Estimation of Source Signals Number and Underdetermined Blind Separation Based on Sparse Representation

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Abstract. In this paper, we propose a new two-step algorithm (PDTA) to solve the problem of underdetermined blind separation, where the number of sensors is less than that of source signals. Unlike the usual two-step algorithm, our algorithm's first step is to estimate the number of source signals and the mixture matrix instead of K-mean clustering algorithm, in which people often suppose that the number of source signals is known when they estimate the mixture matrix. After the mixture matrix is estimated by PDTA, the short path algorithm is used to recover source signals. The last simulations show the good performance of estimation the number of source signals and recovering source signals.

1 Introduction

The blind source separation (BSS) problem is currently receiving increased interests [1],[2],[3],[4],[5] in numerous engineering applications. Blind separation comes from cocktail problem [6], and it consists in restoring n unknown, statistically independent random sources from m available observations that are linear combinations of these sources, but we know little about mixture channel and source signals' distribution. In recent years, blind sources separation has been a hot topic in signal processing field and neural networks field, furthermore, it has been applied to many fields from its appearance to now, such as, wireless communication, radar, image processing, array processing and biomedicine, and so on.

Specially, the authors of paper [1] discussed separability of blind source separation in the linear mixture case. By using the information of the mixing matrix, the authors obtained the results about when the source signals can be extracted or not and how many source signals can be extracted. This paper can enrich the separability theory of blind source separation.

At the same time, in the paper [7], Xie's conjecture corrected the famous Stone's conjecture. BSS algorithms based on Xie's conjecture should be without suspicion in basic theory. From now on, researches have a reliable basis to study BSS both in theory and algorithm design.

Blind separation problem is to restore source signals in unknown mixture parameters, so the mathematics model of blind separation is

$$X(t) = AS(t) + N(t) \quad t = 1 \cdots T \tag{1}$$

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where $X(t) = [x_1(t), x_2(t) \cdots x_m(t)]^T$ are sensor signals, $A \in R^{m \times n}$ is the mixture matrix, and $S(t) = [s_1(t), s_2(t) \cdots s_n(t)]^T$ are source signals, and $N(t) = [n_1(t), n_2(t) \cdots n_m(t)]^T$ is noise. Blind separation aims at restoring source signals only by known sensor signals, so blind separation has two uncertainties, scales uncertainty and permutations uncertainty, but these are allowed in blind separation as a result of the information of source signals in their waveforms. Generally, we suppose noise doesn't exist.

In general, if m is more than or equal to n, just to say, the number of sensor signals is more than that of source signals, which is overdetermined blind separation. We consider m is less than n in this paper, namely, underdetermined blind separation. Although it is difficult to restore source signals, we can use some other information, such as, sparseness of source signals, to restore source signals, and if some source signals aren't sparse in time-field, we can make them sparse through some transformation, such as, fourier transformation or wavelet transformation [8],[17], so blind separation model also written as:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_n(t) \end{bmatrix} \tag{2}$$

where $m < n$, or written with vector format:

$$x(t) = a_1 s_1(t) + a_2 s_2(t) + \cdots a_n s_n(t) \quad t = 1 \cdots T \tag{3}$$

Up to now, the two-step algorithms are general methods for underdetermined blind separation based on sparse representation [15],[16]. The two-step algorithms include two steps, the first step is K-mean clustering algorithm for estimating mixture matrix and the second step is the short-path algorithm for restoring source signals, so we call the two-step algorithms KTA(K-mean Two-step Approach). As it mentioned above, the K-mean clustering algorithm has a key station in KTA and will have an important influence on the next work. When the mixture matrix is estimated, the source signals can be restored through linear programming. In this paper, two-step algorithms still be adopted, but it is different from KTA, and we call the new two-step algorithms PDTA(Probability Distribution Two-step Approach). In the paper, we will estimate the number of source signals first, and the mixture matrix also can be gotten accurately, finally, the work is same to KTA for restoring source signals by linear programming.

2 Sparse Representation of Underdetermined Blind Separation

To underdetermined blind separation, generally, some blind extraction algorithms are taken [9],[10] in past, but the algorithms can't realize restoring all source signals. In order to restore all source signals in underdetermined blind separation, researchers make use of some characteristics of signals, for example, sparse analysis is adopted to make signals sparse presentation, so some underdetermined blind separation are successfully. The good algorithms include in Belouchrani's [11] Maximum likelihood algorithm for discrete sources, Zibulevsky's sparse decomposition algorithm [12], Lee [13], Lewicki [14] and Li' overcomplete representation algorithms [15] and Bofill' sparse representation in frequency domain [16].

Generally, sparse signal is that whose most sample points are zero or are near to zero, and a little points are far from zero. Contrast to Gaussian signal's, sparse signal's distribution function tends to Laplace distribution, namely, there is only one kurtosis in zero point, but it doesn't tend to zero less fastly than that of Gaussian signal and represents super-gaussian, so it is less possible for two source signals have large numbers in the same time, but only one sample point has a large number in almost all time. Here, we suppose that the source signal $s_i(t)$ is nonzero and the other source signals are zero or are near to zero in the time of t. So equation (3) can be written as:

$$x(t) = a_i s_i(t) \tag{4}$$

From the above equation, we can known that a_i and $x(t)$ are collinear, so we can estimate mixture matrix $A = [a_1, a_2, \cdots a_n]$ by clustering $x(t)$ in all time. It is a very important algorithm for sparse component analysis solving underdetermined blind separation, named by K-mean clustering. The algorithm includes two steps, first, cluster centres are estimated by K-mean clustering; secondly, source signals are estimated by known mixture matrix through linear programming.

3 Model of Underdetermined Blind Separation Based on Sparse Representation

All that proposed algorithms can't separate source signals directly in underdetermined blind separation, which includes JADE algorithms, ICA algorithms and H-J algorithms and so on. But the algorithms can resolve the problem based on sparse representation, so the sparse blind separation problem comes down to solving the following optimization [16],

$$\min_{A,S} \frac{1}{2\sigma^2} \|AS - X\|^2 + \sum_{i,t} |s_i(t)| \tag{5}$$

where σ^2 is noise variance, so the equation (5) is optimization problem with multi-variables, which is difficult to resolve directly. We suppose that mixture matrix A is known in advance, then the model is also denoted concisely as,

$$\min_{s(t)} \frac{1}{2\sigma^2} \|As(t) - x(t)\|^2 + \sum_{i}^{n} |s_i(t)|, \quad t = 1,2 \cdots T \tag{6}$$

If noise doesn't taken into account, the equation (6) turns to

$$\begin{cases} \min\limits_{s(t)} \sum\limits_{i}^{n} |s_i(t)| \\ s.t.:\ As(t) = x(t),\ t = 1,2 \cdots T \end{cases} \tag{7}$$

From the equation (6) and the equation (7), we know that there is an optimization problem in every time t based on known mixture matrix A, so optimization problem (7) can be divided into T easy optimization problems.

Generally, the two-step algorithms are fast, but the estimated mixture matrix isn't rigorous as a result of unknown number of the source signals, so the effect of blind separation isn't good. In this paper, we propose a new PDTA algorithm to resolve the underdetermined blind separation problem.

For the sake of simplicity, we suppose $m = 2$, namely, the number of sensors is two, to explain the PDTA algorithms. The sensor signals can be regarded as a point in the 2-dimension plane from equation (4), and they are collinear with the columns of the mixture matrix in the 2-dimension plane. Our PDTA algorithms also include two steps, and the first step is to estimate the number of source signals and estimate the mixture matrix based on sensors signals distributions. In order to analyze the data, we initialize the sensor data first, and the method will be introduced next.

We suppose that $\hat{x}(t) = [\hat{x}_1(t), \hat{x}_2(t)]^T, t = 1,2 \cdots T$ are initialized data, so

$$\hat{x}(t) = \begin{cases} \dfrac{x(t)}{\|x(t)\|}, & if \quad x_2(t) \geq 0 \\ -\dfrac{x(t)}{\|x(t)\|}, & if \quad x_2(t) < 0 \end{cases} \tag{8}$$

and we know that the data $\hat{x}(t)$ will locate in the upper half unit circle.

4 Sparse Blind Separation Algorithms of PDTA

In order to restore source signals, the mixture matrix should be estimated first, and this paper also will estimate it first.

4.1 Estimation of Number of Source Signals and Identification of the Mixture Matrix

In past KTA algorithms, because we don't know the number of source signals, so there is a lot of illegibility in the KTA algorithms and identification of number of source signals has a key effect on blind separation. From the above initialized sensor data, we know that the data points locate in the upper unit circle, so we can compute the arc distance between every point in the unit circle and the point whose coordinate is (1,0),

$$dist(t) = \begin{cases} \arctan(\dfrac{x_2(t)}{x_1(t)}), & if \quad x_1(t) > 0; \\ \arctan(\dfrac{x_2(t)}{x_1(t)}) + \pi, & if \quad x_1(t) < 0; \qquad t = 1,2 \cdots T \\ \dfrac{\pi}{2}, & if \quad x_1(t) = 0. \end{cases} \tag{9}$$

Because the points which are collinear in the 2-dimension plane or near in the unit circle should belong to the same cluster and the points numbers will be enough big, so we can distinguish the number of source signals from the number of columns of mixture matrix, which can be gotten from the distribution of $dist(t)$.

In order to get the distribution of $dist(t)$, we let $a = \min\{dist(t)\}, t = 1,2\cdots T$ and $b = \max\{dist(t)\}, t = 1,2\cdots T$, The interval $[a,b]$ is then divided equally into M subintervals which are $[a+i\delta, a+(i+1)\delta]$ $i = 0,1\cdots M-2$, and $[a+(M-1)\delta, b]$,where $\delta = \frac{b-a}{M}$,and M is a sufficiently large. By estimating the number of sample points of $dist(t)$ in each interval denoted by m_i for the i-th interval, the probability for $dist(t)$ belonging to the i-th interval can be obtained, that is,

$$P_i = \frac{m_i}{T}, i = 1,2\cdots M \tag{10}$$

To make the pdf smooth, we use the following filter,

$$\hat{P}_k = \frac{1}{16}(P_{k-2} + 4P_{k-1} + 6P_k + 4P_{k+1} + P_{k+2}) \tag{11}$$

We want to get the number of source signals, namely, to get the number of peaks in the pdf of $dist(t)$.

Definition 1. if $\hat{P}_k > \hat{P}_{k-1}$, $\hat{P}_k > \hat{P}_{k+1}$ and $\hat{P}_k > \varepsilon_j, k = 2,3,\cdots M-1$, we suppose that there is a peak in the pdf of $dist(t)$, and if $k = 1, M$,we only consider $\hat{P}_k > \varepsilon_j$ .where ε_j is a prior threshold value.

According to the definition 1, we will get the number of peaks denoted as $peaknum$, which also is the estimation of number of source signals.

Next, we will get the estimation of the mixture matrix by the method above, because we get any peak which is identified by $\hat{P}_k$, if $\hat{P}_k > \hat{P}_{k-1}$, $\hat{P}_k > \hat{P}_{k+1}$ and $\hat{P}_k > \varepsilon_j$, so we can find the every $\hat{P}_k$ which is related to a peak and also get the interval of $\hat{P}_k$, denoted as $[a+(k-1)\delta, a+k\delta]$. We let

$$\begin{aligned} length_i &= (a + (k-1)\delta + a + k\delta)/2 \\ &= a + (2k-1)\delta/2, \qquad i = 1,2,\cdots, peaknum. \end{aligned} \tag{12}$$

where $length_i$ denotes the arc distance between the center of the i th cluster of sensor data and the point whose coordinate is (1,0). Because the arc distance is radian in unit circle, and the i th cluster of sensor data is collinear with a column of mixture matrix, so

$$a_i = [\cos(length_i), \sin(length_i)]^T, \quad i = 1,2\cdots peaknum \tag{13}$$

From the above algorithm, the number of source signals and the mixture matrix are both gotten expediently, then, the second step of PDTA algorithms will be used to restore source signals by linear programming.

4.2 Restore Source Signals

From the equation (7), we know that it is a linear programming problem for restoring the source signals, and a_i denotes a column of mixture matrix in the equation (3), so $A=[a_1,a_2,\cdots a_n]$ and a_i has been normalized, namely, $\|a_i\|=1$.

The equation (3) explains that the vector $x(t)$ is composed of the normalized vectors $a_1,a_2\cdots a_n$ or $x(t)=a_1s_1(t)+a_2s_2(t)+\cdots+a_ns_n(t)$, where $s_1(t),s_2(t)\cdots s_n(t)$ are the coefficients. The geometrical graph shows that the vectors $a_1s_1(t),a_2s_2(t)\cdots a_ns_n(t)$ and $x(t)$ can form a close geometrical graph as figure 1, what's more, $\sum_i^n |s_i(t)|$ is the length sum of the vectors $a_1s_1(t),a_2s_2(t)\cdots a_ns_n(t)$ .In underdetermined blind separation, if $m<n$, the solutions of the equations (3) are not single.

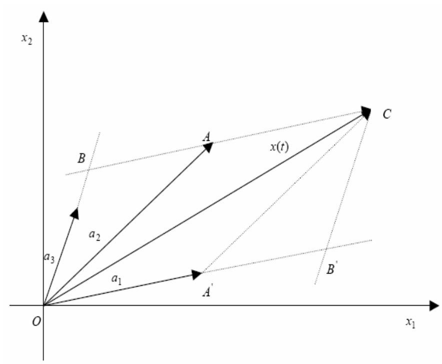

Fig. 1. The illustration of the short path

From the figure 1, we can know that the minimization of $\sum_i^n |s_i(t)|$ which satisfies the equation (7) is equal to find a shortest path from the origin (0,0) to $x(t)$. In the 2-dimension plane, the shortest path of $x(t)$ is composed of the two vectors of a_i and a_j, which are nearest to $x(t)$ respectively.

We let $A_r=[a_i,a_j]$, so $s_r(t)$ is the coefficient of $x(t)$ which is decomposed by a_i and a_j, so the solutions of $s_r(t)$ of the optimization problem (7) is

$$\begin{cases} s_r(t)=A_r^{-1}x(t) \\ s_k(t)=0, k\neq i,j \end{cases} \tag{14}$$

So only the i th source signal and the j th source signal have nonzero values gotten by equation (14) in the time of t, but zero for the other source signals in the time of t.

5 Simulation Results

In the experiment, we take $m=2$, $n=6$, namely there are two sensors and six source signals, and the mixture matrix is randomly taken as $A=\begin{bmatrix} 0.7660 & 0.5000 & 0.2588 & -0.1736 & -0.7071 & -0.9063 \\ 0.6428 & 0.8660 & 0.9659 & 0.9848 & 0.7071 & 0.4226 \end{bmatrix}$, and the initialized sensor data is shown in figure 2. By the method of the equation (9), we calculate the arc distances of $dist(t), t=1,2\cdots T$, and its probability distribution chart is shown the figure 3, which is gotten from the equation (10) and (11), sometimes, the pdf's filter should be used more times. From the definition, we can calculate the *peaknum* is 6, and get the estimated mixture matrix $\hat{A}=\begin{bmatrix} 0.7635 & 0.5029 & 0.2672 & -0.1667 & -0.7007 & -0.9039 \\ 0.6458 & 0.8644 & 0.9636 & 0.9860 & 0.7135 & 0.4277 \end{bmatrix}$ by the (12),(13). According to the estimated mixture matrix $\hat{A}$ and the short path algorithm, the source signals are recovered in the figure 6. What's more, we calculate the correlation coefficient matrix of source signals and restored signals is

$$corrcoef=\begin{bmatrix} 0.9969 & 0.0084 & 0.0014 & -0.0021 & -0.0016 & -0.0071 \\ 0.0168 & 0.9951 & 0.0035 & 0.0009 & -0.0001 & 0.0003 \\ 0.0014 & 0.0246 & 0.9946 & 0.0162 & -0.0000 & 0.0001 \\ -0.0000 & 0.0011 & 0.0250 & 0.9950 & 0.0039 & -0.0025 \\ -0.0009 & 0.0003 & 0.0005 & 0.0202 & 0.9972 & 0.0014 \\ -0.0121 & -0.0008 & -0.0008 & 0.0014 & 0.0288 & 0.9979 \end{bmatrix},$$

which shows that PDTA algorithm is very excellent not only in estimation of the number of source signals and the mixture matrix but also in the restoration of source signals.

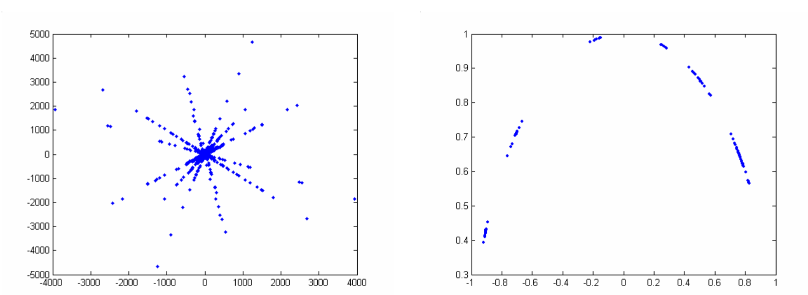

Fig. 2. The sensor signals and their initialized sensor signals

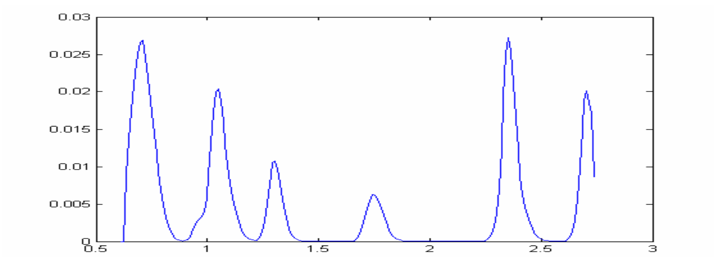

Fig. 3. The $dist(t)$ probability distribution

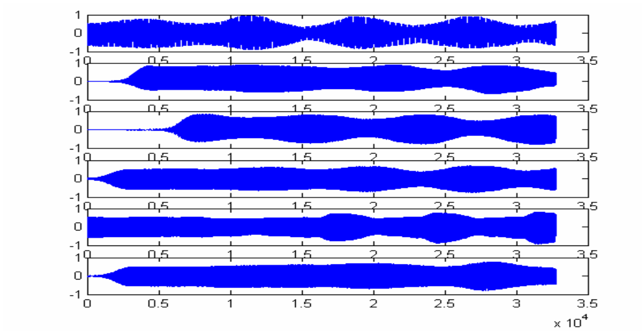

Fig. 4. Six source signals

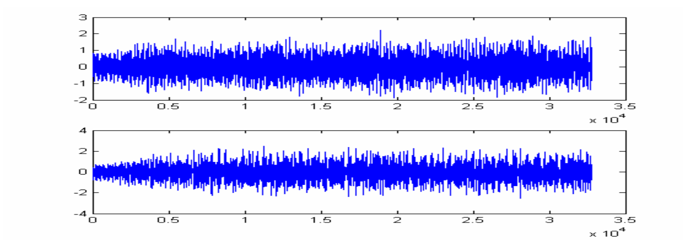

Fig. 5. Two mixture signals

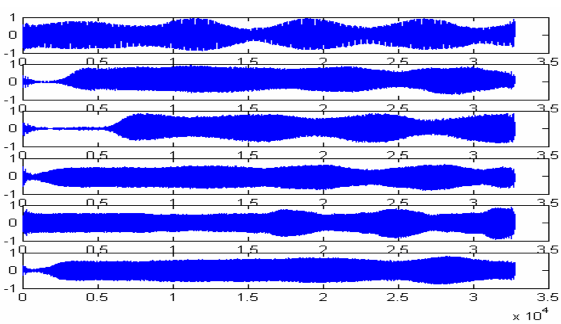

Fig. 6. Six restored source signals

6 Conclusions

In general underdetermined blind separation, source signals are recovered by the two-step algorithms KTA through the information of sparseness of source signals, but there is a big trouble for K-mean clustering algorithm in the two-step algorithms as a result of unknown number of source signals, so we give new two-step algorithms PDTA in this paper, which can estimate the number of source signals in advance by the distribution and get the mixture matrix different from the K-mean clustering algorithm, and the source signals are recovered by the short path algorithm successfully. From the simulation results and the gotten correlation coefficient matrix of source signals and restored signals, the outstanding performance of the PDTA algorithms is expressed. When the sensor number is more than two, it still will be open problem.

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