}$$

with $|R_k| > 0$ and DISCON_k is the set of ordered pairs (x, y) of $Roles_k$ s.t. there is neither a R_k-semipath from x to y nor from y to x, i.e., the set of disconnected ordered pairs of the structural dimension $\langle Roles_k, R_k \rangle$. The condition $|R_k| > 0$ states that the structural dimension k does indeed exist. If the structure does not exist it cannot be measured. As a consequence, $\text{Completeness}_k > 0$. Stating that $\text{Completeness}_k(OS) = 0$ means thus that $R_k = \emptyset$ and hence that no structure at all is given. In practice, formula 1 measures the fraction of the actual links of the dimension $\langle Roles_k, R_k \rangle$ on all the available ones and formula 2 measures how 'not disconnected' that dimension is. With respect to connectedness, an important notion is that of cutpoint or, in an organizational reading, *liason role* [4], i.e., a role whose removal decreases the connectedness of the structure.

The *economy* of a given OS expresses a kind of balance between the two concerns of keeping the structure connected and of minimizing the number of links, i.e., minimizing completeness:

$$\text{Economy}_k(OS) = \\ 1 - \frac{|R_k| - (|Roles_k| - 1)}{|Roles_k| * (|Roles_k| - 1) - (|Roles_k| - 1)} \tag{3}$$

with $|R_k| > 0$. The equation is based on the intuition according to which the most 'economical' digraph of n points consists of $n - 1$ links, i.e., the minimum number of links which is still sufficient to keep the digraph connected. Indeed, the nominator of the fraction, consists of the number of links in the structural dimension k which are in excess or in defect w.r.t. the optimum of $n - 1$ links. The denominator denotes instead the absolute number of links in excess in k. If $|R_k| = n - 1$ then the value of $\text{Economy}_k(OS)$ is optimal, i.e., equal to 1. The equation measures, therefore, how much k is 'not expensive' in terms of links. Notice that $\text{Economy}_k(OS) = 1$ does not imply $\text{Connectedness}_k(OS) = 1$, it does only imply that there are enough links in R_k for it to be possibly connected. If the existence of symmetric links in R_k is assumed, then $n - 1$ links are clearly not enough any more for guaranteeing connectedness. On the other hand, notice also that $\text{Economy}_k(OS)$ can assume a value greater than 1. That indicates a sort of '*over-efficiency*' of k. In this case, it is easy to see that, if $\text{Economy}_k(OS) > 1$ then $\text{Connectedness}_k(OS) < 1$. In other words, if the economy measures of OS is lower than the optimal value 1, then OS has more links than the ones necessary for OS to be connected. If economy is instead higher than the optimal value 1, than there are in OS too few links for it to be connected.

3.2 Unilaterality, Univocity, Flatness

The properties of unilaterality and univocity express the tendency of an OS to display, respectively, an orientation in its links (*unilaterality*), and the absence of redundant links ending up in the same role (*univocity*). Do the links of an OS always have a ‘direction’ or does the OS allow, so to say, ‘peer-to-peer’ connections? And how many of those connections are such that no role has more than one incident link of the same structural dimension?

$$\text{Unilaterality}_k(OS) = 1 - \frac{|\text{SIM}_k|}{|R_k|} \tag{4}$$

$$\text{Univocity}_k(OS) = \frac{|\text{IN}_k|}{|Roles_k|} \tag{5}$$

$$\text{Flatness}_k(OS) = 1 - \frac{|\text{CUT}_k|}{|Roles_k|} \tag{6}$$

with $|R_k| > 0$ and SIM_k denotes the set of links (x, y) in R_k s.t. (y, x) is also in R_k, i.e., $|\text{SIM}_k|$ is twice the number of symmetric links in k; IN_k denotes the set of roles x in $Roles_k$ s.t. $id_k(x) = 1$ or $id_k(x) = 0$, i.e., the set of roles which either have indegree equal to 1 in k or they are a source of k or of some subgraphs of k; and CUT_k denotes the set of roles x s.t. $od_k(x) \leq 1$ and $id_k(x) \leq 1$, that is to say, the set of roles which are at the same time addresser and addressee of k links. Intuitively, equation 4 measures how much asymmetry is present in k, while equation 5 measures how much a dimension k is univocal or “non ambiguous”. The most univocal structures are assumed to be either the ones in which every point, except the source, has one and only one incident link (like in trees), or the ones in which exactly all points have only one incident link (like in cycles). Finally, equation 6 measures the relative amount of points in dimension k which are not intermediate point in a k-path, in other words the amount of points the removal of which would not determine a cut in any k-path. Obviously, the lowest value of flatness is provided by cycles.

Intuitively, unilaterality has to do with the level of subordination present in a structure. Consider the R_{Coord} dimension. The higher the number of unilaterality, the lower the amount of ‘peer-to-peer’ information exchange within OS. Univocity has to do with the level of conflict and redundancies of a given structure. Consider the R_{Pow} dimension. The higher the level of univocity, the more unambiguous is the chain of commands, as well as the more fragile once a link happens to be removed. See also [2] for similar investigations on this issue. Flatness instead, has to do with the length of paths available within a given structure. We will see in Section 4 that long paths of the control dimension can be useful in order to implement levels of control on the controller roles themselves.

3.3 Detour, Overlap, Cover and Chain

The properties we address in this section do not concern structural dimensions taken in isolation, like the one just investigated, but instead how the different dimensions of an OS interact with one another. This constitutes a crucial undertaking, though hardly investigated [4].

The properties we call *detour* and *overlap* regard the degree to which a structural dimension j 'follows' a structural dimension k, meaning by this the degree to which j establishes corresponding paths for each link of k, so that the roles that are related by R_k links are the same as those that are related by R_j-paths.

$$\text{Detour}_{jk}(OS) = \frac{|\text{PATH}_{jk}|}{|R_k|} \tag{7}$$

with $|R_k| > 0$ and the set PATH_{jk} is defined as the set of ordered pairs (x, y) s.t. $(x, y) \in R_k$ and there exists a R_j-path from x to y. Equation 7 measures the relative amount of R_j-paths between the elements of $Roles_k$ which have the same direction of the links in R_k. A special case of detour is the overlap. In fact, to measure how much does a dimension j overlap with a dimension k, it suffices to define a set LINK_{jk} corresponding to a PATH_{jk} where the R_j-paths are of length 1, i.e., simple links, and hence: $\text{LINK}_{jk} \equiv R_k \cap R_j$. A set LINK_{jk} consists then of all the pairs (x, y) which are in R_k and in R_j, that is to say, of all x, y which are linked in R_k and in R_j.

$$\text{Overlap}_{jk}(OS) = \frac{|\text{LINK}_{jk}|}{|R_k|} \tag{8}$$

with $|R_k| > 0$. Intuitively, the more j-pairs correspond to k-pairs, the more j overlaps k in OS.

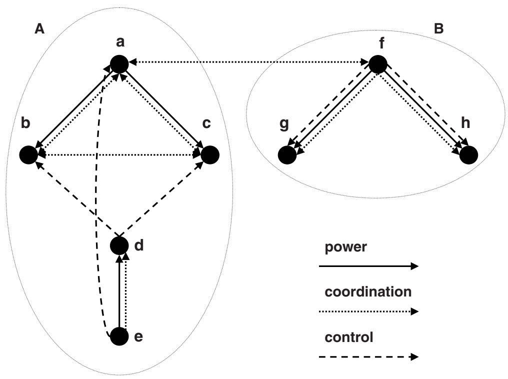
The property we call *in-cover* concerns the extent to which all the incident roles of k are also incident roles of a dimension j. In other words, we say that a dimension j *in-covers* a dimension k if all the roles which are addressees of a k link, are also addressees of a j link.

$$\text{InCover}_{jk}(OS) = \frac{|\text{IN}_j^+ \cap \text{IN}_k^+|}{|\text{IN}_k^+|} \tag{9}$$

with $|R_k| > 0$ and the set IN_i^+ is defined as the set of all elements x in $Roles_i$ such that $id_i(x) \leq 1$. The equation describes then how many of the incident roles of k are also incident roles in j.

The usefulness of these measures for capturing aspects of the structural interplay can already be shown in relation with Definition 2. Readers might have noticed that, via the equations just exposed, it is possible to provide a quantification of the degree to which a given OS adheres to the soundness principle concerning the interplay of the three dimensions of power, coordination and control. In fact, if we have $\text{Detour}_{Coord-Power}(OS) = 1$ and $\text{InCover}_{Contr-Pow}(OS) = 1$ then, following Definition 2, OS is sound. Lower degrees of these measures would thus determine lower adherence to the soundness principle. Notice also that maximum soundness is trivially obtained via an overlap of both coordination and control structures on the power structure: that is to say, if $\text{Overlap}_{Coord-Power}(OS) = 1$ and $\text{Overlap}_{Contr-Power}(OS) = 1$, then OS is (maximally) sound.

Equation 9 can be easily modified in order to capture analogous properties which we call *out-cover* and *chain*. The first one concerns the extent to which all the roles with emanating links in a dimension k are also roles with emanating links in a dimension j.



	Pow	Coord	Contr
Compl.$_{\cdot k}(OS)$	$\frac{5}{56}$	$\frac{11}{56}$	$\frac{5}{56}$
Conn.$_k(OS)$	$\frac{1}{4}$	$\frac{31}{56}$	$\frac{26}{56}$
Econ.$_k(OS)$	$\frac{51}{49}$	$\frac{45}{49}$	$\frac{51}{49}$
Unil.$_k(OS)$	1	$\frac{3}{11}$	1
Univ.$_kOS)$	1	$\frac{5}{8}$	1
Flat.$_kOS)$	1	$\frac{1}{2}$	1

	Coord-Pow	Contr-Pow	Pow-Contr	Coord-Contr	Contr-Coord	Pow-Coord
Detour$_{jk}(OS)$	1	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{9}$	$\frac{4}{9}$
Overlap$_{jk}(OS)$	1	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{9}$	$\frac{5}{9}$
InCover$_{jk}(OS)$	1	$\frac{4}{5}$	$\frac{4}{5}$	1	$\frac{5}{6}$	$\frac{5}{6}$
OutCover$_{jk}(OS)$	1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{3}{5}$
Chain$_{jk}(OS)$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{2}{5}$

Fig. 1. Example of structural measures

The second one concerns the extent to which a dimension j is 'incident' to the emanating links in a dimension k, in the sense that the roles with incident links in j contain the roles with emanating links in k.

$$\text{OutCover}_{jk}(OS) = \frac{|\text{OUT}_j^+ \cap \text{OUT}_k^+|}{|\text{OUT}_k^+|}, \tag{10}$$

$$\text{Chain}_{jk}(OS) = \frac{|\text{IN}_j^+ \cap \text{OUT}_k^+|}{|\text{OUT}_k^+|}, \tag{11}$$

with $|R_k| > 0$, IN_i^+ is as defined above and OUT_i^+ is the set of all elements x in $Roles_i$ such that $od_i(x) \leq 1$. Notice that the chain measure can be viewed as an inter-structural version of the flatness measure.

Before ending the section, it is worth noticing that all structural measures defined above range between 0 and 1 except economy which can get values higher than 1. Despite this, we saw that the optimal value of $\text{Economy}_k(OS)$ is still 1 (higher values determine over-efficiency). Whether a given OS enjoys a property at its optimal level, can therefore be handled as a matter of approximation of the corresponding measure to 1: the more $\text{Economy}_k(OS)$ approximates value 1 the more OS enjoys economy, etc.

3.4 An Example

In order to illustrate the above measures, an example is here provided and discussed. Consider the OS depicted in Figure 1. It is specified as follows:

$Roles = \{a, b, c, d, e, f, g, h\}$,
$R_{Pow} = \{(a, b), (a, c), (e, d), (f, g), (f, h)\}$,
$R_{Coord} = \{(a, b), (a, c), (b, a), (c, a), (b, c), (c, b), (e, d), (f, g), (f, h)\}$,
$R_{Contr} = \{(d, b), (e, a), (d, c), (f, g), (f, h)\}$.
We then have that: $Roles_{Pow} = Roles_{Coord} = Roles_{Contr} = \{a, b, c, d, e, f, g, h\}$.

Such an OS specifies an organization where two substructures A and B are connected via a symmetric coordination link. It is what we may call, following [6], a form of *federation*.

Substructure B is a typical form of highly centralized hierarchy: all connections move from the source f to the subordinated roles g and h. Indeed, it exhibits the optimal level of *efficiency*, *unilaterality*, *univocity* and *flatness* (equal to 1) for all three structural dimensions. Completeness and connectedness are also the same for all three dimensions, respectively equal to $\frac{2}{6}$ and to 1. Besides, there is a full reciprocal *overlap* (equal to 1) of all the three dimensions which, as showed above in Section 3.3, implies the soundness of the structure.

Substructure A, instead, displays a slightly more complex pattern. It hides two disconnected power hierarchies composed by roles a, b and c and, respectively, roles d and e. In fact, we have that $\text{Completeness}_{Pow}(A) = \frac{3}{20}$ and $\text{Connectedness}_{Pow}(A) = \frac{7}{10}$. Besides, the coordination structure is much more complete than the power one ($\text{Completeness}_{Coord}(A) = \frac{7}{20}$). This is due to the full connection holding between roles a, b and c. As to the interplay of the different dimensions in A, it is easily seen that OS is not maximally sound since $\text{InCover}_{Contr-Pow}(A) = \frac{2}{3}$. This is due to the fact that role d is not object of control although it is subordinated, in the power structure, to role e. In case e would delegate to d a task, a failure in accomplishing this task would not be recovered. This would definitely constitute a weak spot in an organization designed according to this structure. Interestingly, there is minimum overlap between R_{Contr} and R_{Pow}: $\text{Overlap}_{Contr-Pow}(A) = 0$. This embodies a sort of complete "*separation of concerns*" between the power and the control dimensions, in the sense that controller roles are never in a power position with respect to the controlled roles. This is obviously a sensible design requirement for preventing connivances between controllers and roles in power positions. On the other hand, $\text{OutCover}_{Pow-Contr}(A) = \frac{1}{2}$ and $\text{OutCover}_{Coord-Contr}(A) = \frac{1}{2}$ show that, although no role is at the same time in a power and in a control position w.r.t. the same roles, there are controllers in A (one out of two) which have the possibility to delegate tasks and communicate with other roles (role e). Worth noticing is also the following: $\text{Chain}_{Contr-Pow}(A) = \frac{1}{2}$, that is, one out of two roles in a power position are subjected to control. Interestingly, the only uncontrolled role in a power position is the controller role e itself, and in fact no control of the controller is implemented: $\text{Flatness}_{Contr}(A) = 1$.

After discussing the two substructures in isolation we focus now on the federation OS emerging by the joining of the two substructures via a symmetric coordination link between roles a and f. The resulting structural measures of OS are the one listed in figure 1. Let us comment upon them. First of all, none of the three dimensions is connected (with coordination being the most connected among the three). This means that within each dimension, there exist unrelated clusters of roles. In particular, the roles in a controlling position within substructure A cannot communicate with the rest of the federation. It follows that all dimensions happen to display high values of economy and even over-efficiency, like in the case of power and control. As to the degree of unilaterality and univocity, power and control enjoy a degree equal to 1, and they thus display typically hierarchical features. On the other hand, coordination is highly reciprocal except, as we saw, within substructure B and it maintains a high degree of univocity keeping therefore a low level of redundancies in coordination as well. As to the interplay between the different dimensions, OS inherits the flaw of substructure A which prevents it from enjoying the maximum degree of measure InCover$_{Contr-Pow}$, jeopardizing soundness. Coordination, instead, fully overlaps power guaranteeing the necessary flow of communication after the delegation activity. In the tables in Figure 1 more measures concerning OS are provided which, for reasons of space limitation, cannot be commented upon here.

4 Criteria and Structure

As the example showed, the structural measures captured in equations (1)-(11) would be already enough for a quantified comparison of organizational structures. What is still lacking, is to give those measures an 'organizational meaning', so to say, in terms of the criteria of robustness, flexibility and efficiency.

In this section we ground such a connection. The structural measures captured in equations (1)-(5) and (7)-(11) are used to provide hints about the adherence of a given organizational structure to criteria commonly utilized for the classification of organizations. Questions we aim at shedding light on are of the type: Is the coordination structure flexible (enough)? Is the power structure efficient (enough)? Is the interplay between power and control structure robust (enough)? etc.

Notice that we do not claim that those notions can be understood only on the basis of structural considerations. We rather address what, just by looking at the structure of an organization, can be said about its robustness, flexibility and efficiency. As a matter of fact, considerations about structure have always been relevant both in organizational sciences and multi-agent systems for explaining why, for instance, a network is more flexible than a hierarchy. Here, we try to ground this kind of considerations on a more solid and fully-fledged base.

Before doing this, it is important to stress that the structural analysis of the general criteria of robustness, flexibility, and efficiency presupposes the semantics of structural links exposed in [3] and summarized in Section 2.

4.1 Robustness

"Robustness is simply a measure of how stable the yield is in the face of anticipated risks. That is, the maintenance of some desired system characteristics despite fluctuations in the behavior of its component parts or its environment [...]. Adding robustness thus adds complexity" [18].

Robustness asks for redundancies in the structural dimensions used for dividing tasks within an organizations, i.e., the power and the coordination structures. Redundancy for a power structure means low values of the Univocity$_{Pow}$ measure, and for a coordination structure also a low degree of the Unilaterality$_{Coord}$ in order to allow for symmetric coordination links. In particular, symmetric coordination links can substitute broken power links allowing for bilateral negotiations of tasks to replace direct delegation. Therefore, a high Overlap$_{Coord-Pow}$ would be a sign of robustness.

For the same reasons the control structure plays an important role for the robustness of an organization allowing for failure detection and reaction. It can be required that each role in the power and coordination structures is controlled, suggesting a high degree of the following measures: Chain$_{Contr-Pow}$, i.e., the control of agents in power positions; Chain$_{Contr-Coord}$, i.e., the control of roles from which coordination links depart; InCover$_{Contr-Coord}$, i.e., the control of roles to which coordination links are directed. Furthermore, every role in the control structure can be required to have a high in-degree (every role is monitored by many other roles), which corresponds to a low level of Univocity$_{Contr}$. The number of control levels can also be increased, so that as many controllers as possible are, in turn, controlled. This has to do with the well-known "control of the controllers" issue which we already touched upon in [3] and corresponds to a low degree of Flatness$_{Contr}$ (long control paths are enabled).

On the other hand, a good control structure is of no use if the controlling roles have no capabilities or no power or coordination connections to follow up on perceived failures. This can be fostered via high values of, respectively, OutCover$_{Pow-Contr}$ and OutCover$_{Pow-Coord}$. In addition, the coordination structure determines how well information can disseminate over the organization. For robustness it is important that information about failures can spread to the roles that can take appropriate action. Also this structure can serve as a back up for a failure of the power structure. So, one can easily claim that the more complete and more connected (Completeness$_{Coord}$ and Connectedness$_{Coord}$) the coordination structure is the more robust the organization is.

To sum up, the level of robustness of an organization, from the point of view of its organizational structure, can be evaluated considering the following structural measures:

Completeness$_{Coord}$	1	Overlap$_{Coord-Pow}$	1
Connectedness$_{Coord}$	1	Chain$_{Contr-Pow}$	1
Univocity$_{Pow}$	0	Chain$_{Contr-Coord}$	1
Unilaterality$_{Coord}$	0	InCover$_{Contr-Coord}$	1
Univocity$_{Contr}$	0	OutCover$_{Pow-Contr}$	1
Flatness$_{Contr}$	0	OutCover$_{Pow-Coord}$	1

The 1 and 0 symbol indicate the value which is considered to maximize robustness with respect to that measure. For instance, the maximum enhancement of robustness obtainable via modification of the connectedness measure is yielded by value 1. In other words, the more Connectedness$_{Coord}$ approximates 1, the more the structure is robust. As to univocity the optimal value for increasing robustness is instead 0.

Getting back to the organizational structure OS discussed in the example above (Section 3.4), we see that the robustness criterion is not its forte. Nevertheless it does score well in the robustness-related measures concerning the interaction between the structures:

$$\text{OutCover}_{Pow-Contr}(OS) = \frac{2}{3}, \text{OutCover}_{Pow-Coord}(OS) = \frac{3}{5},$$
$$\text{Chain}_{Contr-Coord}(OS) = \frac{3}{5} \text{ and } \text{InCover}_{Contr-Coord}(OS) = \frac{5}{6}.$$

4.2 Flexibility

"Flexible organizations are a looser co-operative association than classic hierarchical organizations. [...] Flexible organizations are continually in flux and are able to adapt in a flexible way to changing circumstances" [14].

To make it more concrete, we look at the flexibility of an organizational structure as its ability to cope with changing tasks. It is clear that the capabilities required for the enactment of each role constitute a crucial issue. If all roles require the capability to perform any task at any moment, then all roles would be designed to cope with any different type of task. The actual structure does not really matter in this case, because no matter how a task is distributed over roles and how it is controlled it would be anyway performed. Given that the organization is sound, the information about the task is appropriately distributed, control is properly configured and the organization is thus as flexible as it can be.

Assuming a diversified distribution of capabilities among roles, flexibility of an organization amounts to decomposing a task in subtasks such that for every subtask a role can be found which is held to be capable to perform that subtask. This can be done via delegation through the power structure. However, an articulated power structure hinders flexibility constraining the distribution of tasks to predisposed patterns. This suggests that, for enhancing flexibility at a structural level, low degrees of both $\text{Completeness}_{Pow}$ and $\text{Connectedness}_{Pow}$ are required. Besides, it is worth noticing that a given power structure assumes that the role having the power to delegate a task is at least capable to perform those operations on the task that are needed before it can be delegated. This can be some preprocessing of a task or a decomposition of a task or even just a determination to which role the task should be delegated. Whenever a role does not have the capability to perform this operation for a new task, the processing of the task halts. Even if the subordinate roles could perform the task they would not get it and thus the task would not be performed.

The control structure might alleviate this effect in that it can function as a link between different parts of the power structure. Whenever an agent enacting a role in the power structure fails on (the distribution of) a task, its controller should react and have the power to redistribute the task, structurally: high values of $\text{Chain}_{Contr-Pow}$ and $\text{OutCover}_{Pow-Contr}$.

Network organizations and teams, instead, where no power structure exists, are commonly indicated as the paradigmatic example of flexible organizations [12]. In this type of organizations the specification of the capabilities required for each role cannot be complete since the nature of the tasks the organization has to fulfill is not exhaustively known in advance. What becomes essential is therefore a coordination structure through which the knowledge, concerning which agent might be capable to handle the new task, flows within the whole organization. The more roles are connected through this structure the more likely the right agent can be found to perform a new task. Completeness and connectivity ($\text{Completeness}_{Coord}$ and $\text{Connectedness}_{Coord}$) are thus directly linked also to the enhancement of the flexibility of an organization.

To recapitulate, these are the relevant measures for flexibility:

$\text{Completeness}_{Pow}$	0	$\text{Completeness}_{Coord}$	1
$\text{Connectedness}_{Pow}$	0	$\text{Connectedness}_{Coord}$	1
$\text{Chain}_{Contr-Pow}$	1	$\text{OutCover}_{Pow-Contr}$	1

Again, 1 and 0 indicate the measures' values which are considered to maximize flexibility.

With respect to the flexibility of the structure OS in the example, we see that it has indeed a small power structure (connectedness and completeness are very low) and a reasonably connected coordination structure (= $\frac{31}{56}$). These two aspects both enhance flexibility. This is indeed what we would expect, being OS a form of "federation", that is, a form of organization which retains some purely hierarchical aspects (in its substructures) but exhibiting better flexibility. It scores well also w.r.t. the OutCover measure between power and control: $\text{OutCover}_{Pow-Contr} = \frac{2}{3}$.

4.3 Efficiency

According to [1], efficiency mostly refers to the amount of resources used by the organization to perform its tasks. Organizational structure plays a role in this sense, since "links are not without cost in a social systems" [8].

There is a general assumption that high specialization of roles leads to more efficient performance; it is the old principle of the *division of labour*[2]. Within organizational theory as well as within AOSE (Agent Oriented Software Engineering) it is however known that there is a balance between specializing (and thus creating more roles) and the overhead this generates in the coordination of the tasks. Having less roles in the organizational structure leads to higher efficiency. But having too few roles leads to lower efficiency due to less appropriate performances of the tasks by the roles.

The existence of a power structure guarantees efficient distribution of tasks, and a tree is the most efficient structure to cover all roles. Such a structure is obtained imposing value 1 for all the following measures: $\text{Connectedness}_{Pow}$ (a disconnected power structure generates fragments with independent power), Economy_{Pow} (maximum economy without over-efficiency), $\text{Unilaterality}_{Pow}$ (no peer-to-peer connections) and Univocity_{Pow} (no conflicts in the chain of command). If every role is specialized to an extreme that all the capabilities required by the roles are disjunct, then every task can be distributed in only one way within the organization. Given that there is only one way of distributing the task, one can use a power structure reaching all roles to efficiently effectuate this.

[2] "The greatest improvement in the productive powers of labour [. . .] seem to have been the effects of the division of labour" ([16] p. 9).

As to coordination and control, economy (Economy) should also be required to be 1 in order to minimize the amount of links. Besides, the most efficient way in order to guarantee soundness (Definition 2) consists in mirroring the power dimension, therefore obtaining high levels for all measures of overlap, that is: Overlap w.r.t. the related dimensions of $Coord - Pow$, $Contr - Pow$, as well as $Pow - Coord$ and $Pow - Contr$ (overlap needs to hold in both directions in order to enforce coincidence). This keeps the number of links minimal and avoids the creation of further roles with mere coordination and control tasks. It follows that a fully hierarchical organization (such as substructure B described in the example of Section 3.4) where all structures follow the same pattern forms the most efficient organization possible.

These are the thus the measures we consider to be related to efficiency:

Connectedness$_{Pow}$	1	Unilaterality$_{Pow}$	1
Economy$_{Pow}$	1	Univocity$_{Pow}$	1
Economy$_{Coord}$	1	Economy$_{Contr}$	1
Overlap$_{Coord-Pow}$	1	Overlap$_{Contr-Pow}$	1
Overlap$_{Pow-Coord}$	1	Overlap$_{Pow-Contr}$	1

Again, 1 and 0 indicate the value which is considered to maximize efficiency with respect to the measure at issue.

The structure OS of the example incorporates a very efficient power structure: unilaterality and univocity are optimal (equal to 1) as well as the overlap between coordination and power. On the other hand, the power structure covers only a small fraction of the whole organization (Connectedness$_{Pow}(OS) = \frac{1}{4}$). As a consequence, distribution of tasks via delegation can only partially take place.

5 Tuning Structural Measures to Organizational Properties

At this stage the obvious question is whether organizations can be designed which maximize the adherence to all three properties at the same time. From a structural point of view and as intuition suggests, it is easy to show that this is not possible. Consider, for instance, the coordination structure. In fact, efficiency increases when Economy$_{Coord}$ approximates 1. Maximum robustness and flexibility both require Economy$_{Coord}$ equal to 0, while maximum efficiency requires Economy$_{Coord}$ equal to 1:

	Robust	Flexible	Efficient
Economy$_{Coord}$	0	0	1

Intuitively, both robustness and flexibility increase the number of structural links and thus the costs of the organizational overhead, while efficiency reduces these overhead costs. Similar problems exist, for instance, for the power structure. The robustness criterion requires as many redundancies as possible, and therefore low levels of univocity, while flexibility demands the structure to be as small as possible and therefore with very low degrees of completeness. A number of similar incompatibilities can be detected and mathematically investigated.

Since it is not possible to maximize the adherence to all properties at the same time, the point consists then in finding suitable compromise solutions.

A good option might be, for instance, to maximize all structural features at the same time getting a structure which exhibits Pareto efficiency w.r.t. the allocation of values to equations (1)-(11): an assignment of value to every equation should be found such that no other assignment exists which attributes a better value to one of the equations. This would be a typical compromise solution. Although for many applications this can be a good way to go, such a Pareto efficient structure would adhere to a reasonable extent to each criterion, but it would not exhibit optimal values in any of the investigated measures. A circumstance in which this solution would be sensible is when the environment is expected to change often while the organizational structure is not able to adapt. In that case a middle of the road solution can provide reasonable performances over time.

However, when the environment does not change that frequently (i.e. it is known in which kind of environment the organization should function) the issue amounts to what in organization theory is called "*synthesis problems*", that is, the questions concerning "which structures are best suited to solve optimally certain types of problems" [5]. Should for instance flexibility be privileged over efficiency? In other words, choices should be made between the concurrent criteria. An extensive analysis of the interdependencies between equations (1)-(11) could provide useful insights on this type of issues.

6 Conclusions and Future Work

The work addressed the issue of the influence of organizational structures on the performance of organizations, aiming at providing a rigorous method for analyzing, comparing and evaluating different types of structures. We proceeded as follows. First, making use of graph theory, we provided a number of meaningful measures for quantifying the adherence of organizational structures to specific structural features. Second, these measures have been used to ground a numerical analysis of the key organizational properties of robustness, efficiency and flexibility. Third, it has been shown that such an analysis pose the ground for an exact investigation of the extent to which those properties can conflict with each other, providing interesting information for a more aware design of organizational structures.

In future work we plan to extended this method in order to incorporate more structural measures and to account for more organizational criteria like, for example, the scalability of an organizational structure. Another issue worth a detailed investigation concerns the way the equations proposed in this work are related to each other also constitutes an issue worth pursuing in future researches. Finally, the framework and its results should be compared in details with approaches developed in the field of management sciences, such as [9,10], which bear many similarities both in purposes and technical solutions.

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