Waypoint Selection in Constrained Domains (for Cooperative Systems)

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Abstract. This chapter presents a new framework for multiple vehicle systems modeling and control, emphasizing team behavior in a multi-level, multi-resolution way. To set the common reference trajectory for team vehicles, a waypoint selection strategy is proposed taking into account the dimensions of the free space and practical aspects of motion generation. The multi-vehicle cooperative parking strategy is proposed so that a "class" of problems can be solved by formation reconfiguration. The study focuses on several cases corresponding to different scenarios.

1 Introduction

Coordination of multi-vehicle systems to fulfill a mission will yield more benefits than single vehicles performing solo missions. Recent years have seen much research work on this field [[7], [8], [11], [9]]. One motivation for cooperative autonomous vehicle systems is to follow global trajectories and accomplish task as has been done by single vehicles so that the input trajectory and path following strategy are mainly designed for the leader. Team members will share specific information and achieve the final goal by formation reconfiguration [10].

Usually, the reconfiguration mode will be set corresponding to different type of cooperative control problems. In some of them, rigid formation is kept since the common reference state will be assigned to individual vehicles. In other problems, each vehicle will access the information from its neighbor and the team behavior is determined by recurring local inter-vehicle communication. Also, the combination of the above two kinds of problems exists. That is, the team members have similar missions. They keep rigid formation in some of the scenario and reconfigure their formation for a new situation. Under some circumstances in this process, the temporal/sequenced formation is required.

Our problem falls into the last scope. We are interested in teams of vehicles "going somewhere(transition)", and ultimately, "doing something(ultimate task)". A new model for multiple vehicle systems during transition phase has

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been proposed in [11], emphasizing team behavior in a multi-level, multi-resolution way. This framework integrates issues like team formation and path following, so that tasks can easily be allocated to individual and teams of vehicles. The movement of the leader is modeled as a discrete state system within a cellular map, and the movement of the follower is modeled as a hybrid system, including the leader-follower interface.

To set the common reference trajectory, we present a waypoint selection strategy taking into account the dimensions of the free space and practical aspects of motion generation. A mainline approach and a number of special cases are investigated. The maneuvering task is finished by approaching the target cell and stabilizing to the final parking position. A multi-vehicle cooperative parking framework is proposed based on the above model and the waypoint selection strategy.

This chapter is organized as follows: in Section 2, we give the models of leading vehicle, leader-follower interface and following vehicles. A Waypoint selection strategy in constrained domain and a multi-vehicle cooperative parking framework are introduced in Section 3. After that, we provide simulations with a Dubin's car model in Section 4. Conclusions are drawn in Section 5.

2 A Model During Transition Phase

For autonomous multi-vehicle navigation, coordination of multi-vehicle systems to fulfill a mission will yield more benefits than single vehicles performing solo missions. In this field, how to simplify vehicle dynamics by systems modeling is important for efficiently realizing the transition among different formation modes.

The model framework proposed in [11] embodies the idea of decomposition and synthesis for large scale systems. We repeat the key concepts here for completeness. In the original paper [11] we considered convoy type driving along roadways. Here, we shall consider motion in larger open areas, hence the need for path planning. The hierarchical layered structure is shown in Figure 1. A graph can be used to represent the map where vehicles are moving on, with vertices representing crossings and edges representing roads.

Consider a planar digraph $G = (V, E)$, where V is the vertex set and E is the edge set, and $e^{ij} = (v^i, v^j) \in V \times V$. If at one moment $e_k = e^i$, and at the next moment $e_{k+1} = e^j$, the adjacency matrix A is defined as

$$[A]_{ji} = \begin{cases} 1, \text{if } e_k = e^i \text{ and } e_{k+1} = e^j \\ 0, \text{otherwise} \end{cases}$$

Where A_{ji} means $j-$th row, $i-$th column element of A. Consider a physical road map located in a coordinated system Ω_0 corresponding to the digraph G. Each road can be divided into segments that are connected as a chain, in an approximate sense. We call these segment cells and name these cells along the edge direction as $s_m^{ij} = s_m(e^{ij})$ with ascending subscripts $m \in \{1, 2, \dots, N_{e^{ij}}\}$, where $N_{e^{ij}} \in \mathbb{N}$ is the number of cells on edge e^{ij}.

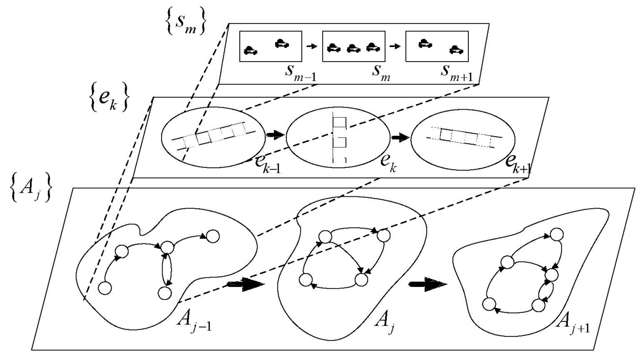

Fig. 1. Hierarchical layered model structure

2.1 Leader Model

With cellular structure modeling, the dynamics of the leader is modeled as a discrete system in a slow time scale. Between "jumps" from one cell to the next, there is a *continuous* movement of the follower in a *fast time scale.*

The cellular movement of the leader is described as

$$\begin{aligned} x_{k+1} &= \varsigma(x_k, u_k) = x_k \oplus u_k \\ &= \begin{cases} (s_k + u_k, e_j), & s_k + u_k \leq N_{e_j} \\ (s_k + u_k - N_{e_j}, e_{j+1}), & \text{otherwise.} \end{cases} \end{aligned} \quad (1)$$

and control variable u_k takes "quantized" values such as

$$u_k = \begin{cases} 0, & \text{stop}, \\ 1, & 1^{st} \text{ speed level}, \\ \cdots & \cdots \\ u^{\max}, & \text{max speed level.} \end{cases} \quad (2)$$

The evolution of road states is as follows.

$$e_{j+1} = A_j e_j \quad (3)$$

2.2 Leader-Follower Interface

An interface is usually needed to connect two types of systems. This operation defines the rotation of each cell and consequently builds a one-to-one mapping for any position between two cells spatially.

Position Mapping. To better describe the movement of the leader and the followers in a common framework, the following notation system is introduced in [4].

Let $T = [t_i, t_f] \subset \mathbb{R}$ be the time zone of interest. Introduce time index

$$\mathcal{T} = \{[\tau'_0, \tau_1], [\tau'_1, \tau_2], \ldots, [\tau'_{n-1}, \tau_n]\} \tag{4}$$

where $\tau_i \in T$ for all i, $\tau'_0 = t_i$, $\tau_n = t_f$, and $\tau_i = \tau'_i \leq \tau_{i+1}$ for all $i = 1, 2, \ldots, n-1$.

Position mapping for $t \in [\tau'_i, \tau_{i+1})$ corresponds to in-cell dynamics while that for $t = \tau_i$ corresponds to inter-cell dynamics.

Coordinate Rotation. A coordinate system $\tilde{\Omega}^{ij}_m$ will be built within each cell s^{ij}_m, with the origin set at the center g^{ij}_m. One of the axes $\mathbf{n}^{ij}_m$ can be chosen as the normal of the arc passing through the cell center, and the other axis, therefore, can be chosen as $\mathbf{n}^{ij\perp}_m$ which rotates $\mathbf{n}^{ij}_m$ by $\pi/2$ counterclockwise, as the tangent of the arc.

The rotation of $\tilde{\Omega}^{ij}_m$ with respect to Ω_0 is recorded in matrix

$$R^{ij}_m = \begin{bmatrix} \cos\phi^{ij}_m & -\sin\phi^{ij}_m \\ \sin\phi^{ij}_m & \cos\phi^{ij}_m \end{bmatrix} \tag{5}$$

where $\phi^{ij}_m = \angle\mathbf{n}^{ij}_m$. As a result, $\hat{\Omega}^{ij}_m$ is the corresponding *upright* coordinate rotated by $\tilde{\Omega}^{ij}_m$ and they are constrained by the relationship, $\tilde{\Omega}^{ij}_m = R^{ij}_m \hat{\Omega}^{ij}_m$. An

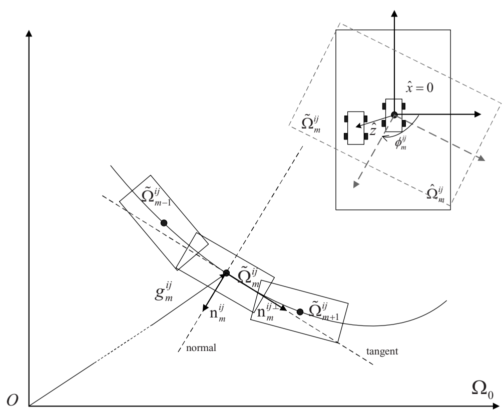

Fig. 2. Illustration for coordinate systems in cells

illustration for the above concept is provided in Figure 2.

Continuous and Discrete Evolution. During the in-cell phase, in the individual up-right coordinate in Figure 2, for $t \in [\tau'_i, \tau_{i+1}]$, $x(t) \equiv x(\tau'_i)$, $g(t) \equiv g(\tau'_i)$, $R^{-1}(\tau'_i) = (R^{ij}_m)^{-1}$. Assume the leading vehicle stays in cell s^{ij}_m. The leader's position in Ω_0 is $g(t) = g^{ij}_m$ and in $\hat{\Omega}^{ij}_m$ is

$$\hat{x}(t) \equiv 0, \forall t \in [\tau'_i, \tau_{i+1}] \tag{6}$$

Let $\hat{z}(t) = (R_m^{ij})^{-1}(z(t) - g(t))$ represent the position of the follower in $\hat{\Omega}_m^{ij}$ with respect to the leader. The movement of followers is simplified from $\dot{z}(t) = \zeta(z(t), g(t))$ to

$$\dot{\hat{z}}(t) = \hat{\zeta}(\hat{z}(t), 0) = \hat{\zeta}(\hat{z}(t)), \forall t \in [\tau_i', \tau_{i+1}] \tag{7}$$

In the global rotated coordinate

$$g(t) \equiv g(\tau_i'), \forall t \in [\tau_i', \tau_{i+1}] \tag{8}$$

$$z(t) = g(t) + R(\tau_i')\hat{\zeta}(R^{-1}(\tau_i')(z(t) - g(t)), 0), \quad \forall t \in [\tau_i', \tau_{i+1}] \tag{9}$$

3 Waypoint Selection for Parking Maneuver

Maximum curvature and space limit are the two most important factors when generating parking trajectories autonomously. Considering the nonholonomic constraint of rolling without slipping, the common reference trajectory for team vehicles is not allowed to make sharp turns. Especially when the operation area is a constrained domain, waypoints should be selected to satisfy both the vehicle dynamics and the area constraints. The problem is related to many others, from a so-called SOFA problem, to path planning algorithms like A-star, to potential field approaches. It has a number of new applications, on the ground with autonomous cars, in the air, with UAV's flying around buildings.

We will consider parking maneuvers in a constrained parking zone. In this zone, upper bound for trajectory curvature is required. The vehicle's pose (position and orientation) should be adjusted to a suitable one before entering the parking bay. We prefer forward maneuvers unless straight reverse is necessary in several special cases. Therefore, the waypoint selection strategy should take into account the dimensions of the free space and practical aspects of motion generation. The strategy is designed in a hybrid framework.

3.1 High Level Decomposition of Configuration Space

Different from the other parking maneuver cases [[5],[6],[2]], which usually concentrate on how to enter the parking bay by robotic behavior from a relatively friendly initial posture, we care how to make use of the open space to adjust a vehicle's posture so that it can enter the parking bay smoothly.

In the higher level, first we divide the whole parking spot into a few cells around the goal parking position and specify the area near the parking bay as the target cell. This cell is a highly constrained area since as long as the vehicle entered this cell the "pose" must meet some requirements so that parking maneuver can be easily finished within this small cell. Secondly, the planar cell structure is extended to three dimensional space in which the orientation is expressed by the third dimension. And now, given a vehicle pose, we can index it by (x, y, z) information of each cubic cell. Finally, for each cell that has a different pose, a corresponding waypoint selection method is developed in the lower level considering vehicle dynamic motion constrains.

The configuration space decomposition is described in Figure 3. We use parking bay fixed coordinate system. The directed arrow denotes the vehicle which has a certain length and direction. The origin (x_p, y_p) is the parking bay position. The constant R is the minimum turning radius for the vehicle. The two gray cells constitute the highly constrained area. In this area, the vehicle's position should not be inside the two quarter circles no matter what its direction is. It should enter the gray rectangular from the upper side.

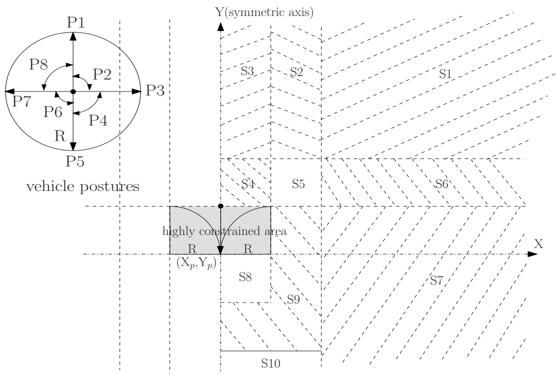

Fig. 3. Configuration space decomposition

Since (x, y) only gives the position information, the third dimension z can denote the heading direction. In the upper left of Figure 3, the z axis is divided to eight levels. That is

$$\begin{array}{llll} P1 = \frac{\pi}{2}, & P2 \in (0, \frac{\pi}{2}), & P3 = 0, & P4 \in (-\frac{\pi}{2}, 0), \\ P5 = -\frac{\pi}{2}, & P6 \in (-\pi, -\frac{\pi}{2}), & P7 = \pi, & P8 \in (\frac{\pi}{2}, \pi). \end{array}$$

Thus for each section in the planar plane, it has eight levels in the 3 dimensional space.

It is worth noting that by this decomposition method, only half plane solution need to be given since the whole open space is symmetry by y axis. We will only analyze the right half plane. In this part, S1 is a friendly area. As long as the vehicle is in this section, it will be easy to find a maneuver adjusting "pose". Our basic idea is to find solution for S1 and drive the vehicle to this section first when the initial position is in other sections. To simplify the algorithm, we try to merge the sections having the same waypoint selection method. In total, ten sections in each z level are needed.

Now the problem can be rephrased as follows. Given a vehicle pose (indexed by x_0, y_0, z_0) and the parking bay (the origin of the configuration space), find a waypoint selection strategy so that the vehicle system will start from (x_0, y_0, z_0) and converge to $(0, 0, 0)$.

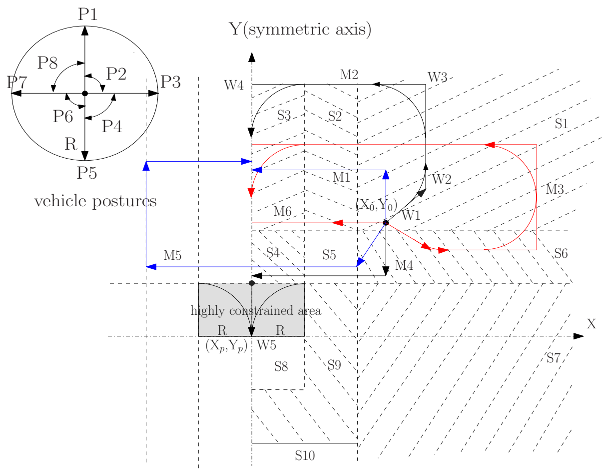

Fig. 4. Six typical maneuvers

3.2 Low Level Waypoint Selection

In the low level, both the vehicle desired (parking) position and its initial position is defined as two points in the configuration space. The maneuvering task is finished by approaching the target cell and stabilizing to the final parking position. A mainline approach and a number of special cases are investigated.

Considering the initial position in S1, we mentioned that this section is a friendly section and the vehicle can drive to the parking bay easily by some maneuver. For different z levels, we define corresponding waypoints and they could be connected by "real-time dynamic trajectory smoothing algorithm[1]" so that a feasible trajectory will be provided to the vehicle. Typical lower level waypoint selection strategy is shown in Table 1. For different initial posture, different selected waypoints are given. The length of the line segment connecting the waypoints is $n * R$ where n is an integer and R is the minimum turning radius, a parameter of the vehicle. The waypoints should be selected so that an obtuse angle or right angle is formed by adjacent line segments connecting waypoints. This method could ensure an arc tangent to both adjacent line segments exists.

A few cubic cells such as S4:P4/P5/P6, S5:P6, S8:P1 and so on, need straight reverse maneuver. Otherwise, there will be no solution.

This method is a resolution-complete algorithm. A solution is guaranteed if it exists at a given resolution when modeling the search space by grids [3]. The completeness of the geometric planner assures the completeness of the algorithm. The resulting trajectory could be tracked by the leader of a team of vehicles.

Table 1. Six typical maneuvers in S1. Note: (x_p, y_p) is the target parking position and θ_0 is the initial yaw angle.

Method	Initial posture	Selected waypoints
M1	P1 $(x_0, y_0, \frac{\pi}{2})$	$(x_0, y_0) \rightarrow (x_0, y_0 + r) \rightarrow (x_p, y_0 + r) \rightarrow (x_p, y_p)$
M2	P2$(x_0, y_0, (0, \frac{\pi}{2}))$ P3$(x_0, y_0, 0)$	$(x_0, y_0) \rightarrow (x_0 + r\cos\theta_0, y_0 + r\sin\theta_0)$ $\rightarrow (x_0 + r\cos\theta_0, y_0 + r\sin\theta_0 + 2r) \rightarrow$ $(x_p, y_0 + r\sin\theta_0 + 2r) \rightarrow (x_p, y_p)$
M3	P4 $(x_0, y_0, (0, -\frac{\pi}{2}))$	$(x_0, y_0) \rightarrow (x_0 + r\cos\theta_0, y_0 + r\sin\theta_0)$ $\rightarrow (x_0 + r\cos\theta_0 + 2r, y_0 + r\sin\theta_0)$ $\rightarrow (x_0 + r\cos\theta_0 + 2r, y_0 + r\sin\theta_0 + 2r)$ $\rightarrow (x_p, y_0 + r\sin\theta_0 + 2r) \rightarrow (x_p, y_p)$
M4	P5 $(x_0, y_0, -\frac{\pi}{2})$	$(x_0, y_0) \rightarrow (x_0 + r\cos\theta_0, y_0 + r\sin\theta_0)$ $\rightarrow (x_p, y_0 + r\sin\theta_0) \rightarrow (x_p, y_p)$
M5	P6$(x_0, y_0, (-\frac{\pi}{2}, -\pi))$ P8$(x_0, y_0, -\pi)$	$(x_0, y_0) \rightarrow (x_0 + r\cos\theta_0, y_0 + r\sin\theta_0) \rightarrow$ $(x_p - 2r, y_0 + r\sin\theta_0) \rightarrow (x_p - 2r, y_0 + r\sin\theta_0 + 2r)$ $\rightarrow (x_p, y_0 + r\sin\theta_0 + 2r) \rightarrow (x_p, y_p)$
M6	P7 (x_0, y_0, π)	$(x_0, y_0) \rightarrow (x_p, y_0) \rightarrow (x_p, y_p)$

3.3 Multi-vehicle Cooperative Parking

Based on the proposed model and the waypoint selection strategy, a multi-vehicle cooperative parking framework is proposed to solve a "class" of problems. Consider several teams of vehicles entering the parking zone by passing a road segment (Figure 5). They aim at different parking bays. Figure 6 and Figure 7 give the function hierarchy and inter car coordination level for the leaders of each team. The leader of the first team L_1 is the master of the communication group and can trigger other leaders. By inter car coordination level, the vehicles will park team by team and the vehicles in the same team will park into by formation reconfiguration. The trajectory for the leader is generated by the waypoint selection method provided before. The transition phase model introduced before will be used here for easy formation reconfiguration. "Shifted" domains can help (plans can be transmitted between leaders).

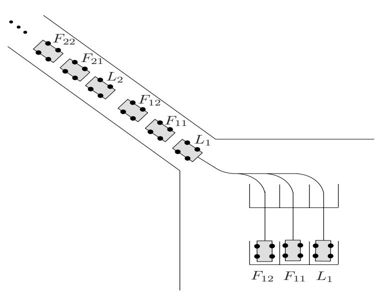

Fig. 5. Cooperative parking

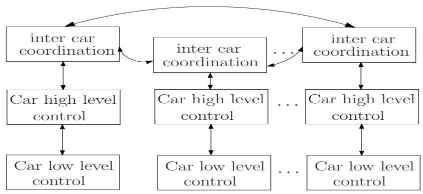

Fig. 6. Functional hierarchy

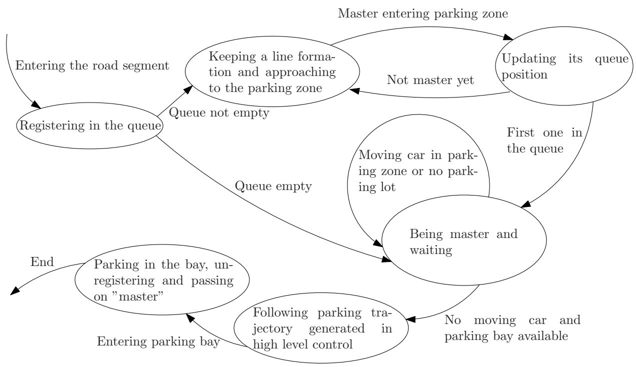

Fig. 7. Inter car coordination level

4 Simulation Results

To illustrate the validity of the above design, several typical parking trajectories are provided. In the following figures, z axis denotes the angle (rad) between vehicle heading direction and parking lot direction. Figure 8 shows the parking trajectory in configuration space and in the $x-y$ plane for initial position in S1 and heading angle $\in$ P2. Vehicle configuration converging to the origin means it enters the parking bay.

Figure 9 shows two different cases in which the initial position is in S1 and S6 respectively. In the first case, the vehicle entered the parking bay by method M1. While in the second one, the vehicle traveled to left half plane first. By symmetry, similar maneuver methods are defined in this area just as M1 to M6 in right half plane. Once the vehicle entered the left half plane, the waypoints will be selected by searching the maneuver in corresponding indexed cubic cell. Figure 10 shows the parking process in configuration space. The vehicle made use of the open area to adjust its heading direction along its trajectory and entered the parking bay at the required angle.

Two other cases are shown in Figure 11. One of them is a special case in which straight reverse is unavoidable. At the intersection point, though the two trajectories have the same position, they are in different z level (different heading

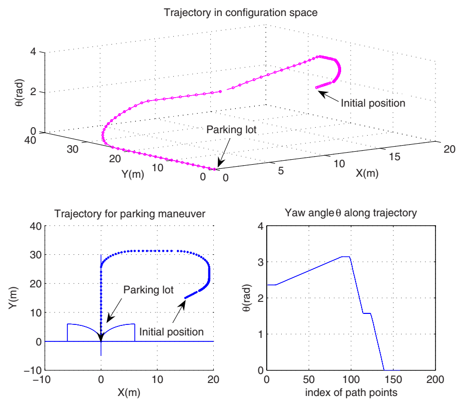

Fig. 8. Parking maneuver

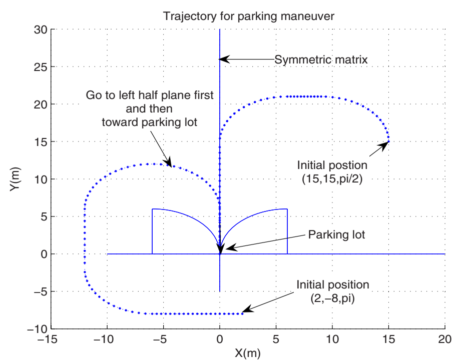

Fig. 9. Waypoint selection in x-y plane

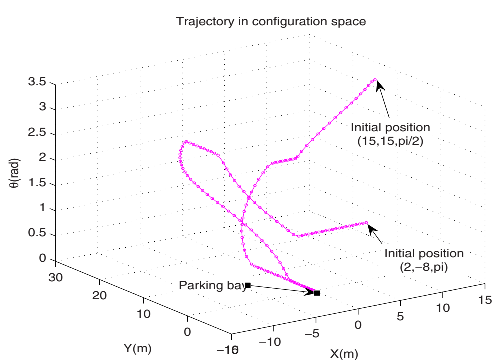

Fig. 10. Parking maneuver configuration space

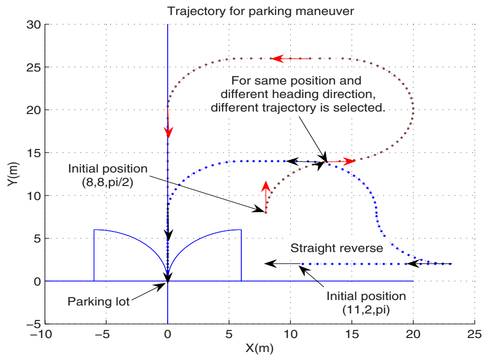

Fig. 11. Waypoint selection in x-y plane

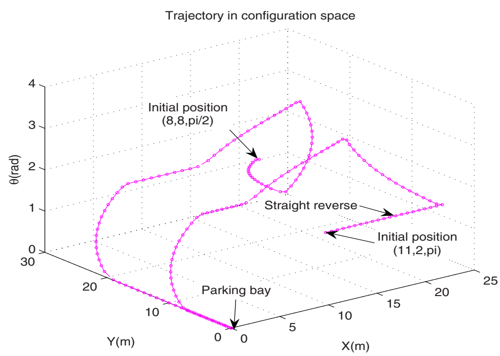

Fig. 12. Parking maneuver configuration space

direction) and will have different waypoint selection method which can be seen from configuration space clearly (Figure 12).

5 Conclusions

This paper presents a new framework for multiple vehicle system modeling and control, emphasizing the team behavior in a multi-level, multi-resolution way. After the search space is modeled by a coarse grid, with fixed cells, a complete algorithm is developed for waypoint selection taking into account the dimensions of the free space and practical aspects of motion generation. The trajectory generated from these waypoints can be fed as reference inputs to leader vehicles of each team. Multiple vehicle cooperative parking is among the most meaningful application of the proposed algorithm. Following a typical hybrid system design procedure as illustrated, a "class" of problems can be solved by formation reconfiguration based on the proposed transition phase model and waypoint selection methods.

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