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Inflationary Cosmology

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Abstract. This chapter presents a general review of the history of inflationary cosmology and of its present status.[1]

1.1 Brief History of Inflation

Since the inflationary theory is more than 25 years old, perhaps it is not inappropriate to start this chapter with a brief history of its development, and some personal recollections.

Several ingredients of inflationary cosmology were discovered in the beginning of the 1970s. The first realization was that the energy density of a scalar field plays the role of the vacuum energy/cosmological constant [1], which was changing during the cosmological phase transitions [2]. In certain cases these changes occur discontinuously, due to first-order phase transitions from a supercooled vacuum state (false vacuum) [3].

In 1978, we with Gennady Chibisov tried to use these facts to construct a cosmological model involving exponential expansion of the universe in the supercooled vacuum as a source of the entropy of the universe, but we immediately realized that the universe becomes very inhomogeneous after the bubble wall collisions. I mentioned our work in my review article [4], but did not pursue this idea any further.

The first semi-realistic model of inflationary type was proposed by Alexei Starobinsky in 1979–1980 [5]. It was based on the investigation of a conformal anomaly in quantum gravity. His model was rather complicated, and its goal was somewhat different from the goals of inflationary cosmology. Instead of attempting to solve the homogeneity and isotropy problems, Starobinsky considered the model of the universe which was homogeneous and isotropic from the very beginning, and emphasized that his scenario was "the extreme opposite of Misner's initial 'chaos'."

[1] Based on a talk given at the 22nd IAP Colloquium, "Inflation+25", Paris, June 2006.

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On the other hand, the Starobinsky model did not suffer from the graceful exit problem, and it was the first model to predict gravitational waves with a flat spectrum [5]. The first mechanism of production of adiabatic perturbations of the metric with a flat spectrum, which are responsible for galaxy production, and which were found by the observations of the CMB anisotropy, was proposed by Mukhanov and Chibisov [6] in the context of this model.

A much simpler inflationary model with a very clear physical motivation was proposed by Alan Guth in 1981 [7]. His model, which is now called "old inflation," was based on the theory of supercooling during the cosmological phase transitions [3]. Even though this scenario did not work, it played a profound role in the development of inflationary cosmology since it contained a very clear explanation of how inflation may solve the major cosmological problems.

According to this scenario, inflation is described by the exponential expansion of the universe in a supercooled false vacuum state. False vacuum is a metastable state without any fields or particles but with a large energy density. Imagine a universe filled with such "heavy nothing." When the universe expands, empty space remains empty, so its energy density does not change. The universe with a constant energy density expands exponentially, thus we have inflation in the false vacuum. This expansion makes the universe very big and very flat. Then the false vacuum decays, the bubbles of the new phase collide, and our universe becomes hot.

Unfortunately, this simple and intuitive picture of inflation in the false vacuum state is somewhat misleading. If the probability of the bubble formation is large, bubbles of the new phase are formed near each other, inflation is too short to solve any problems, and the bubble wall collisions make the universe extremely inhomogeneous. If they are formed far away from each other, which is the case if the probability of their formation is small and inflation is long, each of these bubbles represents a separate open universe with a vanishingly small Ω. Both options are unacceptable, which has lead to the conclusion that this scenario does not work and cannot be improved (graceful exit problem) [7, 8, 9].

The solution was found in 1981–1982 with the invention of the new inflationary theory [10], see also [11]. In this theory, inflation may begin either in the false vacuum, or in an unstable state at the top of the effective potential. Then the inflaton field ϕ slowly rolls down to the minimum of its effective potential. The motion of the field away from the false vacuum is of crucial importance: density perturbations produced during the slow-roll inflation are inversely proportional to $\dot{\phi}$ [6, 12, 13]. Thus the key difference between the new inflationary scenario and the old one is that the useful part of inflation in the new scenario, which is responsible for the homogeneity of our universe, does *not* occur in the false vacuum state, where $\dot{\phi} = 0$.

Soon after the invention of the new inflationary scenario it became so popular that even now most of the textbooks on astrophysics incorrectly describe inflation as an exponential expansion in a supercooled false vacuum state during the cosmological phase transitions in grand unified theories. Unfortunately, this scenario was plagued by its own problems. It works only if the effective potential of the field ϕ has a very flat plateau near $\phi = 0$, which is somewhat artificial. In most versions of this scenario the inflaton field has an extremely small coupling constant, so it could not be in thermal equilibrium with other matter fields. The theory of cosmological phase transitions, which was the basis for old and new inflation, did not work in such a situation. Moreover, thermal equilibrium requires many particles interacting with each other. This means that new inflation could explain why our universe was so large only if it was very large and contained many particles from the very beginning [14].

Old and new inflation represented a substantial but incomplete modification of the big bang theory. It was still assumed that the universe was in a state of thermal equilibrium from the very beginning, that it was relatively homogeneous and large enough to survive until the beginning of inflation, and that the stage of inflation was just an intermediate stage of the evolution of the universe. In the beginning of the 1980s these assumptions seemed most natural and practically unavoidable. On the basis of all available observations (CMB, abundance of light elements) everybody believed that the universe was created in a hot big bang. That is why it was so difficult to overcome a certain psychological barrier and abandon all of these assumptions. This was done in 1983 with the invention of the chaotic inflation scenario [15]. This scenario resolved all problems of old and new inflation. According to this scenario, inflation may begin even if there was no thermal equilibrium in the early universe, and it may occur even in the theories with simplest potentials such as $V(\phi) \sim \phi^2$. But it is not limited to the theories with polynomial potentials: chaotic inflation occurs in *any* theory where the potential has a sufficiently flat region, which allows the existence of the slow-roll regime [15].

1.2 Chaotic Inflation

1.2.1 Basic Model

Consider the simplest model of a scalar field ϕ with a mass m and with the potential energy density $V(\phi) = \frac{m^2}{2}\phi^2$. Since this function has a minimum at $\phi = 0$, one may expect that the scalar field ϕ should oscillate near this minimum. This is indeed the case if the universe does not expand, in which case the equation of motion for the scalar field coincides with the equation for the harmonic oscillator, $\ddot{\phi} = -m^2\phi$.

However, because of the expansion of the universe with Hubble constant $H = \dot{a}/a$, an additional term $3H\dot{\phi}$ appears in the harmonic oscillator equation:

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi \, . \tag{1.1}$$

The term $3H\dot\phi$ can be interpreted as a friction term. The Einstein equation for a homogeneous universe containing a scalar field ϕ looks as follows:

$$H^2 + \frac{k}{a^2} = \frac{1}{6}\left(\dot\phi^2 + m^2\phi^2\right) . \tag{1.2}$$

Here $k = -1, 0, 1$ for an open, flat or closed universe respectively. We work in units $M_{\rm pl}^{-2} = 8\pi G = 1$.

If the scalar field ϕ initially was large, the Hubble parameter H was large too, according to the second equation. This means that the friction term $3H\dot\phi$ was very large, and therefore the scalar field was moving very slowly, as a ball in a viscous liquid. Therefore at this stage the energy density of the scalar field, unlike the density of ordinary matter, remained almost constant, and the expansion of the universe continued at a much greater speed than in the old cosmological theory. Due to the rapid growth of the scale of the universe and the slow motion of the field ϕ, soon after the beginning of this regime one has $\ddot\phi \ll 3H\dot\phi$, $H^2 \gg \frac{k}{a^2}$, $\dot\phi^2 \ll m^2\phi^2$, so the system of equations can be simplified:

$$H = \frac{\dot a}{a} = \frac{m\phi}{\sqrt{6}} , \qquad \dot\phi = -m\,\sqrt{\frac{2}{3}} . \tag{1.3}$$

The first equation shows that if the field ϕ changes slowly, the size of the universe in this regime grows approximately as e^{Ht}, where $H = \frac{m\phi}{\sqrt{6}}$. This is the stage of inflation, which ends when the field ϕ becomes much smaller than $M_{\rm Pl} = 1$. The solution to these equations shows that after a long stage of inflation the universe initially filled with the field $\phi \gg 1$ grows exponentially [14],

$$a = a_0\, e^{\phi^2/4} . \tag{1.4}$$

Thus, inflation does not require an initial state of thermal equilibrium, supercooling and tunneling from the false vacuum. It appears in the theories that can be as simple as a theory of a harmonic oscillator [15]. Only when it was realized, it became clear that inflation is not just a trick necessary to fix problems of the old big bang theory, but a generic cosmological regime.

1.2.2 Initial Conditions

But what is about the initial conditions required for chaotic inflation? Let us consider first a closed universe of initial size $l \sim 1$ (in Planck units), which emerges from the space–time foam, or from singularity, or from "nothing" in a state with the Planck density $\rho \sim 1$. Only starting from this moment, i.e. at $\rho \lesssim 1$, can we describe this domain as a *classical* universe. Thus, at this initial moment the sum of the kinetic energy density, gradient energy density, and the potential energy density is of the order unity: $\frac{1}{2}\dot\phi^2 + \frac{1}{2}(\partial_i\phi)^2 + V(\phi) \sim 1$ (Fig. 1.1).

We wish to emphasize, that there are no a priori constraints on the initial value of the scalar field in this domain, except for the constraint

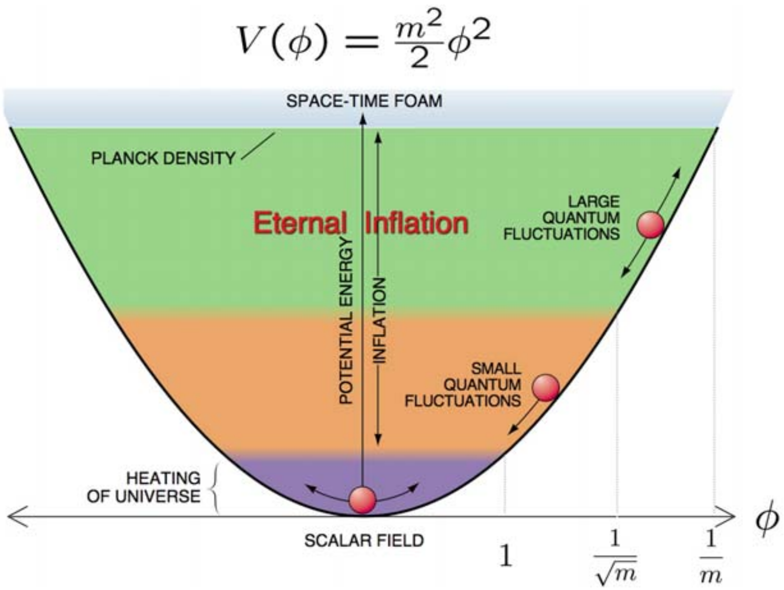

Fig. 1.1. Motion of the scalar field in the theory with $V(\phi) = \frac{m^2}{2}\phi^2$. Several different regimes are possible, depending on the value of the field ϕ. If the potential energy density of the field is greater than the Planck density $M_{\rm Pl}^4 = 1$, $\phi \gtrsim m^{-1}$, the quantum fluctuations of space–time are so strong that one cannot describe it in usual terms. Such a state is called space–time foam. At a somewhat smaller energy density (for $m \lesssim V(\phi) \lesssim 1,\ m^{-1/2} \lesssim \phi \lesssim m^{-1}$) the quantum fluctuations of space–time are small, but the quantum fluctuations of the scalar field ϕ may be large. Jumps of the scalar field due to quantum fluctuations lead to a process of eternal self-reproduction of inflationary universe which we are going to discuss later. At even smaller values of $V(\phi)$ (for $m^2 \lesssim V(\phi) \lesssim m,\ 1 \lesssim \phi \lesssim m^{-1/2}$) fluctuations of the field ϕ are small; it slowly moves down as a ball in a viscous liquid. Inflation occurs for $1 \lesssim \phi \lesssim m^{-1}$. Finally, near the minimum of $V(\phi)$ (for $\phi \lesssim 1$) the scalar field rapidly oscillates, creates pairs of elementary particles, and the universe becomes hot

$\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i\phi)^2 + V(\phi) \sim 1$. Let us consider for a moment a theory with $V(\phi) = \text{const}$. This theory is invariant under the *shift symmetry* $\phi \to \phi + c$. Therefore, in such a theory *all* initial values of the homogeneous component of the scalar field ϕ are equally probable.

The only constraint on the amplitude of the field appears if the effective potential is not constant, but grows and becomes greater than the Planck density at $\phi > \phi_{\rm p}$, where $V(\phi_{\rm p}) = 1$. This constraint implies that $\phi \lesssim \phi_{\rm p}$, but there is no reason to expect that initially ϕ must be much smaller than $\phi_{\rm p}$. This suggests that the typical initial value of the field ϕ in such a theory is $\phi \sim \phi_{\rm p}$.

Thus, we expect that typical initial conditions correspond to $\frac{1}{2}\dot{\phi}^2 \sim \frac{1}{2}(\partial_i\phi)^2 \sim V(\phi) = O(1)$. If $\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i\phi)^2 \lesssim V(\phi)$ in the domain under consideration, then inflation begins, and then within the Planck time the terms $\frac{1}{2}\dot{\phi}^2$ and $\frac{1}{2}(\partial_i\phi)^2$ become much smaller than $V(\phi)$, which ensures continuation of inflation. It seems therefore that chaotic inflation occurs under rather natural initial conditions, if it can begin at $V(\phi) \sim 1$ [14, 16].

One can get a different perspective on this issue by studying the probability of quantum creation of the universe from "nothing." The basic idea is that quantum fluctuations can create a small universe from nothing if it can be done quickly, in agreement with the quantum uncertainty relation $\Delta E \cdot \Delta t \lesssim 1$. The total energy of scalar field in a closed inflationary universe is proportional to its minimal volume $H^{-3} \sim V^{-3/2}$ multiplied by the energy density $V(\phi)$: $E \sim V^{-1/2}$. Therefore such a universe can appear quantum mechanically within the time $\Delta t \gtrsim 1$ if $V(\phi)$ is not too much smaller than the Planck density O(1).

This qualitative conclusion agrees with the result of the investigation in the context of quantum cosmology. Indeed, according to [17, 18], the probability of quantum creation of a closed universe is proportional to

$$P \sim \exp\left(-\frac{24\pi^2}{V}\right) , \tag{1.5}$$

which means that the universe can be created if V is not too much smaller than the Planck density. The Euclidean approach to the quantum creation of the universe is based on the analytical continuation of the Euclidean de Sitter solution to the real time. This continuation is possible if $\dot{\phi} = 0$ at the moment of quantum creation of the universe. Thus in the simplest chaotic inflation model with $V(\phi) = \frac{m^2}{2}\phi^2$ the universe is created in a state with $V(\phi) \sim 1$, $\phi \sim m^{-1} \gg 1$ and $\dot{\phi} = 0$, which is a perfect initial condition for inflation in this model [14, 17].

One should note that there are many other attempts to evaluate the probability of initial conditions for inflation (see Chap. 5 in this volume). For example, if one interprets the square of the Hartle–Hawking wave function [19] as a probability of initial condition, one obtains a paradoxical answer $P \sim \exp(\frac{24\pi^2}{V})$, which could seem to imply that it is easier to create the universe with $V \to 0$ and with an infinitely large total energy $E \sim V^{-1/2} \to \infty$. There were many attempts to improve this anti-intuitive answer, but from my perspective these attempts were misplaced: the Hartle–Hawking wave function was derived in [19] as a wave function for the *ground state of the universe*, and therefore it describes the most probable *final* state of the universe, instead of the probability of initial conditions; see a discussion of this issue in [14, 20, 21].

Another recent attempt to study this problem was made by Gibbons and Turok [22]. They studied classical solutions describing a combined evolution of a scalar field and the scale factor of the universe, and imposed "initial conditions" not at the beginning of inflation but at its end. Since one can always reverse the direction of time in the solutions, one can always relate the conditions at the end of inflation to the conditions at its beginning. If one assumes that certain conditions at the end of inflation are equally probable, then one may conclude that the probability of initial conditions suitable for inflation must be very small [22].

From our perspective [23, 24], we have here the same paradox which is encountered in the discussion of the growth of entropy. If one starts with a well ordered system, its entropy will always grow. However, if we make a movie of this process, and play it back starting from the end of the process, then the final conditions for the original system become the initial conditions for the time-reversed system, and we will see the entropy decreasing. That is why replacing initial conditions by final conditions can be very misleading. An advantage of the inflationary regime is that it is an attractor (i.e. the most probable regime) for the family of solutions describing an expanding universe. But if one replaces initial conditions by the final conditions at the end of the process and then studies the same process back in time, the same trajectory will look like a repulsor. This is the main reason of the negative conclusion of [22].

The main problem in [22] is that the methods developed there are valid for the classical evolution of the universe, but the initial conditions for the classical evolution are determined by the processes at the quantum epoch near the singularity, where the methods of [22] are inapplicable. It is not surprising, therefore, that the results of [22] imply that initially $\dot{\phi}^2 \gg V(\phi)$. This result contradicts the results of the Euclidean approach to quantum creation of the universe [17, 18, 19] which require that initially $\dot{\phi} = 0$, see a discussion above.

As we will show in a separate publication [24], if one further develops the methods of [22], but imposes the initial conditions at the beginning of inflation, rather than at its end, one finds that inflation is most probable, in agreement with the arguments given in the first part of this section.

The discussion of initial conditions in this section was limited to the simplest versions of chaotic inflation which allow inflation at the very high energy densities, such as the models with $V \sim \phi^n$. We will return to the discussion of the problem of initial conditions in inflationary cosmology in Sects. 1.13 and 1.14, where we will analyze it in the context of more complicated inflationary models.

1.2.3 Solving the Cosmological Problems

As we will see shortly, the realistic value of the mass m is about 3×10^{-6}, in Planck units. Therefore, according to (1.4), the total amount of inflation achieved starting from $V(\phi) \sim 1$ is of the order $10^{10^{10}}$. The total duration of inflation in this model is about 10^{-30} s. When inflation ends, the scalar field ϕ begins to oscillate near the minimum of $V(\phi)$. As any rapidly oscillating classical field, it looses its energy by creating pairs of elementary particles. These particles interact with each other and come to a state of thermal equilibrium with some temperature $T_{\rm rh}$ [25, 26, 27, 28, 29, 30, 31]. From this time on, the universe can be described by the usual big bang theory.

The main difference between inflationary theory and the old cosmology becomes clear when one calculates the size of a typical inflationary domain at the end of inflation. The investigation of this question shows that even if the initial size of inflationary universe was as small as the Planck size $l_{\rm P} \sim 10^{-33}$ cm, after 10^{-30} s of inflation the universe acquires a huge size of $l \sim 10^{10^{10}}$ cm! This number is model-dependent, but in all realistic models the size of the universe after inflation appears to be many orders of magnitude greater than the size of the part of the universe which we can see now, $l \sim 10^{28}$ cm. This immediately solves most of the problems of the old cosmological theory [14, 15].

Our universe is almost exactly homogeneous on large scales because all inhomogeneities were exponentially stretched during inflation. The density of primordial monopoles and other undesirable "defects" becomes exponentially diluted by inflation. The universe becomes enormously large. Even if it was a closed universe of a size $\sim 10^{-33}$ cm, after inflation the distance between its "South" and "North" poles becomes many orders of magnitude greater than 10^{28} cm. We see only a tiny part of the huge cosmic balloon. That is why nobody has ever seen how parallel lines cross. That is why the universe looks so flat.

If our universe initially consisted of many domains with chaotically distributed scalar field ϕ (or if one considers different universes with different values of the field), then domains in which the scalar field was too small never inflated. The main contribution to the total volume of the universe will be given by those domains which originally contained a large scalar field ϕ. Inflation of such domains creates huge homogeneous islands out of initial chaos. (That is why I called this scenario "chaotic inflation.") Each homogeneous domain in this scenario is much greater than the size of the observable part of the universe.

1.2.4 Chaotic Inflation Versus New Inflation

The first models of chaotic inflation were based on the theories with polynomial potentials, such as $V(\phi) = \pm\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$. But, as was emphasized in [15], the main idea of this scenario is quite generic. One should consider any particular potential $V(\phi)$, polynomial or not, with or without spontaneous symmetry breaking, and study all possible initial conditions without assuming that the universe was in a state of thermal equilibrium, and that the field ϕ was in the minimum of its effective potential from the very beginning.

This scenario strongly deviated from the standard lore of the hot big bang theory and was psychologically difficult to accept. Therefore during the first few years after the invention of chaotic inflation many authors claimed that the idea of chaotic initial conditions is unnatural, and made attempts to realize the new inflation scenario based on the theory of high-temperature phase transitions, despite numerous problems associated with it. Some authors believed that the theory must satisfy the so-called "thermal constraints" which were necessary to ensure that the minimum of the effective potential at large T should be at $\phi = 0$ [32], even though the scalar field in the models they considered was not in a state of thermal equilibrium with other particles.

The issue of thermal initial conditions played the central role in the long debate about new inflation versus chaotic inflation in the 1980s. This debate continued for many years, and a significant part of my book [14] was dedicated to it. By now the debate is over: no realistic versions of new inflation based on the theory of thermal phase transitions and supercooling have been proposed so far. Gradually it became clear that the idea of chaotic initial conditions is most general, and it is much easier to construct a consistent cosmological theory without making unnecessary assumptions about thermal equilibrium and high-temperature phase transitions in the early universe.

As a result, the corresponding terminology changed. Chaotic inflation, as defined in [15], occurs in *all* models with sufficiently flat potentials, including the potentials with a flat maximum, originally used in new inflation [33]. Now the versions of inflationary scenario with such potentials for simplicity are often called "new inflation," even though inflation begins there not as in the original new inflation scenario, but as in the chaotic inflation scenario. To avoid this terminological misunderstanding, some authors call the version of chaotic inflation scenario, where inflation occurs near the top of the scalar potential, a "hilltop inflation" [34].

1.3 Hybrid Inflation

The simplest models of inflation involve just one scalar field. However, in supergravity and string theory there are many different scalar fields, so it does make sense to study models with several different scalar fields, especially if they have some qualitatively new properties. Here we will consider one of these models, hybrid inflation [35].

The simplest version of hybrid inflation describes the theory of two scalar fields with the effective potential

$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2 \ . \qquad (1.6)$$

The effective mass squared of the field σ is equal to $-M^2 + g^2\phi^2$. Therefore for $\phi > \phi_c = M/g$ the only minimum of the effective potential $V(\sigma,\phi)$ is at $\sigma = 0$. The curvature of the effective potential in the σ-direction is much greater than in the ϕ-direction. Thus at the first stages of expansion of the universe the field σ rolled down to $\sigma = 0$, whereas the field ϕ could remain large for a much longer time.

At the moment when the inflaton field ϕ becomes smaller than $\phi_c = M/g$, the phase transition with the symmetry breaking occurs. The fields rapidly fall to the absolute minimum of the potential at $\phi = 0, \sigma^2 = M^2/\lambda$. If $m^2\phi_c^2 = m^2M^2/g^2 \ll M^4/\lambda$, the Hubble constant at the time of the phase transition is given by $H^2 = \frac{M^4}{12\lambda}$ (in units $M_{\rm Pl} = 1$). If $M^2 \gg \frac{\lambda m^2}{g^2}$ and $m^2 \ll H^2$, then the universe at $\phi > \phi_c$ undergoes a stage of inflation, which abruptly ends at $\phi = \phi_c$.

Note that hybrid inflation is also a version of the chaotic inflation scenario: i am unaware of any way to realize this model in the context of the theory of high-temperature phase transitions. The main difference between this scenario and the simplest versions of the one-field chaotic inflation is in the way inflation ends. In the theory with a single field, inflation ends when the potential of this field becomes steep. In hybrid inflation, the structure of the universe depends on the way one of the fields moves, but inflation ends when the potential of the second field becomes steep. This fact allows much greater flexibility of construction of inflationary models. Several extensions of this scenario became quite popular in the context of supergravity and string cosmology, which we will discuss later.

1.4 Quantum Fluctuations and Density Perturbations

The average amplitude of inflationary perturbations generated during a typical time interval H^{-1} is given by [36, 37]

$$|\delta\phi(x)| \approx \frac{H}{2\pi} \ . \tag{1.7}$$

These fluctuations lead to density perturbations that later produce galaxies (see Chap. 6 in this volume). The theory of this effect is very complicated [6, 12], and it was fully understood only in the second part of the 1980s [13]. The main idea can be described as follows.

Fluctuations of the field ϕ lead to a local delay of the time of the end of inflation, $\delta t = \frac{\delta\phi}{\dot\phi} \sim \frac{H}{2\pi\dot\phi}$. Once the usual post-inflationary stage begins, the density of the universe starts to decrease as $\rho = 3H^2$, where $H \sim t^{-1}$. Therefore a local delay of expansion leads to a local density increase δ_H such that $\delta_{\rm H} \sim \delta\rho/\rho \sim \delta t/t$. Combining these estimates together yields the famous result [6, 12, 13]

$$\delta_{\rm H} \sim \frac{\delta\rho}{\rho} \sim \frac{H^2}{2\pi\dot\phi} \ . \tag{1.8}$$

The field ϕ during inflation changes very slowly, so the quantity $\frac{H^2}{2\pi\dot\phi}$ remains almost constant over an exponentially large range of wavelengths. This means that the spectrum of perturbations of the metric is flat.

A detailed calculation in our simplest chaotic inflation model of the amplitude of perturbations gives

$$\delta_{\rm H} \sim \frac{m\phi^2}{5\pi\sqrt{6}} \ . \tag{1.9}$$

The perturbations on the scale of the horizon were produced at $\phi_{\rm H} \sim 15$ [14]. This, together with the COBE normalization $\delta_{\rm H} \sim 2\times10^{-5}$ gives $m \sim 3\times10^{-6}$, in Planck units, which is approximately equivalent to 7×10^{12} GeV. An exact value of m depends on $\phi_{\rm H}$, which in its turn depends slightly on the subsequent thermal history of the universe.

When the fluctuations of the scalar field ϕ are first produced (frozen), their wavelength is given by $H(\phi)^{-1}$. At the end of inflation, the wavelength grows by the factor of $e^{\phi^2/4}$, see (1.4). In other words, the logarithm of the wavelength l of the perturbations of metric is proportional to the value of ϕ^2 at the moment when these perturbations were produced. As a result, according to (1.9), the amplitude of the perturbations of the metric depends logarithmically on the wavelength: $\delta_{\rm H} \sim m \ln l$. A similar logarithmic dependence (with different powers of the logarithm) appears in other versions of chaotic inflation with $V \sim \phi^n$ and in the simplest versions of new inflation.

At first glance, this logarithmic deviation from scale invariance could seem inconsequential, but in a certain sense it is similar to the famous logarithmic dependence of the coupling constants in QCD, where it leads to asymptotic freedom at high energies, instead of simple scaling invariance [38, 39]. In QCD, the slow growth of the coupling constants at small momenta/large distances is responsible for nonperturbative effects resulting in quark confinement. In inflationary theory, the slow growth of the amplitude of perturbations of metric at large distances is equally important. It leads to the existence of the regime of eternal inflation and to the fractal structure of the universe on super-large scales, see Sect. 1.6.

Since the observations provide us with information about a rather limited range of l, it is often possible to parametrize the scale dependence of density perturbations by a simple power law, $\delta_{\rm H} \sim l^{(1-n_{\rm s})/2}$. An exactly flat spectrum, called Harrison–Zeldovich spectrum, would correspond to $n_{\rm s} = 1$.

The amplitude of the scalar perturbations of the metric can be characterized either by $\delta_{\rm H}$, or by a closely related quantity $\Delta_{\mathcal{R}}$ [40]. Similarly, the amplitude of tensor perturbations is given by Δ_h. Following [40, 41], one can represent these quantities as

$$\Delta^2_{\mathcal{R}}(k) = \Delta^2_{\mathcal{R}}(k_0)\left(\frac{k}{k_0}\right)^{n_{\rm s}-1}, \tag{1.10}$$

$$\Delta^2_h(k) = \Delta^2_h(k_0)\left(\frac{k}{k_0}\right)^{n_{\rm t}}, \tag{1.11}$$

where $\Delta^2(k_0)$ is a normalization constant, and k_0 is a normalization point. Here we ignored running of the indexes $n_{\rm s}$ and $n_{\rm t}$ since there is no observational evidence that it is significant.

One can also introduce the tensor/scalar ratio r, the relative amplitude of the tensor to scalar modes,

$$r \equiv \frac{\Delta^2_h(k_0)}{\Delta^2_{\mathcal{R}}(k_0)}. \tag{1.12}$$

There are three slow-roll parameters [40]

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \quad \eta = \frac{V''}{V}, \quad \xi = \frac{V'V'''}{V^2} , \tag{1.13}$$

where prime denotes derivatives with respect to the field ϕ. All parameters must be smaller than one for the slow-roll approximation to be valid.

A standard slow roll analysis gives observable quantities in terms of the slow-roll parameters to first order as

$$\Delta_{\mathcal{R}}^2 = \frac{V}{24\pi^2\epsilon} = \frac{V^3}{12\pi^2(V')^2} , \tag{1.14}$$

$$n_{\mathrm{s}} - 1 = -6\epsilon + 2\eta , \tag{1.15}$$

$$r = 16\epsilon , \tag{1.16}$$

$$n_{\mathrm{t}} = -2\epsilon = -\frac{r}{8} . \tag{1.17}$$

The equation $n_{\mathrm{t}} = -r/8$ is known as the consistency relation for single-field inflation models; it becomes an inequality for multi-field inflation models. If V during inflation is sufficiently large, as in the simplest models of chaotic inflation, one may have a chance to find the tensor contribution to the CMB anisotropy. The possibility to determine n_t is less certain. The most important information which can be obtained now from the cosmological observations at present is related to (1.14) and (1.15).

Following notational conventions in [41], we use $A(k_0)$ for the scalar power spectrum amplitude, where $A(k_0)$ and $\Delta_{\mathcal{R}}^2(k_0)$ are related through

$$\Delta_{\mathcal{R}}^2(k_0) \simeq 3 \times 10^{-9} A(k_0) . \tag{1.18}$$

The parameter A is often normalized at $k_0 \sim 0.05/\mathrm{Mpc}$; its observational value is about 0.8 [41, 42, 43] (see also Chap. 6 in this volume). This leads to the observational constraint on $V(\phi)$ and on r following from the normalization of the spectrum of the large-scale density perturbations:

$$\frac{V^{3/2}}{V'} \simeq 5 \times 10^{-4} . \tag{1.19}$$

Here $V(\phi)$ should be evaluated for the value of the field ϕ which is determined by the condition that the perturbations produced at the moment when the field was equal ϕ evolve into the present time perturbations with momentum $k_0 \sim 0.05/\mathrm{Mpc}$. In the first approximation, one can find the corresponding moment by assuming that it happened 60 e-foldings before the end of inflation. The number of e-foldings can be calculated in the slow roll approximation using the relation

$$N \simeq \int_{\phi_{\mathrm{end}}}^{\phi} \frac{V}{V'} \mathrm{d}\phi . \tag{1.20}$$

Equation (1.19) leads to the relation between r, V and H, in Planck units:

$$r \approx 3 \times 10^7 \; V \approx 10^8 \; H^2 \; . \tag{1.21}$$

Finally, recent observational data suggest [42] that

$$n_{\rm s} = 1 - 3\left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V} = 0.95 \pm 0.016 \; , \tag{1.22}$$

for $r \ll 0.1$. These relations are very useful for comparing inflationary models with observations. In particular, the simplest versions of chaotic and new inflation predict $n_{\rm s} < 1$, whereas in hybrid inflation one may have either $n_{\rm s} < 1$ or $n_{\rm s} > 1$, depending on the model. A more accurate representation of observational constraints can be found in Sect. 1.7.

Until now we have discussed the standard mechanism of generation of perturbations of metric. However, if the model is sufficiently complicated, other mechanisms become possible. For example, one may consider a theory of two scalar fields, ϕ and σ, and assume that inflation was driven by the field ϕ, and the field σ was very light during inflation and did not contribute much to the total energy density. Therefore its quantum fluctuations also did not contribute much to the amplitude of perturbations of metric during inflation (isocurvature perturbations).

After inflation the field ϕ decays. If the products of its decay rapidly loose energy, the field σ may dominate the energy density of the universe and its perturbations suddenly become important. If, in its turn, the field σ decays, its perturbations under certain conditions can be converted into the usual adiabatic perturbations of metric. If this conversion is incomplete, one obtains a theory at odds with recent observational data [44, 45]. On the other hand, if the conversion is complete, one obtains a novel mechanism of generation of adiabatic density perturbations, which is called the curvaton mechanism [46, 47, 48, 49]. A closely related but different mechanism was also proposed in [50]. See Chap. 8 in this volume for a detailed discussion.

These mechanisms are much more complicated than the original one, but one should keep them in mind since they sometimes work in the situations where the standard one does not. Therefore they can give us an additional freedom in finding realistic models of inflationary cosmology.

1.5 Creation of Matter After Inflation: Reheating and Preheating

The theory of reheating of the universe after inflation is the most important application of the quantum theory of particle creation, since almost all matter constituting the universe was created during this process.

At the stage of inflation all energy is concentrated in a classical slowly moving inflaton field ϕ. Soon after the end of inflation this field begins to oscillate near the minimum of its effective potential. Eventually it produces many elementary particles, they interact with each other and come to a state of thermal equilibrium with some temperature T_r.

Early discussions of reheating of the universe after inflation [25] were based on the idea that the homogeneous inflaton field can be represented as a collection of the particles of the field ϕ. Each of these particles decayed independently. This process can be studied by the usual perturbative approach to particle decay. Typically, it takes thousands of oscillations of the inflaton field until it decays into usual elementary particles by this mechanism. More recently, however, it was discovered that coherent field effects such as parametric resonance can lead to the decay of the homogeneous field much faster than would have been predicted by perturbative methods, within a few dozen oscillations [26]. These coherent effects produce high energy, nonthermal fluctuations that could have significance for understanding developments in the early universe, such as baryogenesis. This early stage of rapid nonperturbative decay was called "preheating." In [27] it was found that another effect known as tachyonic preheating can lead to even faster decay than parametric resonance. This effect occurs whenever the homogeneous field rolls down a tachyonic ($V'' < 0$) region of its potential. When that occurs, a tachyonic, or spinodal instability leads to exponentially rapid growth of all long wavelength modes with $k^2 < |V''|$. This growth can often drain all of the energy from the homogeneous field within a single oscillation.

We are now in a position to classify the dominant mechanisms by which the homogeneous inflaton field decays in different classes of inflationary models. Even though all of these models, strictly speaking, belong to the general class of chaotic inflation (none of them is based on the theory of thermal initial conditions), one can break them into three classes: small field, or new inflation models [10], large field, or chaotic inflation models of the type of the model $m^2\phi^2/2$ [15], and multi-field, or hybrid models [35]. This classification is incomplete, but still rather helpful.

In the simplest versions of chaotic inflation, the stage of preheating is generally dominated by parametric resonance, although there are parameter ranges where this cannot occur [26]. In [27], it was shown that tachyonic preheating dominates the preheating phase in hybrid models of inflation. New inflation in this respect occupies an intermediate position between chaotic inflation and hybrid inflation: If spontaneous symmetry breaking in this scenario is very large, reheating occurs due to parametric resonance and perturbative decay. However, for the models with spontaneous symmetry breaking at or below the GUT scale, $\phi \ll 10^{-2} M_{\rm Pl}$, preheating occurs due to a combination of tachyonic preheating and parametric resonance. The resulting effect is very strong, so that the homogeneous mode of the inflaton field typically decays within few oscillations [28].

A detailed investigation of preheating usually requires lattice simulations, which can be achieved following [29, 30]. Note that preheating is not the last stage of reheating; it is followed by a period of turbulence [31], by a much slower perturbative decay described by the methods developed in [25], and by eventual thermalization.

1.6 Eternal Inflation

A significant step in the development of inflationary theory was the discovery of the process of self-reproduction of inflationary universe. This process was known to exist in old inflationary theory [7] and in the new one [51, 52, 53], but its significance was fully realized only after the discovery of the regime of eternal inflation in the simplest versions of the chaotic inflation scenario [54, 55]. It appears that in many inflationary models large quantum fluctuations produced during inflation may significantly increase the value of the energy density in some parts of the universe. These regions expand at a greater rate than their parent domains, and quantum fluctuations inside them lead to production of new inflationary domains which expand even faster. This leads to an eternal process of self-reproduction of the universe.

To understand the mechanism of self-reproduction one should remember that processes separated by distances l greater than H^{-1} proceed independently of one another. This is so because during exponential expansion the distance between any two objects separated by more than H^{-1} is growing with a speed exceeding the speed of light. As a result, an observer in the inflationary universe can see only the processes occurring inside the horizon of the radius H^{-1}. An important consequence of this general result is that the process of inflation in any spatial domain of radius H^{-1} occurs independently of any events outside it. In this sense any inflationary domain of initial radius exceeding H^{-1} can be considered as a separate mini-universe.

To investigate the behavior of such a mini-universe, with an account taken of quantum fluctuations, let us consider an inflationary domain of initial radius H^{-1} containing sufficiently homogeneous field with initial value $\phi \gg M_{\rm Pl}$. Equation (1.3) implies that during a typical time interval $\Delta t = H^{-1}$ the field inside this domain will be reduced by $\Delta\phi = \frac{2}{\phi}$. By comparison this expression with $|\delta\phi(x)| \approx \frac{H}{2\pi} = \frac{m\phi}{2\pi\sqrt{6}}$ one can easily see that if ϕ is much less than $\phi^* \sim \frac{5}{\sqrt{m}}$, then the decrease of the field ϕ due to its classical motion is much greater than the average amplitude of the quantum fluctuations $\delta\phi$ generated during the same time. But for $\phi \gg \phi^*$ one has $\delta\phi(x) \gg \Delta\phi$. Because the typical wavelength of the fluctuations $\delta\phi(x)$ generated during the time is H^{-1}, the whole domain after $\Delta t = H^{-1}$ effectively becomes divided into $e^3 \sim 20$ separate domains (mini-universes) of radius H^{-1}, each containing almost homogeneous field $\phi - \Delta\phi + \delta\phi$. In almost a half of these domains the field ϕ grows by $|\delta\phi(x)| - \Delta\phi \approx |\delta\phi(x)| = H/2\pi$, rather than decreases.

This means that the total volume of the universe containing the *growing* field ϕ increases 10 times. During the next time interval $\Delta t = H^{-1}$ this process repeats itself. Thus, after the two time intervals H^{-1} the total volume of the universe containing the growing scalar field increases 100 times, etc. The universe enters the eternal process of self-reproduction.

The existence of this process implies that the universe will never disappear as a whole. Some of its parts may collapse, the life in our part of the universe may perish, but there always will be some other parts of the universe where life will appear again and again, in all of its possible forms.

One should be careful, however, with the interpretation of these results. There is still an ongoing debate of whether eternal inflation is eternal only in the future or also in the past. In order to understand what is going on, let us consider any particular time-like geodesic line at the stage of inflation. One can show that for any given observer following this geodesic, the duration $t_{\rm i}$ of the stage of inflation on this geodesic will be finite. One the other hand, eternal inflation implies that if one takes all such geodesics and calculate the time $t_{\rm i}$ for each of them, then there will be no upper bound for $t_{\rm i}$, i.e. for each time T there will exist geodesics which experience inflation for a time $t_{\rm i} > T$. Even though the relative number of long geodesics can be very small, exponential expansion of space surrounding them will lead to an eternal exponential growth of the total volume of the inflationary parts of the universe.

Similarly, if one concentrates on any particular geodesic in the past time direction, one can prove that it has finite length [56], i.e. inflation in any particular point of the universe should have a beginning at some time τ_i. However, there is no reason to expect that there is an upper bound for all τ_i on all geodesics. If this upper bound does not exist, then eternal inflation is eternal not only in the future but also in the past.

In other words, there was a beginning for each part of the universe, and there will be an end for inflation at any particular point. But there will be no end for the evolution of the universe *as a whole* in the eternal inflation scenario, and at present we do not have any reason to believe that there was a single beginning of the evolution of the whole universe at some moment $t = 0$, which was traditionally associated with the big bang.

To illustrate the process of eternal inflation, we present here the results of computer simulations of evolution of a system of two scalar fields during inflation. The field ϕ is the inflaton field driving inflation; it is shown by the height of the distribution of the field $\phi(x, y)$ in a two-dimensional slice of the universe. The second field, Φ, determines the type of spontaneous symmetry breaking which may occur in the theory. We paint the surface in red, green or blue corresponding to three different minima of the potential of the field Φ. Different colors correspond to different types of spontaneous symmetry breaking, and therefore to different sets of laws of low-energy physics in different exponentially large parts of the universe.

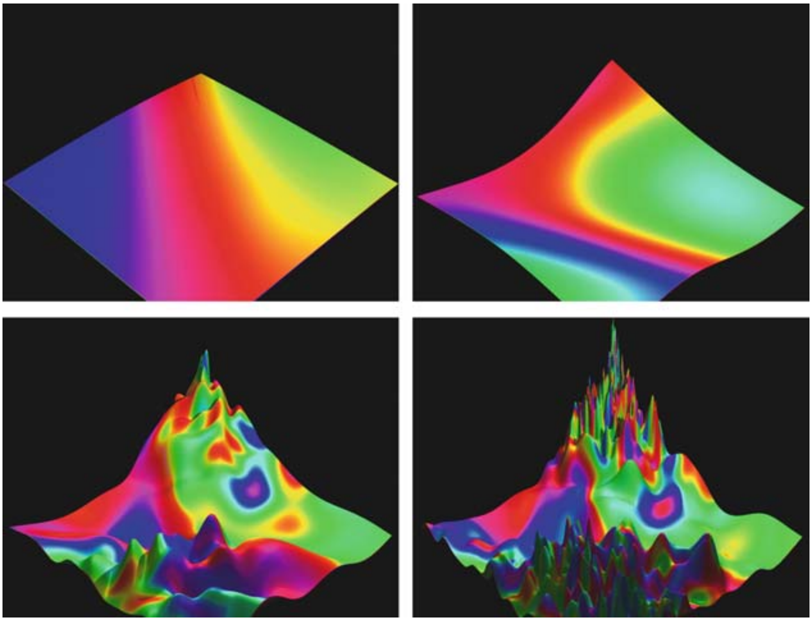

Fig. 1.2. Evolution of scalar fields ϕ and Φ during the process of self-reproduction of the universe. The height of the distribution shows the value of the field ϕ which drives inflation. The surface is painted in red (medium), green (dark) or blue (light) corresponding to three different minima of the potential of the field Φ. The laws of low-energy physics are different in the regions of different color. The peaks of the "mountains" correspond to places where quantum fluctuations bring the scalar fields back to the Planck density. Each of such places in a certain sense can be considered as a beginning of a new big bang

In the beginning of the process the whole inflationary domain is red, and the distribution of both fields is very homogeneous. Then the domain became exponentially large (but it has the same size in comoving coordinates, as shown in Fig. 1.2). Each peak of the mountains corresponds to nearly Planckian density and can be interpreted as a beginning of a new "big bang." The laws of physics are rapidly changing there, as indicated by changing colors, but they become fixed in the parts of the universe where the field ϕ becomes small. These parts correspond to valleys in Fig. 1.2. Thus quantum fluctuations of the scalar fields divide the universe into exponentially large domains with different laws of low-energy physics, and with different values of energy density.

Eternal inflation scenario was extensively studied during the last 20 years. I should mention, in particular, the discovery of the topological eternal inflation [57] and the calculation of the fractal dimension of the universe [58, 55]. The most interesting consequences of the theory of eternal inflation are related to the theory of inflationary multiverse and string theory landscape. We will discuss these subjects in Sect. 1.14.

1.7 Inflation and Observations

Inflation is not just an interesting theory that can resolve many difficult problems of the standard big bang cosmology. This theory made several predictions which can be tested by cosmological observations. Here are the most important predictions:

(1) The universe must be flat. In most models $\Omega_{\rm total} = 1 \pm 10^{-4}$.
(2) Perturbations of the metric produced during inflation are adiabatic.
(3) Inflationary perturbations have a nearly flat spectrum. In most inflationary models the spectral index $n_{\rm s} = 1 \pm 0.2$ ($n_{\rm s} = 1$ means totally flat).
(4) The spectrum of inflationary perturbations should be slightly non-flat. (It is very difficult to construct a model with $n_{\rm s} = 1$.)
(5) These perturbations are gaussian.
(6) Perturbations of the metric could be scalar, vector or tensor. Inflation mostly produces scalar perturbations, but it also produces tensor perturbations with a nearly flat spectrum, and it does *not* produce vector perturbations. There are certain relations between the properties of scalar and tensor perturbations produced by inflation.
(7) Inflationary perturbations produce specific peaks in the spectrum of CMB radiation. (For a simple pedagogical interpretation of this effect see e.g. [59]; a detailed theoretical description can be found in [60].)

It is possible to violate each of these predictions if one makes the inflationary theory sufficiently complicated. For example, it is possible to produce vector perturbations of the metric in the models where cosmic strings are produced at the end of inflation, which is the case in some versions of hybrid inflation. It is possible to have an open or closed inflationary universe, or even a small periodic inflationary universe, it is possible to have models with non-gaussian isocurvature fluctuations with a non-flat spectrum. However, it is difficult to do so, and most of the inflationary models obey the simple rules given above.

It is not easy to test all of these predictions. The major breakthrough in this direction was achieved due to the recent measurements of the CMB anisotropy. The latest results based on the WMAP experiment, in combination with the Sloan Digital Sky Survey, are consistent with predictions of the simplest inflationary models with adiabatic gaussian perturbations, with $\Omega = 1.003 \pm 0.01$, and $n_{\rm s} = 0.95 \pm 0.016$ [42].

There are still some question marks to be examined, such as an unexpectedly small anisotropy of the CMB at large angles [41, 61] and possible correlations between low multipoles; for a recent discussion see e.g. [62, 63] and references therein (Fig. 1.3).

The observational status and interpretation of these effects is still uncertain, but if one takes these effects seriously, one may try to look for some theoretical explanations. For example, there are several ways to suppress the large angle anisotropy, see e.g. [64]. The situation with correlations between

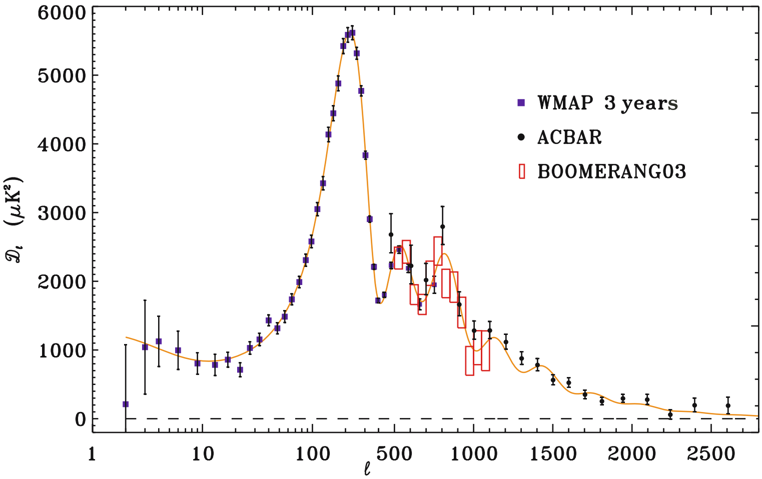

Fig. 1.3. CMB data (WMAP3, BOOMERANG03, ACBAR) versus the predictions of one of the simplest inflationary models with $\Omega = 1$ (*solid red line*), according to [43]

low multipoles requires more work. In particular, it would be interesting to study effects related to relatively light domain walls [65, 66, 67]. Another possibility is to analyze the possible effects on the CMB anisotropy which can be produced by the cosmic web structure of the perturbations in the curvaton scenario [46]. Some other possibilities are mentioned in [63]. One way or another, it is quite significant that all proposed explanations of these anomalies are based on inflationary cosmology.

One of the interesting issues to be probed by future observations is the possible existence of gravitational waves produced during inflation. The present upper bound on the tensor to scalar ratio r is not very strict, $r \lesssim 0.3$. However, new observations may either find the tensor modes or push the bound on r much further, towards $r \lesssim 10^{-2}$ or even $r \lesssim 10^{-3}$.

In the simplest monomial versions of chaotic inflation with $V \sim \phi^n$ one find the following (approximate) result: $r = 4n/N$. Here N is the number of e-folds of inflation corresponding to the wavelength equal to the present size of the observable part of our universe; typically N can be in the range of 50–60; its value depends on the mechanism of reheating. For the simplest model with $n = 2$ and $N \sim 60$ one has $r \sim 0.13 - 0.14$. On the other hand, for most of the other models, including the original version of new inflation, hybrid inflation, and many versions of string theory inflation, r is extremely small, which makes the observation of gravitational waves in such models very difficult.

One may wonder whether there are any sufficiently simple and natural models with intermediate values of r? This is an important question for those who are planning a new generation of CMB experiments. The answer to this question is positive: In the versions of chaotic inflation with potentials like

$\pm m^2\phi^2 + \lambda\phi^4$, as well as in the natural inflation scenario, one can easily obtain any value of r from 0.3 to 10^{-2}. I will illustrate it with two figures. The first one shows the graph of possible values of n_s and r in the standard symmetry breaking model with the potential

$$V = -m^2\phi^2/2 + \lambda\phi^4/4 + m^4/4\lambda = \frac{\lambda}{4}(\phi^2 - v^2)^2 \ , \qquad (1.23)$$

where $v = m/\sqrt{\lambda}$ is the amplitude of spontaneous symmetry breaking.

If v is very large, $v \gtrsim 10^2$, inflation occurs near the minimum of the potential, and all properties of inflation are the same as in the simplest chaotic inflation model with quadratic potential $m^2\phi^2$. If $v \ll 10$, inflation occurs as in the theory $\lambda\phi^4/4$, which leads to $r \sim 0.28$. If v takes some intermediate values, such as $v = \mathrm{O}(10)$, then two different inflationary regimes are possible in this model: at large ϕ and at small ϕ. In the first case r interpolates between its value in the theory $\lambda\phi^4/4$ and the theory $m^2\phi^2$ (i.e. between 0.28 and 0.14). In the second case, r can take any value from 0.14 to 10^{-2}, see Fig. 1.4 [68, 69].

If one considers chaotic inflation with the potential including terms ϕ^2, ϕ^3 and ϕ^4, one can considerably alter the properties of inflationary perturbations [70]. Depending on the values of parameters, initial conditions and the required number of e-foldings N, this relatively simple class of models covers almost all parts of the area in the (r, n_s) plane allowed by the latest observational data [71], see Fig. 1.5.

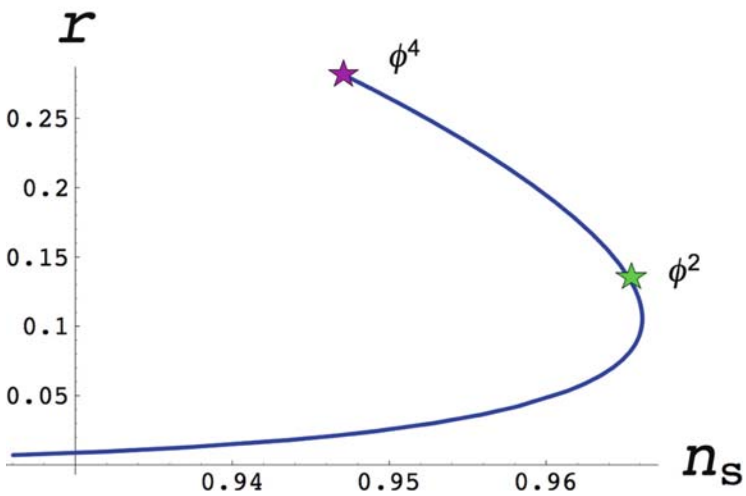

Fig. 1.4. Possible values of r and n_s in the theory $\frac{\lambda}{4}(\phi^2 - v^2)^2$ for different initial conditions and different v, for $N = 60$. In the small v limit, the model has the same predictions as the theory $\lambda\phi^4/4$. In the large v limit it has the same predictions as the theory $m^2\phi^2$. The upper branch, above the first star from below (marked as ϕ^2), corresponds to inflation which occurs while the field rolls down from large ϕ; the lower branch corresponds to the motion from $\phi = 0$

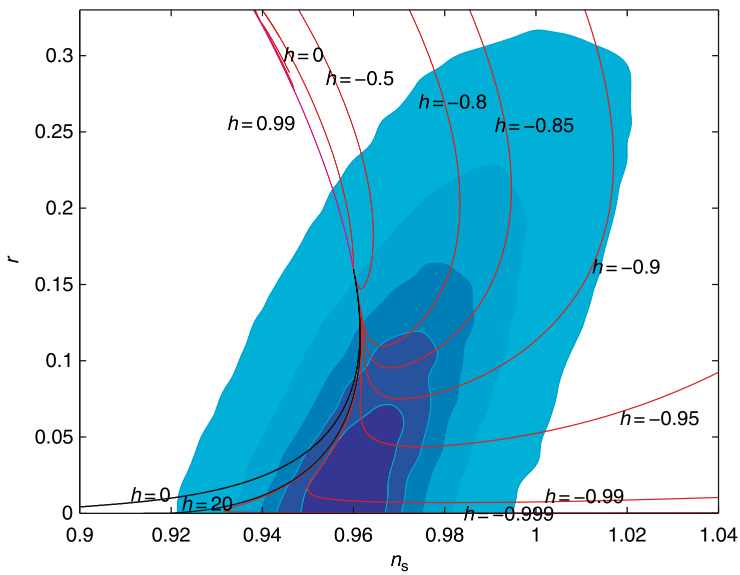

Fig. 1.5. Possible values of r and $n_{\rm s}$ for chaotic inflation with a potential including terms ϕ^2, ϕ^3 and ϕ^4 for N = 50, according to [71]. The color-filled areas correspond to 12%, 27%, 45%, 68% and 95% confidence levels according to the WMAP3 and SDSS data

Note that for all versions of the model shown in Figs. 1.4 and 1.5 the range of the cosmological evolution of the fields is $\Delta\phi > 1$, so formally these models can be called the large field models. And yet they have dramatically different properties, which do not fit into the often-used scheme dividing all models into small field models, large field models and hybrid inflation models.

1.8 Alternatives to Inflation?

The inflationary scenario is very versatile, and now, after 25 years of persistent attempts of many physicists to propose an alternative to inflation, we still do not know any other way to construct a consistent cosmological theory. Indeed, in order to compete with inflation a new theory should make similar predictions and should offer an alternative solution to many difficult cosmological problems. Let us look at these problems before starting a discussion.

(1) The homogeneity problem. Before even starting an investigation of density perturbations and structure formation, one should explain why the universe is nearly homogeneous on the horizon scale.
(2) The isotropy problem. We need to understand why all directions in the universe are similar to each other, why there is no overall rotation of the universe, etc...

(3) The horizon problem. This one is closely related to the homogeneity problem. If different parts of the universe have not been in a causal contact when the universe was born, why do they look so similar?
(4) The flatness problem. Why $\Omega \approx 1$? Why parallel lines do not intersect?
(5) The total entropy problem. The total entropy of the observable part of the universe is greater than 10^{87}. Where did this huge number come from? Note that the lifetime of a closed universe filled with hot gas with total entropy S is $S^{2/3} \times 10^{-43}$ s [14]. Thus S must be huge. Why?
(6) The total mass problem. The total mass of the observable part of the universe has mass $\sim 10^{60} M_{\rm Pl}$. Note also that the lifetime of a closed universe filled with nonrelativistic particles of total mass M is $\frac{M}{M_{\rm P}} \times 10^{-43}$ s. Thus M must be huge. But why?
(7) The structure formation problem. If we manage to explain the homogeneity of the universe, how can we explain the origin of inhomogeneities required for the large scale structure formation?
(8) The monopole problem, gravitino problem, etc.

This list is very long. That is why it was not easy to propose any alternative to inflation even before we learned that $\Omega \approx 1$, $n_{\rm s} \approx 1$, and that the perturbations responsible for galaxy formation are mostly adiabatic, in agreement with the predictions of the simplest inflationary models.

There were many attempts to propose an alternative to inflation in recent years. In general, this could be a very healthy tendency. If one of these attempts will succeed, it will be of great importance. If none of them are successful, it will be an additional demonstration of the advantages of inflationary cosmology. However, since the stakes are high, we are witnessing a growing number of premature announcements of success in developing an alternative cosmological theory (see Chap. 11 in this volume for an alternative discussion).

1.8.1 Cosmic Strings and Textures

Fifteen years ago the models of structure formation due to topological defects or textures were advertised in popular press as the models that "match the explanatory triumphs of inflation while rectifying its major failings" [72]. However, it was clear from the very beginning that these theories at best could solve only one problem (structure formation) out of the eight problems mentioned above. The true question was not whether one can replace inflation by the theory of cosmic strings/textures, but whether inflation with cosmic strings/textures is better than inflation without cosmic strings/textures. Recent observational data favor the simplest version of inflationary theory, without topological defects, or with an extremely small (few percent) admixture of the effects due to cosmic strings.

1.8.2 Pre-big Bang

An attempt to avoid the use of the standard inflationary mechanism (though still use a stage of inflation prior to the big bang) was made in the pre-big bang scenario [73]. This scenario is based on the assumption that eventually one will find a solution of the cosmological singularity problem and learn how one could transfer small perturbations of the metric through the singularity. This problem still remains unsolved, see e.g. [74]. Moreover, a detailed investigation of the homogeneity, isotropy and flatness problems in the pre-big bang scenario demonstrated that the stage of the pre-big bang inflation introduced in [73] is insufficient to solve the major cosmological problems [75].

1.8.3 Ekpyrotic/Cyclic Scenario

A similar situation emerged with the introduction of the ekpyrotic scenario [76]. The original version of this theory claimed that this scenario can solve all cosmological problems without using the stage of inflation, i.e. without a prolonged stage of an accelerated expansion of the universe, which was called in [76] "superluminal expansion." However, the original ekpyrotic scenario contained many significant errors and did not work. It is sufficient to say that instead of the big bang expected in [76], there was a big crunch [77, 78].

The ekpyrotic scenario was replaced by the cyclic scenario, which used an infinite number of periods of expansion and contraction of the universe [79]. The origin of the required scalar field potential in this model remains unclear, and the very existence of the cycles postulated in [79] have not been demonstrated. When we analyzed this scenario using the particular potential given in [79], and took into account the effect of particle production in the early universe, we found a very different cosmological regime [80, 81].

The original version of the cyclic scenario relied on the existence of an infinite number of very long stages of "superluminal expansion," i.e. inflation, in order to solve the major cosmological problems. In this sense, the original version of the cyclic scenario was not a true alternative to inflationary scenario, but its rather peculiar version. The main difference between the usual inflation and the cyclic inflation, just as in the case of topological defects and textures, was the mechanism of generation of density perturbations. However, since the theory of density perturbations in cyclic inflation requires a solution of the cosmological singularity problem [82, 83], it is difficult to say anything definite about it.

Most of the authors believe that even if the singularity problem were solved, the spectrum of perturbations in the standard version of this scenario involving only one scalar field after the singularity would be very non-flat. One may introduce more complicated versions of this scenario, involving many scalar fields. In this case, under certain assumptions about the way the universe passes through the singularity, one may find a special regime where isocurvature perturbations in one of these fields are converted into adiabatic perturbations with a nearly flat spectrum. A recent discussion of this scenario shows that this regime requires an extreme fine-tuning of initial conditions [84]. Moreover, the instability of the solutions in this regime, which was found in [84], implies that it may be very easy to switch from one regime to another under the influence of small perturbations. This may lead to a domain-like structure of the universe and large perturbations of the metric [85]. If this is the case, no fine-tuning of initial conditions could help.

One of the latest versions of the cyclic scenario attempted to avoid the long stage of accelerated expansion (low-scale inflation) and to make the universe homogeneous using some specific features of the ekpyrotic collapse [86]. The authors assumed that the universe was homogeneous prior to its collapse on the scale that becomes greater than the scale of the observable part of the universe during the next cycle. Under this assumption, they argued that the perturbations of metric produced during each subsequent cycle do not interfere with the perturbations of metric produced in the next cycle. As a result, if the universe has been homogeneous from the very beginning, it remains homogeneous on the cosmologically interesting scales in all subsequent cycles.

Is this a real solution of the homogeneity problem? The initial size of the part of the universe, which is required to be homogeneous in this scenario prior to the collapse, was many orders of magnitude greater than the Planck scale. How homogeneous should it be? If we want the inhomogeneities to be produced due to amplification of quantum perturbations, then the initial classical perturbations of the field responsible for the isocurvature perturbations must be incredibly small, smaller than its quantum fluctuations. Otherwise the initial classical inhomogeneities of this field will be amplified by the same processes that amplified its quantum fluctuations and will dominate the spectrum of perturbations after the bounce [77]. This problem is closely related to the problem mentioned above [84, 85].

Recently there was an attempt to revive the original (non-cyclic) version of the ekpyrotic scenario by involving a nonsingular bounce. This regime requires violating the null energy condition [78], which usually leads to a catastrophic vacuum instability and/or causality violation. One may hope to avoid these problems in the ghost condensate theory [87]; see a series of recent papers on this subject [88, 89, 90]. However, even the authors of the ghost condensate theory emphasize that a fully consistent version of this theory is yet to be constructed [91], and that it may be incompatible with basic gravitational principles [92].

In addition, just as the ekpyrotic scenario with the singularity [84], the new version of the ekpyrotic theory requires two fields, and a conversion of the isocurvature perturbations to adiabatic perturbations [93]. Once again, the initial state of the universe in this scenario must be extremely homogeneous: the initial classical perturbations of the field responsible for the isocurvature perturbations must be smaller than its quantum fluctuations. It does not seem possible to solve this problem without further extending this exotic model and making it a part of an even more complicated scenario.

1.8.4 String Gas Scenario

Another attempt to solve some of the cosmological problems without using inflation has been proposed by Brandenberger et al. in the context of string gas cosmology [94, 95]. The authors admitted that their model did not solve the flatness problem, so it was not a real alternative to inflation. However, they claimed that their model provided a non-inflationary mechanism of production of metric perturbations with a flat spectrum.

It would be quite interesting and important to have a new mechanism of generation of metric perturbations based on string theory. Unfortunately, a detailed analysis of the scenario proposed in [94, 95] revealed that some of its essential ingredients were either unproven or incorrect [96]. For example, the theory of generation of metric perturbations used in [94] was formulated in the Einstein frame, where the usual Einstein equations are valid. On the other hand, the bounce and the string gas cosmology were described in string frame. Then both of these results were combined without distinguishing between different frames and a proper translation from one frame to another.

If one makes all calculations carefully (ignoring other unsolved problems of this scenario), one finds that the perturbations generated in their scenario have a blue spectrum with $n = 5$, which is ruled out by cosmological observations [96]. After the conference "Inflation + 25" where this issue was actively debated, the authors of [94, 95] issued two new papers reiterating their claims [97, 98], but eventually they agreed with our conclusion expressed at this conference: the spectrum of perturbations of metric in this scenario is blue, with $n = 5$, see (43) of [99]. This rules out the models proposed in [94, 95, 97, 98]. Nevertheless, as often happens with various alternatives to inflation, some of the authors of [94, 95, 97, 98] still claim that their basic scenario remains intact and propose its further modifications [99, 100, 101].

1.8.5 Mirage Bounce

Paradoxes with the choice of frames appear in other works on bounces in cosmology as well. For example, in [102] it was claimed that one can solve all cosmological problems in the context of mirage cosmology. However, as explained in [103], in the Einstein frame in this scenario the universe does not evolve at all.

To clarify the situation without going to technical details, one may consider the following analogy. We know that all particles in our body get their masses due to spontaneous symmetry breaking in the standard model. Suppose that the Higgs field initially was out of the minimum of its potential, and experienced oscillations. During these oscillations the masses of electrons and protons also oscillated. If one measures the size of the universe in units of the (time-dependent) Compton wavelengths of the electron (which could seem to be a good idea), one would think that the scale factor of the universe oscillates (bounces) with the frequency equal to the Higgs boson mass. And yet, this "cosmological evolution" with bounces of the scale factor is an illusion, which disappears if one measures the distances in units of the Planck length M_{p}^{-1} (the Einstein frame).

In addition, the mechanism of generation of density perturbations used in [102] was borrowed from the paper by Hollands and Wald [104], who suggested yet another alternative mechanism of generation of metric perturbations. However, this mechanism requires investigating thermal processes at the density 90 orders of magnitude greater than the Planck density, which makes all calculations unreliable [23].

1.8.6 Bounce in Quantum Cosmology

Finally, I should mention [105], where it was argued that under certain conditions one can have a bouncing universe and produce metric perturbations with a flat spectrum in the context of quantum cosmology. However, the model of [105] does not solve the flatness and homogeneity problems. A more detailed analysis revealed that the wave function of the universe proposed in [105] makes the probability of a bounce of a large universe exponentially small [106]. The authors are working on a modification of their model, which, as they hope, will not suffer from this problem.

To conclude, at the moment it is hard to see any real alternative to inflationary cosmology, despite an active search for such alternatives. All of the proposed alternatives are based on various attempts to solve the singularity problem: one should either construct a bouncing nonsingular cosmological solution, or learn what happens to the universe when it goes through the singularity. This problem bothered cosmologists for nearly a century, so it would be great to find its solution, quite independently of the possibility to find an alternative to inflation. None of the proposed alternatives can be consistently formulated until this problem is solved.

In this respect, inflationary theory has a very important advantage: it works practically independently of the solution of the singularity problem. It can work equally well after the singularity, or after the bounce, or after the quantum creation of the universe. This fact is especially clear in the eternal inflation scenario: eternal inflation makes the processes which occurred near the big bang practically irrelevant for the subsequent evolution of the universe.

1.9 Naturalness of Chaotic Inflation

Now we will return to the discussion of various versions of inflationary theory. Most of them are based on the idea of chaotic initial conditions, which is the trademark of the chaotic inflation scenario. In the simplest versions of chaotic inflation scenario with the potentials $V \sim \phi^n$, the process of inflation occurs at $\phi > 1$, in Planck units. Meanwhile, there are many other models where inflation may occur at $\phi \ll 1$.

There are several reasons why this difference may be important. First of all, some authors argue that the generic expression for the effective potential can be cast in the form

$$V(\phi) = V_0 + \alpha\phi + \frac{m^2}{2}\phi^2 + \frac{\beta}{3}\phi^3 + \frac{\lambda}{4}\phi^4 + \sum_n \lambda_n \frac{\phi^{4+n}}{{M_{\rm Pl}}^n} \,, \tag{1.24}$$

and then they assume that generically $\lambda_n = {\rm O}(1)$, see e.g. (128) in [107]. If this assumption were correct, one would have little control over the behavior of $V(\phi)$ at $\phi > M_{\rm Pl}$.

Here we have written $M_{\rm Pl}$ explicitly, to expose the implicit assumption made in [107]. Why do we write $M_{\rm Pl}$ in the denominator, instead of $1000 M_{\rm Pl}$? An intuitive reason is that quantum gravity is non-renormalizable, so one should introduce a cut-off at momenta $k \sim M_{\rm Pl}$. This is a reasonable assumption, but it does not imply the validity of (1.24). Indeed, the constant part of the scalar field appears in the gravitational diagrams not directly, but only via its effective potential $V(\phi)$ and the masses of particles interacting with the scalar field ϕ. As a result, the terms induced by quantum gravity effects are suppressed not by factors $\frac{\phi^n}{{M_{\rm Pl}}^n}$, but by factors $\frac{V}{{M_{\rm Pl}}^4}$ and $\frac{m^2(\phi)}{{M_{\rm Pl}}^2}$ [14]. Consequently, quantum gravity corrections to $V(\phi)$ become large not at $\phi > M_{\rm Pl}$, as one could infer from (1.24), but only at super-Planckian energy density, or for super-Planckian masses. This justifies our use of the simplest chaotic inflation models.

The simplest way to understand this argument is to consider the case where the potential of the field ϕ is a constant, $V = V_0$. Then the theory has a *shift symmetry*, $\phi \to \phi + c$. This symmetry is not broken by perturbative quantum gravity corrections, so no such terms as $\sum_n \lambda_n \frac{\phi^{4+n}}{{M_{\rm Pl}}^n}$ are generated. This symmetry may be broken by nonperturbative quantum gravity effects (wormholes? virtual black holes?), but such effects, even if they exist, can be made exponentially small [108].

On the other hand, one may still wonder whether there is any reason not to add terms like $\lambda_n \frac{\phi^{4+n}}{{M_{\rm Pl}}^n}$ with $\lambda = {\rm O}(1)$ to the theory. Here I will make a simple argument which may help to explain it. I am not sure whether this argument should be taken too seriously, but I find it quite amusing and unexpected.

Let us consider a theory with the potential

$$V(\phi) = V_0 + \alpha\phi + \frac{m^2}{2}\phi^2 + \lambda_n \frac{\phi^{4+n}}{{M_{\rm Pl}}^n} + \frac{\xi}{2}R\phi^2 \,. \tag{1.25}$$

The last term is added to increase the generality of our discussion by considering fields non-minimally coupled to gravity, including the conformal fields with $\xi = 1/6$.

Suppose first that $m^2 = \lambda_n = 0$. Then the theory can describe our ground state with a slowly changing vacuum energy only if $V_0 + \alpha\phi < 10^{-120}$, $\alpha < 10^{-120}$ [109]. This theory cannot describe inflation because α is too small to produce the required density perturbations.

Let us now add the quadratic term. Without loss of generality one can make a redefinition of the field ϕ and V_0 to remove the linear term:

$$V(\phi) = V_0 + \frac{m^2}{2}\phi^2 \,. \tag{1.26}$$

This is the simplest version of chaotic inflation. The maximal value of the field ϕ in this scenario is given by the condition $\frac{m^2}{2}\phi^2 \sim 1$ (Planckian density), so the maximal amount of inflation in this model is $\sim e^{\phi^2/4} \sim e^{1/m^2}$.

If, instead, we considered a more general case with the three terms $\frac{m^2}{2}\phi^2 + \lambda_n \frac{\phi^{4+n}}{M_{\rm Pl}{}^n} + \frac{\xi}{2}R\phi^2$, the maximal amount of inflation would be

$$N < \exp\left[\min\{m^{-2}, \lambda_n^{-2/n}, \xi^{-1}\}\right] . \tag{1.27}$$

The last constraint appears because the effective gravitational constant becomes singular at $\phi^2 \sim \xi^{-1}$.

Thus, if any of the constants $\lambda_n^{2/n}$ or ξ is greater than m^2, the total amount of inflation will be exponentially smaller than in the simplest theory $\frac{m^2}{2}\phi^2$. Therefore one could argue that if one has a possibility to choose between different inflationary theories, as in the string theory landscape, then the largest fraction of the volume of the universe will be in the parts of the multiverse with $\lambda_n^{2/n}, \xi \ll m^2$. One can easily check that for $\lambda_n^{2/n}, \xi \lesssim m^2$ the higher order terms can be ignored at the last stages of inflation, where $\phi = \mathrm{O}(1)$. In other words, the theory behaves as purely quadratic during the last stages of inflation when the observable part of the universe was formed.

One can come to the same conclusion if one takes into account only the part of inflation at smaller values of the field ϕ, when the stage of eternal inflation is over. This suggests that the simplest version of chaotic inflation scenario is the best.

Of course, this is just an argument. Our main goal here was not to promote the model $\frac{m^2}{2}\phi^2$, but to demonstrate that the considerations of naturalness (e.g. an assumption that all λ_n should be large) depend quite crucially on the underlying assumptions. In the example given above, a very simple change of these assumptions (the emphasis on the total volume of the post-inflationary universe) was sufficient to explain the naturalness of the simplest model $\frac{m^2}{2}\phi^2$. However, the situation may become quite different if instead of the simplest theory of a scalar field combined with general relativity one starts to investigate more complicated models, such as supergravity and string theory.

1.10 Chaotic Inflation in Supergravity

In the simplest models of inflation, the field ϕ itself does not have any direct physical meaning; everything depends only on its functions such as the masses of particles and the scalar potential. However, in more complicated theories the scalar field ϕ itself may have a physical (geometrical) meaning, which may constrain the possible values of the fields during inflation. The most important example is given by $N = 1$ supergravity.

The F-term potential of the complex scalar field Φ in supergravity is given by the well-known expression (in units $M_{\rm Pl} = 1$):

$$V = \mathrm{e}^K \left[K^{-1}_{\Phi\bar{\Phi}} |D_\Phi W|^2 - 3|W|^2 \right] . \tag{1.28}$$

Here $W(\Phi)$ is the superpotential, Φ denotes the scalar component of the superfield Φ; $D_\Phi W = \frac{\partial W}{\partial \Phi} + \frac{\partial K}{\partial \Phi} W$. The kinetic term of the scalar field is given by $K_{\Phi\bar{\Phi}}\, \partial_\mu \Phi \partial_\mu \bar{\Phi}$. The standard textbook choice of the Kähler potential corresponding to the canonically normalized fields Φ and $\bar{\Phi}$ is $K = \Phi\bar{\Phi}$, so that $K_{\Phi\bar{\Phi}} = 1$.

This immediately reveals a problem: At $\Phi > 1$ the potential is extremely steep. It blows up as $\mathrm{e}^{|\Phi|^2}$, which makes it very difficult to realize chaotic inflation in supergravity at $\phi \equiv \sqrt{2}|\Phi| > 1$. Moreover, the problem persists even at small ϕ. If, for example, one considers the simplest case when there are many other scalar fields in the theory and the superpotential does not depend on the inflaton field ϕ, then (1.28) implies that at $\phi \ll 1$ the effective mass of the inflaton field is $m^2_\phi = 3H^2$. This violates the condition $m^2_\phi \ll H^2$ required for successful slow-roll inflation (the so-called η-problem).

The major progress in SUGRA inflation during the last decade was achieved in the context of the models of the hybrid inflation type, where inflation may occur at $\phi \ll 1$. Among the best models are the F-term inflation, where different contributions to the effective mass term m^2_ϕ cancel [110], and D-term inflation [111], where the dangerous term e^K does not affect the potential in the inflaton direction. A detailed discussion of various versions of hybrid inflation in supersymmetric theories can be found in the Chaps. 3 and 4 in this volume, see also [107, 112, 113].

However, hybrid inflation occurs only on a relatively small energy scale, and many of its versions do not lead to eternal inflation. Therefore it would be nice to obtain inflation in a context of a more general class of supergravity models.

This goal seemed very difficult to achieve; it took almost 20 years to find a natural realization of the chaotic inflation model in supergravity. Kawasaki, Yamaguchi and Yanagida suggested to take the Kähler potential

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + X\bar{X} \tag{1.29}$$

of the fields Φ and X, with the superpotential $m\Phi X$ [114].

At first glance, this Kähler potential may seem somewhat unusual. However, it can be obtained from the standard Kähler potential $K = \Phi\bar{\Phi} + X\bar{X}$ by adding terms $\Phi^2/2 + \bar{\Phi}^2/2$, which do not give any contribution to the kinetic term of the scalar fields $K_{\Phi\bar{\Phi}}\, \partial_\mu \Phi \partial_\mu \bar{\Phi}$. In other words, the new Kähler potential, just as the old one, leads to canonical kinetic terms for the fields Φ and X, so it is as simple and legitimate as the standard textbook Kähler potential. However, instead of the U(1) symmetry with respect to rotation of the field Φ in the complex plane, the new Kähler potential has a *shift symmetry*; it does not depend on the imaginary part of the field Φ. The shift symmetry is broken only by the superpotential.

This leads to a profound change of the potential (1.28): the dangerous term e^{K} continues growing exponentially in the direction $(\Phi + \bar{\Phi})$, but it remains constant in the direction $(\Phi - \bar{\Phi})$. Decomposing the complex field Φ into two real scalar fields, $\Phi = \frac{1}{\sqrt{2}}(\eta + i\phi)$, one can find the resulting potential $V(\phi, \eta, X)$ for $\eta, |X| \ll 1$:

$$V = \frac{m^2}{2}\phi^2(1+\eta^2) + m^2|X|^2 \ . \tag{1.30}$$

This potential has a deep valley, with a minimum at $\eta = X = 0$. At $\eta, |X| > 1$ the potential grows up exponentially. Therefore the fields η and X rapidly fall down towards $\eta = X = 0$, after which the potential for the field ϕ becomes $V = \frac{m^2}{2}\phi^2$. This provides a very simple realization of eternal chaotic inflation scenario in supergravity [114]. This model can be extended to include theories with different power-law potentials, or models where inflation begins as in the simplest versions of chaotic inflation scenario, but ends as in new or hybrid inflation, see e.g. [115, 116].

The existence of the shift symmetry was also the basis of the natural inflation scenario [117]. The basic assumption of this scenario was that the axion field in the first approximation is massless because the flatness of the axion direction is protected by U(1) symmetry. Nonperturbative corrections lead to the axion potential $V(\phi) = V_0(1 + \cos(\phi/f_{\mathrm{a}}))$. If the 'radius' of the axion potential f_{a} is sufficiently large, $f_{\mathrm{a}} \gtrsim 3$, inflation near the top of the potential becomes possible. For much greater values of f_{a} one can have inflation near the minimum of the axion potential, where the potential is quadratic [118].

The natural inflation scenario was proposed back in 1990, but until now all attempts to realize this scenario in supergravity have failed. First of all, it has been difficult to find theories with large f_{a}. More importantly, it has been difficult to stabilize the radial part of the axion field. A possible model of natural inflation in supergravity was constructed only very recently, see Chap. 4 in this volume.

Unfortunately, we still do not know how one could incorporate the models discussed in this section in string theory. We will briefly describe some features of inflation in string theory, and refer the readers to a more detailed presentation in Chap. 4 in this volume.

1.11 Towards Inflation in String Theory

1.11.1 de Sitter Vacua in String Theory

For a long time, it had seemed rather difficult to obtain inflation in M/string theory. The main problem here was the stability of compactification of internal dimensions. For example, ignoring non-perturbative effects to be discussed below, a typical effective potential of the effective four-dimensional theory obtained by compactification in string theory of type IIB can be represented in the following form:

$$V(\varphi, \rho, \phi) \sim \mathrm{e}^{\sqrt{2}\varphi - \sqrt{6}\rho}\ \tilde{V}(\phi) \tag{1.31}$$

Here φ and ρ are canonically normalized fields representing the dilaton field and the volume of the compactified space; ϕ stays for all other fields, including the inflaton field.

If φ and ρ were constant, then the potential $\tilde{V}(\phi)$ could drive inflation. However, this does not happen because of the steep exponent $\mathrm{e}^{\sqrt{2}\varphi-\sqrt{6}\rho}$, which rapidly pushes the dilaton field φ to $-\infty$, and the volume modulus ρ to $+\infty$. As a result, the radius of compactification becomes infinite; instead of inflating, four-dimensional space decompactifies and becomes 10-dimensional.

Thus in order to describe inflation one should first learn how to stabilize the dilaton and the volume modulus. The dilaton stabilization was achieved in [119]. The most difficult problem was to stabilize the volume. The solution of this problem was found in [120] (KKLT construction). It consists of two steps.

First, due to a combination of effects related to the warped geometry of the compactified space and nonperturbative effects calculated directly in four-dimensional (instead of being obtained by compactification), it was possible to obtain a supersymmetric AdS minimum of the effective potential for ρ. In the original version of the KKLT scenario, it was done in the theory with the Kähler potential

$$K = -3\log(\rho + \bar{\rho})\ , \tag{1.32}$$

and with the nonperturbative superpotential of the form

$$W = W_0 + A\mathrm{e}^{-a\rho}\ , \tag{1.33}$$

with $a = 2\pi/N$. The corresponding effective potential for the complex field $\rho = \sigma + i\alpha$ had a minimum at finite, moderately large values of the volume modulus field σ_0, which fixed the volume modulus in a state with a negative vacuum energy. Then an anti-$D3$ brane with the positive energy $\sim \sigma^{-2}$ was added. This addition uplifted the minimum of the potential to the state with a positive vacuum energy, see Fig. 1.6.

Instead of adding an anti-$D3$ brane, which explicitly breaks supersymmetry, one can add a D7 brane with fluxes. This results in the appearance of a D-term which has a similar dependence on ρ, but leads to spontaneous supersymmetry breaking [121]. In either case, one ends up with a metastable dS state which can decay by tunneling and formation of bubbles of 10d space with vanishing vacuum energy density. The decay rate is extremely small [120], so for all practical purposes, one obtains an exponentially expanding de Sitter space with the stabilized volume of the internal space.[2]

1.11.2 Inflation in String Theory

There are two different versions of string inflation. In the first version, which we will call modular inflation, the inflaton field is associated with one of the

[2] It is also possible to find de Sitter solutions in noncritical string theory [122].

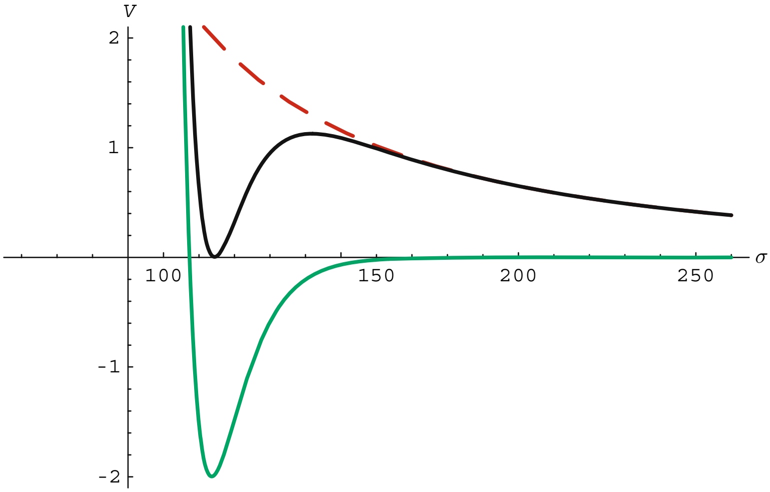

Fig. 1.6. KKLT potential as a function of $\sigma = \mathrm{Re}\,\rho$. The thin *green (lower) line* corresponds to the AdS stabilized potential for $W_0 = -10^{-4}$, $A = 1$, $a = 0.1$. The *dashed line* shows the additional term, which appears either due to the contribution of a $\overline{D3}$ brane or of a D7 brane. The thick *black line* shows the resulting potential with a very small but positive value of V in the minimum. The potential is shown multiplied by 10^{15}

moduli, the scalar fields which are already present in the KKLT construction. In the second version, the inflaton is related to the distance between branes moving in the compactified space. (This scenario should not be confused with inflation in the brane world scenario [123, 124]. This is a separate interesting subject, which we are not going to discuss in this chapter.)

Modular Inflation

An example of the KKLT-based modular inflation is provided by the racetrack inflation model of [125]. It uses a slightly more complicated superpotential

$$W = W_0 + Ae^{-a\rho} + Be^{-b\rho} \,. \tag{1.34}$$

The potential of this theory has a saddle point as a function of the real and the complex part of the volume modulus: it has a local minimum in the direction $\mathrm{Re}\,\rho$, which is simultaneously a very flat maximum with respect to $\mathrm{Im}\,\rho$. Inflation occurs during a slow rolling of the field $\mathrm{Im}\,\rho$ away from this maximum (i.e. from the saddle point). The existence of this regime requires a significant fine-tuning of parameters of the superpotential. However, in the context of the string landscape scenario describing from 10^{100} to 10^{1000} different vacua (see below), this may not be such a big issue. A nice feature of this model is that it does not require adding any new branes to the original KKLT scenario, i.e. it is rather economical (Fig. 1.7.)

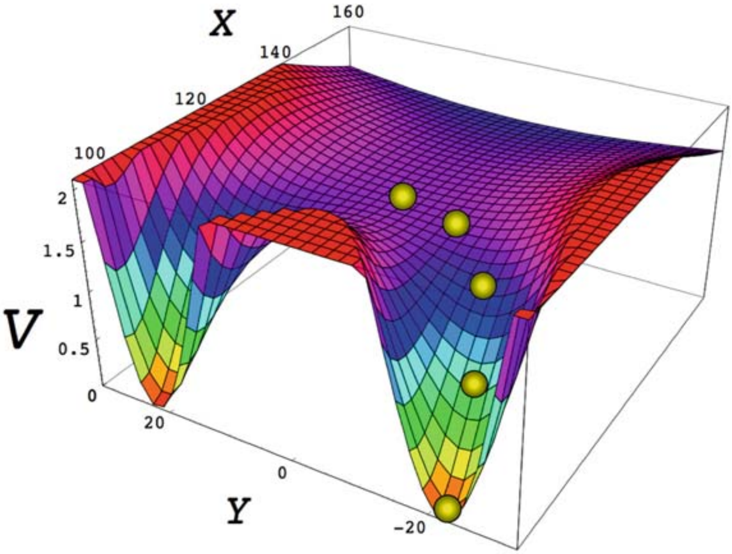

Fig. 1.7. Plot for the potential in the racetrack model (rescaled by 10^{16}). Here X stays for $\sigma = \mathrm{Re}\,\rho$ and Y stays for $\alpha = \mathrm{Im}\,\rho$. Inflation begins in a vicinity of the saddle point at $X_{\mathrm{saddle}} = 123.22$, $Y_{\mathrm{saddle}} = 0$. Units are $M_{\mathrm{Pl}} = 1$

Other interesting models of moduli inflation were developed in [126, 127, 128, 129]. An interesting property of all of these models is the existence of the regime of eternal slow-roll inflation. This property distinguishes modular inflation from the brane inflation scenario to be discussed below.

Brane Inflation

During the last few years, there were many suggestions on how to obtain hybrid inflation in string theory by considering motion of branes in the compactified space, see [130, 131] and references therein. The main problem of all of these models was the absence of stabilization of the compactified space. Once this problem was solved for dS space [120], one could try to revisit these models and develop models of brane inflation compatible with the volume stabilization.

The first idea [132] was to consider a pair of D3 and anti-D3 branes in the warped geometry studied in [120]. The role of the inflaton field ϕ in this model, which is known as the KKLMMT model, could be played by the interbrane separation. A description of this situation in terms of the effective four-dimensional supergravity involved Kähler potential

$$K = -3\log(\rho + \bar{\rho} - k(\phi, \bar{\phi})) , \qquad (1.35)$$

where the function $k(\phi, \bar{\phi})$ for the inflaton field ϕ, at small ϕ, was taken in the simplest form $k(\phi, \bar{\phi}) = \phi\bar{\phi}$. If one makes the simplest assumption that the superpotential does not depend on ϕ, then the ϕ dependence of the potential (1.28) comes from the term $\mathrm{e}^K = (\rho + \bar\rho - \phi\bar\phi)^{-3}$. Expanding this term near the stabilization point $\rho = \rho_0$, one finds that the inflaton field has a mass $m_\phi^2 = 2H^2$. Just like the similar relation $m_\phi^2 = 3H^2$ in the simplest models of supergravity, this is not what we want for inflation.

One way to solve this problem is to consider ϕ-dependent superpotentials. By doing so, one may fine-tune m_ϕ^2 to be $\mathrm{O}(10^{-2})H^2$ in a vicinity of the point where inflation occurs [132]. Whereas fine-tuning is certainly undesirable, in the context of string cosmology it may not be a serious drawback. Indeed, if there exist many realizations of string theory (see Sect. 1.14), then one might argue that all realizations not leading to inflation can be discarded, because they do not describe a universe in which we could live. This makes the issue of fine-tuning less problematic. Inflation in the KKLMMT model and its generalizations were studied by many authors; see Chap. 4 in this volume and references therein.

Can we avoid fine-tuning altogether? One of the possible ideas is to find theories with some kind of shift symmetry. Another possibility is to construct something like D-term inflation, where the flatness of the potential is not spoiled by the term e^K. Both of these ideas were combined together in Ref. [133] based on the model of D3/D7 inflation in string theory [134]. In this model the Kähler potential is given by

$$K = -3\log(\rho + \bar\rho) - \frac{1}{2}(\phi - \bar\phi)^2\ , \tag{1.36}$$

and the superpotential depends only on ρ. The role of the inflaton field is played by the field $s = \mathrm{Re}\,\phi$, which represents the distance between the D3 and D7 branes. The shift symmetry $s \to s + c$ in this model is related to the requirement of unbroken supersymmetry of branes in a BPS state.

The effective potential with respect to the field ρ in this model coincides with the KKLT potential [120, 121]. The potential is exactly flat in the direction of the inflaton field s, until one adds a hypermultiplet of other fields $\phi_\pm$, which break this flatness due to quantum corrections and produce a logarithmic potential for the field s. The resulting potential with respect to the fields s and $\phi_\pm$ is very similar to the potential of D-term hybrid inflation [111].

During inflation, $\phi_\pm = 0$, and the field s slowly rolls down to its smaller values. When it becomes sufficiently small, the theory becomes unstable with respect to the generation of the field ϕ_+, see Fig. 1.8. The fields s and ϕ_+ roll down to the KKLT minimum, and inflation ends. For the latest developments in D3/D7 inflation see [135, 136].

All inflationary models discussed above were formulated in the context of Type IIB string theory with the KKLT stabilization. A discussion of the possibility to obtain inflation in the heterotic string theory with stable compactification can be found in [137, 138].

Finally, we should mention that making the effective potential flat is not the only way to achieve inflation. There are some models with nontrivial

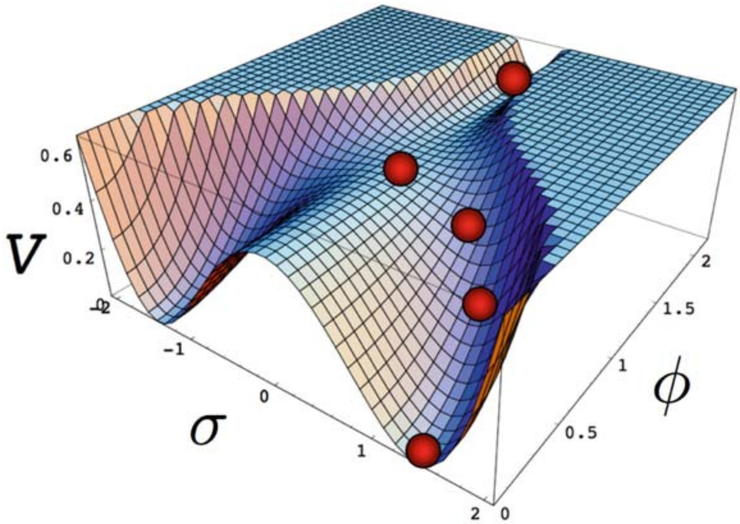

Fig. 1.8. The inflationary potential as a function of the inflaton field s and $\mathrm{Re}\,\phi_+$. In the beginning, the field s rolls along the valley $\phi_+ = 0$, and then it falls down to the KKLT minimum

kinetic terms where inflation may occur even without any potential [139]. One may also consider models with steep potentials but with anomalously large kinetic terms for the scalar fields see e.g. [140]. In application to string theory, such models, called "DBI inflation," were developed in [141].

In contrast to the moduli inflation, none of the existing versions of the brane inflation allow the slow-roll eternal inflation [142].

1.12 Scale of Inflation, the Gravitino Mass, and the Amplitude of the Gravitational Waves

So far, we did not discuss the relation of the new class of models with particle phenomenology. This relation is rather unexpected and may impose strong constraints on particle phenomenology and on inflationary models: In the simplest models based on the KKLT mechanism the Hubble constant H and the inflaton mass m_ϕ are smaller than the gravitino mass [143],

$$m_\phi \ll H \lesssim m_{3/2} \,. \tag{1.37}$$

The reason for the constraint $H \lesssim m_{3/2}$ is that the height of the barrier stabilizing the KKLT minimum is $\mathrm{O}(m_{3/2}^2)$. Adding a large vacuum energy density to the KKLT potential, which is required for inflation, may destabilize it, see Fig. 1.9. The constraint $m_\phi \ll H$ is a consequence of the slow-roll conditions.

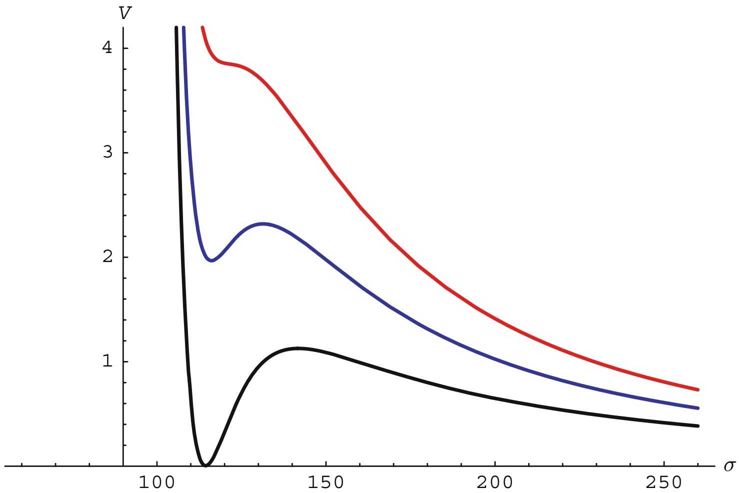

Fig. 1.9. The lowest curve with dS minimum is the one from the KKLT model. The height of the barrier in this potential is of the order $m_{3/2}^2$. The second line shows the σ-dependence of the inflaton potential. When one adds it to the theory, it always appears divided by σ^n, where in the simplest cases $n = 2$ or 3. Therefore an addition of the inflationary potential lifts up the potential at small σ. The top curve shows that when the inflation potential becomes too large, the barrier disappears, and the internal space decompactifies. This explains the origin of the constraint $H \lesssim m_{3/2}$

Therefore if one believes in the standard SUSY phenomenology with $m_{3/2} \lesssim$ O(1) TeV, one should find a realistic particle physics model where inflation occurs at a density at least 30 orders of magnitude below the Planck energy density. Such models are possible, but their parameters should be substantially different from the parameters used in all presently existing models of string theory inflation.

An interesting observational consequence of this result is that the amplitude of the gravitational waves in all string inflation models of this type should be extremely small. Indeed, according to (1.21), one has $r \approx 3 \times 10^7\ V \approx 10^8\ H^2$, which implies that

$$r \lesssim 10^8\ m_{3/2}^2\ , \tag{1.38}$$

in Planck units. In particular, for $m_{3/2} \lesssim 1$ TeV $\sim 4 \times 10^{-16}\ M_p$, which is in the range most often discussed by SUSY phenomenology, one has [144]

$$r \lesssim 10^{-24}\ . \tag{1.39}$$

If CMB experiments find that $r \gtrsim 10^{-2}$, then this will imply, in the class of theories described above, that

$$m_{3/2} \gtrsim 10^{-5}\ M_p \sim 2.4 \times 10^{13}\ \text{GeV}\ , \tag{1.40}$$

which is 10 orders of magnitude greater than the standard gravitino mass range discussed by particle phenomenologists.

There are several different ways to address this problem. First of all, one may try to construct realistic particle physics models with superheavy gravitinos [145, 146].

Another possibility is to consider models with the racetrack superpotential containing at least two exponents (1.34) and find parameters such that the supersymmetric minimum of the potential even before the uplifting occurs at zero energy density [143], which would mean $m_{3/2} = 0$, see Fig. 1.10. Then, by a slight change of parameters one can get the gravitino mass squared much smaller than the height of the barrier, which removes the constraint $H \lesssim m_{3/2}$.

Note, however, that in order to have $H^2 \sim V \sim 10^{-10}$ with $m_{3/2} \lesssim 1$ TeV $\sim 4 \times 10^{-16}\ M_{\rm p}$ in the model of [143] one would need to fine-tune the parameters of the theory with an incredible precision. This observation further strengthens the results of [147, 148], which imply that the tensor perturbations produced in all known versions of string theory inflation are undetectably small.

One could argue that since the existing versions of string theory inflation predict tensor modes with an extremely small amplitude, there is no sense to even try to detect them. From our perspective, however, the attitude should be opposite. There is a class of inflationary models that predict r in the range from 0.3 to 10^{-2}, see Sect. 1.7, so it makes a lot of sense to test this range of r even though the corresponding models have not been constructed as yet in the context of string theory.

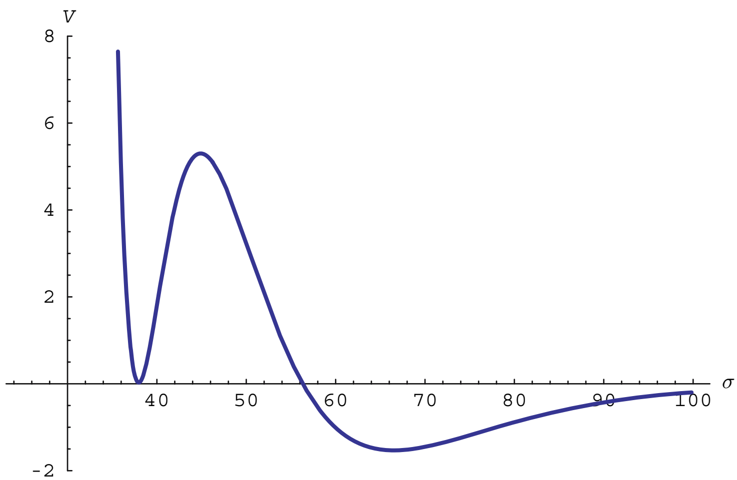

Fig. 1.10. The potential in the theory (1.34) for $A = 1,\ B = -5,\ a = 2\pi/100,\ b = 2\pi/50,\ W_0 = -0.05$. A Minkowski minimum at $V = 0$ stabilizes the volume at $\sigma_0 \approx 37$. The height of the barrier in this model is not correlated with the gravitino mass, which vanishes if the system is trapped in Minkowski vacuum. Therefore, in this model one can avoid the constraint $H \lesssim m_{3/2}$ [143]

If the tensor modes is found, the resulting situation will be similar to the situation with the discovery of the acceleration of the universe. This discovery initially puzzled string theorists, since none of the versions of string theory which existed 5 years ago could describe an accelerating universe in a stable vacuum state with a positive energy density. Eventually this problem was resolved with the development of the KKLT construction.

A possible discovery of tensor modes could lead to another constructive crisis since it may rule out many existing versions of string inflation and string phenomenology, and it may imply that the gravitino must be superheavy. Thus, investigation of gravitational waves produced during inflation may serve as a unique source of information about string theory and fundamental physics in general [144].

1.13 Initial Conditions for the Low-Scale Inflation and Topology of the Universe

One of the advantages of the simplest versions of the chaotic inflation scenario is that inflation may begin in the universe immediately after its creation at the largest possible energy density $M_{\rm Pl}^4$, of a smallest possible size (Planck length), with the smallest possible mass $M \sim M_{\rm Pl}$ and with the smallest possible entropy $S = \mathrm{O}(1)$. This provides a true solution to the flatness, horizon, homogeneity, mass and entropy problems [14].

Meanwhile, in the new inflation scenario (more accurately, in the hilltop version of the chaotic inflation scenario), inflation occurs on the mass scale 3 orders of magnitude below $M_{\rm Pl}$, when the total size of the universe was very large. If, for example, the universe is closed, its total mass at the beginning of new inflation must be greater than $10^6 M_{\rm Pl}$, and its total entropy must be greater than 10^9. In other words, in order to explain why the entropy of the universe at present is greater than 10^{87} one should assume that it was extremely large from the very beginning. Then it becomes difficult to understand why such a large universe was homogeneous. This does not look like a real solution of the problem of initial conditions.

Thus one may wonder whether it possible to solve the problem of initial conditions for the low-scale inflation? The answer to this question is positive though perhaps somewhat unexpected: the simplest way to solve the problem of initial conditions for the low-scale inflation is to consider a compact flat or open universe with nontrivial topology (usual flat or open universes are infinite). The universe may initially look like a nearly homogeneous torus of a Planckian size containing just one or two photons or gravitons. It can be shown that such a universe continues expanding and remains homogeneous until the onset of inflation, even if inflation occurs only on a very low energy scale [149, 150, 151, 152, 153].

Consider, e.g. a flat compact universe having the topology of a torus, S_1^3,

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_i^2(t)\,\mathrm{d}x_i^2 \tag{1.41}$$

with identification $x_i + 1 = x_i$ for each of the three dimensions. Suppose for simplicity that $a_1 = a_2 = a_3 = a(t)$. In this case the curvature of the universe and the Einstein equations written in terms of $a(t)$ will be the same as in the infinite flat Friedmann universe with metric $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\,\mathrm{d}\mathbf{x^2}$. In our notation, the scale factor $a(t)$ is equal to the size of the universe in Planck units $M_\mathrm{p}^{-1} = 1$.

Let us assume, that at the Planck time $t_\mathrm{p} \sim M_\mathrm{Pl}^{-1} = 1$ the universe was radiation dominated, $V \ll T^4 = \mathrm{O}(1)$. Let us also assume that at the Planck time the total size of the box was Planckian, $a(t_\mathrm{p}) = \mathrm{O}(1)$. In such case, the whole universe initially contained only O(1) relativistic particles such as photons or gravitons, so that the total entropy of the whole universe was O(1).

The size of the universe dominated by relativistic particles was growing as $a(t) \sim \sqrt{t}$, whereas the mean free path of the gravitons was growing as $H^{-1} \sim t$. If the initial size of the universe was O(1), then at the time $t \gg 1$ each particle (or a gravitational perturbation of the metric) within one cosmological time would run all over the torus many times, appearing in all of its parts with nearly equal probability. This effect, called "chaotic mixing," should lead to a rapid homogenization of the universe [150, 151]. Note, that to achieve a modest degree of homogeneity required for inflation to start when the density of ordinary matter drops down, we do not even need chaotic mixing. Indeed, density perturbations do not grow in a universe dominated by ultrarelativistic particles if the size of the universe is smaller than H^{-1}. This is exactly what happens in our model. Therefore the universe should remain relatively homogeneous until the thermal energy density drops below V and inflation begins. And once it happens, the universe rapidly becomes very homogeneous.

Thus we see that in this scenario, just as in the simplest chaotic inflation scenario, inflation begins if we had a sufficiently homogeneous domain of the smallest possible size (Planck scale), with the smallest possible mass (Planck mass), and with the total entropy O(1). The only additional requirement is that this domain should have identified sides, in order to make a flat or open universe compact. We see no reason to expect that the probability of formation of such domains is strongly suppressed.

One can come to a similar conclusion from a completely different point of view. The investigation of the quantum creation of a closed or an infinite open inflationary universe with $V \ll 1$ shows that this process is forbidden at the classical level, and therefore it occurs only due to tunneling. As a result, the probability of this process is exponentially suppressed [17, 18, 20]. Meanwhile, creation of the flat or open universe is possible without any need for the tunneling, and therefore there is no exponential suppression for the probability of quantum creation of a topologically nontrivial compact flat or open inflationary universe [149, 152, 153].

These results suggest that if inflation can occur only much below the Planck density, then the compact topologically nontrivial flat or open universes should be much more probable than the standard Friedmann universes described in every textbook on cosmology. This possibility is quite natural in the context of string theory, where all internal dimensions are supposed to be compact. Note, however, that if the stage of inflation is sufficiently long, it should make the observable part of the universe so large that its topology does not affect observational data.

The problem of initial conditions in string cosmology has several other interesting features. The most important one is the existence of an enormously large number of metastable de Sitter vacuum states, which makes the stage of exponential expansion of the universe almost inevitable. We will discuss this issue in the next section.

1.14 Inflationary Multiverse, String Theory Landscape and the Anthropic Principle

For many decades, people have tried to explain strange correlations between the properties of our universe, the masses of elementary particles, their coupling constants, and the fact of our existence. We know that we could not live in a five-dimensional universe, or in a universe where the electromagnetic coupling constant, or the masses of electrons and protons would be just a few times greater or smaller than their present values. These and other similar observations have formed the basis for the anthropic principle. However, for a long time many scientists believed that the universe was given to us as a single copy, and therefore speculations about these magic coincidences could not have any scientific meaning. Moreover, it would require a wild stretch of imagination and a certain degree of arrogance to assume that somebody was creating one universe after another, changing their parameters and fine-tuning their design, doing all of that for the sole purpose of making the universe suitable for our existence.

The situation changed dramatically with the invention of inflationary cosmology. It was realized that inflation may divide our universe into many exponentially large domains corresponding to different metastable vacuum states, forming a huge inflationary multiverse [52, 54, 154]. The total number of such vacuum states in string theory can be enormously large, in the range of 10^{100} or 10^{1000} [120, 155, 156, 157]. A combination of these two facts gave rise to what the experts in inflation call "the inflationary multiverse," [14, 55, 158] and string theorists call "the string theory landscape" [159].

This leads to an interesting twist in the theory of initial conditions. Let us assume first that we live in one of the many metastable de Sitter minima, say, $\mathrm{d}S_i$. Eventually this dS state decays, and each of the *points* belonging to this initial state jumps to another vacuum state, which may have either a smaller vacuum energy, or a greater vacuum energy (transitions of the second type are possible because of the gravitational effects). But if the decay probability is not too large, then the total *volume* of the universe remaining in the state $\mathrm{d}S_i$ continues growing exponentially [9]. This is eternal inflation of the old inflation type. If the bubbles of the new phase correspond to another de Sitter space, dS_j, then some parts of the space dS_j may jump back to the state dS_i. On the other hand, if the tunneling goes to a Minkowski vacuum, such as the uncompactified 10-dimensional vacuum corresponding to the state with $\sigma \to \infty$ in Fig. 1.6, the subsequent jumps to dS states no longer occur. Similarly, if the tunneling goes to the state with a negative vacuum energy, such as the AdS vacuum in Fig. 1.10, the interior of the bubble of the new vacuum rapidly collapses. Minkowski and AdS vacua of such type are called terminal vacua, or sinks.

If initial conditions in a certain part of the universe are such that it goes directly to the sink, without an intermediate stage of inflation, then it will never return back, we will be unable to live there; so for all practical purposes such initial conditions (or such parts of the universe) can be discarded (ignoring for the moment the possibility of the resurrection of the universe after the collapse). On the other hand, if some other part of the universe goes to one of the dS states, the process of eternal inflation begins, which eventually produces an inflationary multiverse consisting of all possible dS states. This suggests that all initial conditions that allow life as we know it to exist, inevitably lead to formation of an eternal inflationary multiverse.

This scenario assumes that the vacuum transitions may bring us from any part of the string theory landscape to any other part. Here we should note that the theory of such transitions accompanied by the change of fluxes was developed for the case where dS states are not stabilized [156, 160]. A generalization of this theory for the string landscape scenario based on the KKLT mechanism of vacuum stabilization is rather nontrivial. As of now, the theory of such transitions was fully developed only for the transitions where the scalar fields change but the fluxes remain unchanged [161]. It might happen that the landscape is divided into separate totally disconnected islands, but this does not seem likely [162]. Even if the landscape is not fully transversable, one may probe all parts of the inflationary multiverse by considering the wave function of the universe corresponding to the possibility of its quantum creation in the states with different values of fluxes [163, 164].

The string theory landscape describes an incredibly large set of *discrete* parameters. However, the theory of inflationary multiverse goes even further. Some of the features of our world are determined not by the final values of the fields in the minima of their potential in the landscape, but by the dynamical, time-dependent values, which these fields were taking at different stages of the evolution of the inflationary universe. This introduces a large set of *continuous* parameters, which may take different values in different parts of the universe. For example, in the theory of dark energy, inflationary fluctuations may divide the universe into exponentially large parts with the effective value of the cosmological constant taking a continuous range of values [109]. In such models, the effective cosmological constant Λ becomes a continuous parameter. Similarly, inflationary fluctuations of the axion field make the density of dark matter a continuous parameter, which takes different values in different parts of the universe [165, 166]. Another example of a continuous parameter is the baryon asymmetry n_b/n_γ, which can take different values in different parts of the universe in the Affleck–Dine scenario of baryogenesis [167, 168].

This means that the same physical theory may yield exponentially large parts of the universe that have diverse properties. This provided the first scientific justification of the anthropic principle: We find ourselves inside a part of the universe with our kind of physical laws not because the parts with different properties are impossible or improbable, but simply because we cannot live there [52, 154].

This fact can help us understand many otherwise mysterious features of our world. The simplest example concerns the dimensionality of our universe. String theorists usually assume that the universe is 10- or 11-dimensional, so why do we live in the universe where only 4 dimensions of space–time are large? There have been many attempts to address this question, but no convincing answer has been found. This question became even more urgent after the development of the KKLT construction. Now we know that all de Sitter states, including the state in which we live now, are either unstable or metastable. They tend to decay by producing bubbles of a collapsing space, or of a 10-dimensional Minkowski space. So what is wrong about the 10-dimensional universe if it is so naturally appears in string theory?

The answer to this question was given in 1917 by Paul Ehrenfest [169]: in space–time with dimensionality $d > 4$, gravitational forces between distant bodies fall off faster than r^{-2}, and in space–time with $d < 4$, the general theory of relativity tells us that such forces are absent altogether. This rules out the existence of stable planetary systems for $d \neq 4$. A similar conclusion is valid for atoms: stable atomic systems could not exist for $d > 4$. This means that we do not need to prove that the four-dimensional space–time is a *necessary* outcome of string cosmology (in fact, it does not seem to be the case). Instead of that, we only need to make sure that the four-dimensional space–time is *possible*.

Anthropic considerations may help us to understand why the amount of dark matter is approximately five times greater than the amount of normal matter [165, 166] and why the baryon asymmetry is so small, $n_b/n_\gamma \sim 10^{-10}$ [168]. But perhaps the most famous example of this type is related to the cosmological constant problem.

Naively, one could expect the vacuum energy to be equal to the Planck density, $\rho_\Lambda \sim 1$, whereas the recent observational data show that $\rho_\Lambda \sim 10^{-120}$, in Planck units, which is approximately three times greater than the density of other matter in the universe. Why is it so small but nonzero? Why ρ_Λ constitutes is about three times greater than the density of other types of matter in the universe now? Note that long ago the density of matter was much greater than ρ_Λ, and in the future it will be much smaller.

The first anthropic solution to the cosmological constant problem in the context of inflationary cosmology was proposed in 1984 [163]. The basic assumption was that the vacuum energy density is a sum of the scalar field potential $V(\phi)$ and the energy of fluxes $V(F)$. According to [17], quantum creation of the universe is not suppressed if the universe is created at the Planck energy density, $V(\phi) + V(F) = \mathrm{O}(1)$, in Planck units. Eventually the field ϕ rolls to its minimum at some value ϕ_0, and the vacuum energy becomes $\Lambda = V(\phi_0) + V(F)$. Since initially $V(\phi)$ and $V(F)$ could take any values with nearly equal probability, under the condition $V(\phi) + V(F) = \mathrm{O}(1)$, we get a flat probability distribution to find a universe with a given value of the cosmological constant after inflation, $\Lambda = V(\phi_0) + V(F)$, for $\Lambda \ll 1$. The flatness of this probability distribution is crucial, because it allows us to study the probability of emergence of life for different Λ. Finally, it was argued in [163] that life as we know it is possible only for $|\Lambda| \lesssim \rho_0$, where $\rho_0 \sim 10^{-120}$ is the present energy density of the universe. This fact, in combination with inflation, which makes such universes exponentially large, provided a possible solution of the cosmological constant problem.

Shortly after that, several other anthropic solutions to the cosmological constant problem were proposed [170]. All of them were based on the assumption that life as we know it is possible only for $-\rho_0 \lesssim \rho_\Lambda \lesssim \rho_0$. This bound seemed almost self-evident to many of us at that time, and therefore in [163, 170] we concentrated on the development of the theoretical framework where the anthropic arguments could be applied to the cosmological constant.

The fact that ρ_Λ could not be much smaller than $-\rho_0$ was indeed quite obvious, since such a universe would rapidly collapse. However, the origin of the constraint $\rho_\Lambda \lesssim \rho_0$ was much less trivial. The first attempt to justify it was made in 1987 in the famous paper by Weinberg [171], but the constraint obtained there allowed the cosmological constant to be three orders of magnitude greater than its present value.

Since that time, the anthropic approach to the cosmological constant problem developed in two different directions. First of all, it became possible, under certain assumptions, to significantly strengthen the constraint on the positive cosmological constant, see e.g. [172, 173, 174, 175]. The final result of these investigations, $|\Lambda| \lesssim O(10)\ \rho_0 \sim 10^{-119}$, is very similar to the bound used in [163].

Simultaneously, new models have been developed which may allow us to put an anthropic approach to the cosmological constant problem on a firm ground. In particular, the existence of a huge number of vacuum states in string theory implies that in different parts of our universe, or in its different quantum states, the cosmological constant may take all of its possible values, from -1 to $+1$, with an increment which may be as small as 10^{-1000}. If the prior probability to be in each of these vacua does not depend strongly on Λ, one can justify the anthropic bound on Λ using the methods of [172, 173, 174, 175, 176].

However, the issue of probabilities in eternal inflation is very delicate, so one should approach anthropic arguments with some care. For example, one may try to calculate the probability to be born in a part of the universe with given properties *at a given point.* One can do this using comoving coordinates, which are not expanding during inflation [162, 177, 178, 179, 180, 181]. However, it is not obvious whether the calculation of the probabilities of physical processes at a given point, ignoring the expansion of the universe, should be used in anthropic considerations. Most of the physical entities which could be associated with "points" did not even exist before the beginning of inflation: protons did not exist, photons did not exist, galaxies did not exist. They appeared only after inflation, and their total number, and the total number of observers, is proportional to the growth of volume during inflation.

This leads to the volume-weighted [55, 182, 183, 184], or pocket-weighted [184, 185, 186] probability measures [187]. The main problem with this approach is the embarrassment of riches: the total volume of the universe occupied by any particular vacuum state, integrated over the indefinitely long history of the eternally inflating universe, is infinitely large. Thus we need to compare infinities, which is a very ambiguous task, with the answer depending on the choice of the cut-off procedure.

The volume-weighted probability measure proposed in [55] is based on the calculation of the ratio of the volumes of the parts of the universe with different properties. This is possible because if we wait long enough, eternal inflation approaches a stationary regime. Different parts of the universe expand and transform to each other. As a result, the total volumes of all parts of the universe of each particular type grow at the same rate, and the ratio of their volumes becomes time-independent [55].

This method is very good for describing the map of the inflationary multiverse, but in order to use it in anthropic considerations one should make some additional steps. According to [182], instead of calculating the ratio of volumes in different vacuum states at different densities and temperatures, we should calculate the total volume of *new* parts of the universe where life becomes possible. This ratio is related to the incoming probability current through the hypersurface of the end of inflation, or the hypersurface of a fixed density or temperature. If one uses the probability measure of [55] for anthropic considerations (which was *not* proposed in [55]), one may encounter the so-called youngness paradox [188, 189]. If one uses the prescription of [182], this paradox does not appear [21].

The results of the calculations by this method are very sensitive to the choice of the time parametrization [182, 21]. However, a recent investigation of this issue indicates that it may be possible to resolve this problem [190]. The main idea is that the parts of the universe with different properties approach the stationary regime of eternal inflation at different times. This fact was not taken into account in our earlier papers [55, 182]; the calculations of the probabilities started everywhere at the same time, even if the corresponding parts of the universe did not yet approach the stationary regime. If we start comparing the volumes of different part of the universe not at the same time after the beginning of inflation, but at the same time since the beginning of the stationarity regime, the dependence on the time parametrization disappears, at least in the simple cases where we could verify this property [190].

As we already mentioned, there are many other proposals for the calculations of probabilities in an inflationary multiverse, see e.g. [184, 186]. The results of some of these methods are not sensitive to the choice of time parametrization, but they do depend on the choice of the cut-off. A detailed discussion of this series of proposals can be found in [185, 191] and in Chap. 5 in this volume.

While discussing all of these approaches one should keep in mind yet another possibility: it is quite possible that it does not make much sense to compare infinities and talk about the probability of events that already happened. Instead of doing it, one should simply study our part of the universe, take these data as an initial input for all subsequent calculations, and study conditional probabilities for the quantities which we did not measure yet [21]. This is a standard approach used by experimentalists who continuously re-evaluate the probability of various outcomes of their future experiments on the basis of other experimental data. The non-standard part is that we should be allowed to use *all* of our observations, including our knowledge of our own properties, for the calculation of conditional probabilities.

Let us apply this limited approach to the cosmological constant problem. Twenty years ago, we already knew that our life is carbon-based, and that the amplitude of density perturbations required for the formation of galaxies was about 10^{-5}. We did not know yet what was the vacuum energy, and the prevailing idea was that we did not have much choice anyway. But with the discovery of inflation, we learned that the universe could be created differently, with different values of the cosmological constant in each of its parts created by eternal inflation. This allowed us to propose several different anthropic solutions to the cosmological constant problem based on the assumption that, for the given value of the amplitude of density perturbations and other already measured parameters, *we* cannot live in a universe with $|\Lambda| \gg 10^{-120}$. If observations would show that the cosmological constant were a million times smaller than the anthropic bound, then we would be surprised, and a theoretical explanation of this anomaly would be in order. As of now, the small value of the cosmological constant does not look too surprising, so for a while we can concentrate on solving many other problems which cannot be addressed by anthropic considerations.

Within this approach, one should not vary the constants of nature that were already known at the time when the predictions were made. In doing so, one faces the risk of repeating the old argument that the bomb does not hit the same spot twice: it is correct only until the first hit, after which the probabilities should be re-evaluated. Similarly, one should not omit the word "anthropic" from the "anthropic principle" and should not replace the investigation of the probability of *our* life with the study of life in general: we are trying to explain *our* observations rather than the possible observations made by some abstract information-processing devices. This can help us to avoid some paradoxes recently discussed in the literature [192, 193, 194].

From this discussion it should be clear that we do not really know yet which of the recently developed approaches to the theory of the inflationary multiverse is going to be more fruitful, and how far we will be able to go in this direction. One way or another, it would be very difficult to forget about what we have just learned and return to our search for the theory which unambiguously explains all parameters of our world. Now we know that some features of our part of the universe may have an unambiguous explanation, whereas some others can be purely environmental and closely correlated with our own existence.

When the inflationary theory was first proposed, its main goal was to address many problems which at that time could seem rather metaphysical: why is our universe so big? Why is it so uniform? Why parallel lines do not intersect? It took some time before we got used to the idea that the large size, flatness and uniformity of the universe should not be dismissed as trivial facts of life. Instead of that, they should be considered as observational data requiring an explanation.

Similarly, the existence of an amazingly strong correlation between our own properties and the values of many parameters of our world, such as the masses and charges of the electron and the proton, the value of the gravitational constant, the amplitude of spontaneous symmetry breaking in the electroweak theory, the value of the vacuum energy, and the dimensionality of our world, is an experimental fact requiring an explanation. A combination of the theory of inflationary multiverse and the string theory landscape provide us with a unique framework where this explanation can possibly be found.

1.15 Conclusions

Twenty five years ago, the inflationary theory looked like an exotic product of vivid scientific imagination. Some of us believed that it possessed such a great explanatory potential that it had to be correct; some others thought that it was too good to be true. Not many expected that it would be possible to verify any of its predictions in our lifetime. Thanks to the enthusiastic work of many scientists, the inflationary theory is gradually becoming a widely accepted cosmological paradigm, with many of its predictions being confirmed by observational data.

However, while the basic principles of inflationary cosmology are rather well established, many of its details keep changing with each new change of the theory of all fundamental interactions. The investigation of the inflationary multiverse and the string theory landscape force us to think about problems which sometimes go beyond the well established boundaries of physics. This makes our life difficult, sometimes quite frustrating, but also very interesting, which is perhaps the best thing that one could expect from the branch of science we have been trying to develop during the last quarter of a century.

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