

# Image-Based Robust Control of Robot Manipulators Under Jacobian Uncertainty

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**Abstract.** In this paper, an image-based robust controller for tracking control of robot manipulators using a single camera is proposed. The proposed controller has robustness to parametric uncertainties of the robot manipulator and compensation for uncertainties included in the image Jacobian. The stability of the closed-loop system is proved by Lyapunov approach. The performance of the proposed method is demonstrated by experiments on a 5-link robot manipulator with two degree of freedom.

**Keywords:** robust control, image-based control, Jacobian uncertainty, robot manipulator.

## 1 Introduction

Robot control using vision based controller has been regarded as one of highly attracted research area these days. Among them, visual feedback technique based on the robot control was applied to the robot working in unstructured environments so that this application was received attention from many researchers as a solution for the position and motion control of robot manipulators under the such environment.

Kelly proposed an image-based direct visual servo controller for camera-in-hand robot manipulators, which is a simple structure based on a transpose Jacobian term plus gravity compensation[1]. In the camera-in-hand configuration, the camera supplies visual information of the object to the controller. The objective of this visual feedback control scheme is to move the robot manipulator to the desired in the image plane obtained by the camera [2-3]. However since robot dynamics was not included in the control input, the visual servo controller has a drawback that is sensitive to the load variations and disturbances. In that reason, we consider applying the robust control schemes to compensate such uncertainties.

Robust control schemes for robot manipulators are required to achieve good tracking performance in the presence of parametric uncertainties such as load variations and unmodeled joint friction[4]. A robust tracking controller has been

designed to compensate the uncertainty in the camera orientation and to ensure globally uniformly ultimate boundedness[5-6].

When the robot manipulators recognize the objects through a camera, noises generated by the lighting or the distance errors can cause parameter fluctuation even though there is no actual movement of the objects. If Jacobian uncertainty exists, it is hard to achieve the expected control performance with the robust control method which only considers uncertainty of the robot dynamics. Therefore, in the robot vision system where uncertainty of bounded Jacobian exists, a controller that compensates its uncertainty is required.

In this paper, we propose an image-based robust controller under Jacobian uncertainty to compensate uncertainties of the robot dynamics and Jacobian due to load variations, lighting or disturbances. The closed-loop stability including the whole robot dynamics is shown by the Lyapunov method. The proposed controller is applied on a two degree of freedom 5-link robot manipulator to show the performance of the closed-loop system.

## 2 Robot and Camera Model

In the absence of friction, the dynamic equation of an n-link rigid robot manipulators can be expressed as[7]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (1)$$

where  $q \in R^n$  is the vector of joint displacements,  $\tau \in R^n$  is the vector of torques applied to the joints,  $M(q) \in R^{n \times n}$  is the symmetric positive definite inertia matrix,  $C(q, \dot{q})\dot{q} \in R^n$  is the vector of centripetal and Coriolis torques, and  $G(q) \in R^n$  is the vector of the gravitational torques.

The robot dynamic model (1) has the following properties.

**Property 1:** For the unknown constant parameter vector  $\theta \in R^p$ , the dynamic equation (1) can be expressed linearly

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \ddot{q})\theta = \tau \quad (2)$$

where  $Y(q, \dot{q}, \ddot{q}) \in R^{n \times p}$  is the known regression matrix.

**Property 2:** The matrix  $\dot{M}(q) - 2C(q, \dot{q})$  is skew-symmetric.

We consider a CCD camera mounted at the end-effector of the robot manipulator. Let  ${}^0P_c$  be the origin of the camera coordinate frame with respect to the robot coordinate frame and  ${}^cP_t$  be the position of object with respect to the camera coordinate frame. Then the feature point  ${}^cP_t = \begin{bmatrix} {}^c x_t & {}^c y_t & {}^c z_t \end{bmatrix}^T$  with respect to the camera coordinate frame is represented by

$${}^cP_t = {}^cR_0 ({}^0P_t - {}^0P_c) \quad (3)$$

where  ${}^0R_c$  is the orientation of the camera frame with respect to the robot coordinate frame.

We assume that the feature point  ${}^cP_t$  in the camera frame projects onto the point  $\xi = [x \ y]^T$  on the image plane by a perspective projection as follows[8]

$$\xi = [x \ y]^T = \begin{bmatrix} \frac{f x_t}{\alpha^c z_t} & \frac{f y_t}{\beta^c z_t} \end{bmatrix}^T \quad (4)$$

where  $f$  is the focal length of the camera,  $\alpha$  and  $\beta$  are scaling factors in pixels per meter due to the camera sampling.

### 3 Image-Based Robust Control

In image-based robot control, the control problem is to design a controller to move the end-effector in such a way that the actual image features reach the desired ones specified in the image plane. In this section, we consider the image-based robust control problem of the robot manipulator with bounded Jacobian uncertainties and parametric uncertainties due to unknown payload or unmodeled friction.

The time derivative of (4) yields

$$\dot{\xi} = \begin{bmatrix} \frac{1}{\alpha^c z_t} & 0 & -\frac{c x_t}{\alpha^c z_t^2} \\ 0 & \frac{1}{\beta^c z_t} & -\frac{c y_t}{\beta^c z_t^2} \end{bmatrix} \begin{bmatrix} c \dot{x}_t \\ c \dot{y}_t \\ c \dot{z}_t \end{bmatrix} \quad (5)$$

By substituting the time derivative of (3) into (4), the motion of the image feature point is expressed as a function of the camera velocities

$$\dot{\xi} = J_{img}(q, \xi, {}^c z_t) \begin{bmatrix} v_c^T \\ \omega_c^T \end{bmatrix} \quad (6)$$

where  $v_c$  and  $\omega_c$  are end-effector linear velocity and angular velocity, respectively and the image Jacobian matrix  $J_{img}(q, \xi, {}^c z_t)$  is defined by

$$J_{img}(q, \xi, {}^c z_t) = \begin{bmatrix} -\frac{f}{\alpha^c z_t} & 0 & \frac{x}{c z_t} & \frac{\beta xy}{f} & \frac{\alpha x^2 + f}{f} & \frac{\beta y}{\alpha} \\ 0 & -\frac{f}{\beta^c z_t} & \frac{y}{c z_t} & \frac{\beta y^2 - f}{f} & \frac{\alpha xy}{f} & \frac{\alpha y}{\beta} \end{bmatrix} \begin{bmatrix} {}^0R_c^T & 0 \\ 0 & {}^0R_c^T \end{bmatrix} \quad (7)$$

The differential kinematics of a robot manipulator gives the relationship between the joint velocity  $\dot{q}$  and the corresponding end-effector linear velocity  $v_c$  and angular velocity  $\omega_c$  [9].

$$\begin{bmatrix} v_c^T & \omega_c^T \end{bmatrix}^T = J_g(q)\dot{q} = \begin{bmatrix} I & 0 \\ 0 & T(q) \end{bmatrix} J_A(q)\dot{q} \quad (8)$$

where  $J_g(q)$  is the geometric Jacobian matrix and  $J_A(q)$  is the analytic Jacobian and  $T(q)$  is the rotational matrix of end-effector.

Substituting (8) into (6) yields the motion dynamics of the image feature point described by the robot joint velocity as

$$\dot{\xi} = J(q, \xi, {}^c z_t)\dot{q} \quad (9)$$

where  $J(q, \xi, {}^c z_t)$  is the Jacobian matrix defined by

$$J(q, \xi, {}^c z_t) = J_{img}(q, \xi, {}^c z_t) \begin{bmatrix} I & 0 \\ 0 & T(q) \end{bmatrix} J_A(q) \quad (10)$$

Let us denote with  $\xi_d$  the desired image feature vector which is assumed to be constant. We define the image feature error as

$$\tilde{\xi} = \xi_d - \xi \quad (11)$$

Using (9), the time derivative of (11) can be expressed by

$$\dot{\tilde{\xi}} = -J(q, \xi, {}^c z_t)\dot{q} = -J(q, \xi, {}^c z_t)(\dot{q}_d - \dot{\tilde{q}}) \quad (12)$$

where  $\dot{q}_d$  is the desired joint velocity vector and  $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$ .

We take the desired joint velocity  $\dot{q}_d$  as

$$\dot{q}_d = -J^+(q, \xi, {}^c z_t)K\tilde{\xi} \quad (13)$$

where  $K$  is positive definite gain matrix and  $J^+(q, \xi, {}^c z_t)$  is the pseudo inverse matrix defined by  $J^+(q, \xi, {}^c z_t) = [J^T(q, \xi, {}^c z_t)J(q, \xi, {}^c z_t)]^{-1}J^T(q, \xi, {}^c z_t)$ .

Substituting (13) into (12) leads to

$$\dot{\tilde{\xi}} = -K\tilde{\xi} + J(q, \xi, {}^c z_t)\dot{\tilde{q}} \quad (14)$$

Now, let us start to design an image-based robust controller to compensate the bounded Jacobian and parametric uncertainties of robot manipulators.

First, we assume that the uncertainty of the parameter vector  $\theta$  is bounded as

$$\|\tilde{\theta}\| = \|\theta_0 - \theta\| \leq \rho, \quad \rho > 0 \quad (15)$$

where  $\theta_0$  is the nominal value of  $\theta$  and  $\rho$  is a known value.

And it is also assumed that the uncertainty of the Jacobian matrix  $J(q, \xi, {}^c z_t)$  is bounded as

$$\|\tilde{J}(q, \xi, {}^c z_t)\| = \|J(q, \xi, {}^c z_t) - \hat{J}(q, \xi, {}^c z_t)\| \leq \sigma, \quad \sigma > 0 \tag{16}$$

where  $\hat{J}(q, \xi, {}^c z_t)$  is the estimated value of  $J(q, \xi, {}^c z_t)$  and  $\sigma$  is a known value.

Next, in order to achieve the control objective that performs tracking control of robot manipulator (1) in the presence of bounded parametric uncertainties and bounded Jacobian uncertainties, we propose the image-based robust controller as follows:

$$\tau = Y(q, \dot{q}_d, \ddot{q}_d)(\theta_0 + u) + K_v \dot{\tilde{q}} + \hat{J}^T(q, \xi, {}^c z_t) \tilde{\xi} + k \|\tilde{\xi}\|^2 \dot{\tilde{q}} \tag{17}$$

where  $K_v$  is the symmetric positive definite gain matrix,  $u$  is an additional control input that will be designed to achieve robustness to the parametric uncertainty, the gain  $k$  is selected to compensate Jacobian uncertainty and  $Y(q, \dot{q}_d, \ddot{q}_d)\theta_0$  is represented by nominal values of  $M(q)$ ,  $C(q, \dot{q})$  and  $G(q)$  such that

$$Y(q, \dot{q}_d, \ddot{q}_d)\theta_0 = M_0(q)\ddot{q}_d + C_0(q, \dot{q})\dot{q}_d + G_0(q) \tag{18}$$

Substituting (17) into (1) and defining the state vector as  $[\tilde{\xi}^T \quad \dot{\tilde{q}}^T]^T$ , the closed-loop system equation is obtained

$$\begin{bmatrix} \dot{\tilde{\xi}} \\ \dot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} -K\tilde{\xi} + J\dot{\tilde{q}} \\ M^{-1}(q) \left[ -C(q, \dot{q})\dot{\tilde{q}} + Y(q, \dot{q}_d, \ddot{q}_d)(\tilde{\theta} + u) - K_v\dot{\tilde{q}} - \hat{J}^T\tilde{\xi} - k\|\tilde{\xi}\|^2\dot{\tilde{q}} \right] \end{bmatrix} \tag{19}$$

We propose the additional control input  $u$  and the gain  $k$  as

$$u = \begin{cases} -\rho \frac{Y^T \dot{\tilde{q}}}{\|Y^T \dot{\tilde{q}}\|}, & \|Y^T \dot{\tilde{q}}\| > \varepsilon \\ -\rho \frac{Y^T \dot{\tilde{q}}}{\varepsilon}, & \|Y^T \dot{\tilde{q}}\| \leq \varepsilon \end{cases}, \quad k = \begin{cases} \frac{\sigma}{\|\dot{\tilde{q}} \tilde{\xi}^T\|}, & \|\dot{\tilde{q}} \tilde{\xi}^T\| > \eta \\ \frac{\sigma}{\eta}, & \|\dot{\tilde{q}} \tilde{\xi}^T\| \leq \eta \end{cases} \tag{20}$$

where  $\varepsilon > 0$  and  $\eta > 0$  are design parameters.

Finally, show the robust stability of the proposed control system despite robot parametric and Jacobian uncertainty exists. Consider the Lyapunov function candidate

$$V = \frac{1}{2} \dot{\tilde{q}}^T M(q) \dot{\tilde{q}} + \frac{1}{2} \tilde{\xi}^T \tilde{\xi} \tag{21}$$

Differentiation of  $V$  along the trajectory of (19) yields

$$\dot{V} = -\chi^T W \chi + \dot{\tilde{q}}^T Y(q, \dot{q}_d, \ddot{q}_d) (\tilde{\theta} + u) + \dot{\tilde{q}}^T \tilde{J}^T(q, \xi, {}^c z_t) \tilde{\xi} - k \|\tilde{\xi}\|^2 \|\dot{\tilde{q}}\|^2 \quad (22)$$

where  $\chi = \begin{bmatrix} \tilde{\xi}^T & \dot{\tilde{q}}^T \end{bmatrix}^T$  and  $W = \begin{bmatrix} K^T & 0 \\ 0 & K_v \end{bmatrix}$ .

The first, if  $\|Y^T \dot{\tilde{q}}\| > \varepsilon$  and  $\|\dot{\tilde{q}} \tilde{\xi}^T\| > \eta$ , we have

$$\dot{V} \leq -\lambda_{\min}(W) \|\chi\|^2 + \|\dot{\tilde{q}}\| \left( \|\tilde{J}\| - \sigma \right) \|\tilde{\xi}\| \leq 0 \quad (23)$$

where  $\lambda_{\min}(W)$  denotes the minimum eigenvalue of  $W$ .

The second, if  $\|Y^T \dot{\tilde{q}}\| \leq \varepsilon$  and  $\|\dot{\tilde{q}} \tilde{\xi}^T\| > \eta$ , we have

$$\dot{V} \leq -\chi^T W \chi + \frac{\varepsilon \rho}{4} + \|\dot{\tilde{q}}\| \left( \|\tilde{J}\| - \sigma \right) \|\tilde{\xi}\| \leq -\lambda_{\min}(W) \|\chi\|^2 + \frac{\varepsilon \rho}{4} \quad (24)$$

For  $\|\chi\| \geq \frac{1}{2} \sqrt{\frac{\varepsilon \rho}{\lambda_{\min}(W)}}$ , we have  $\dot{V} \leq 0$ .

The third, if  $\|Y^T \dot{\tilde{q}}\| > \varepsilon$  and  $\|\dot{\tilde{q}} \tilde{\xi}^T\| \leq \eta$ , we have

$$\begin{aligned} \dot{V} &\leq -\chi^T W \chi + \|Y^T \dot{\tilde{q}}\| \left( \|\tilde{\theta}\| - \rho \right) + \sigma \|\dot{\tilde{q}}\| \|\tilde{\xi}\| - \frac{\sigma}{\eta} \|\tilde{\xi}\|^2 \|\dot{\tilde{q}}\|^2 \\ &\leq -\lambda_{\min}(W) \|\chi\|^2 + \frac{\eta \sigma}{4} \end{aligned} \quad (25)$$

For  $\|\chi\| \geq \frac{1}{2} \sqrt{\frac{\eta \sigma}{\lambda_{\min}(W)}}$ , we have  $\dot{V} \leq 0$ .

The last, if  $\|Y^T \dot{\tilde{q}}\| \leq \varepsilon$  and  $\|\dot{\tilde{q}} \tilde{\xi}^T\| \leq \eta$ , we have

$$\begin{aligned} \dot{V} &\leq -\chi^T W \chi + \|Y^T \dot{\tilde{q}}\|^T \rho - \frac{\rho}{\varepsilon} \|Y^T \dot{\tilde{q}}\|^2 + \sigma \|\dot{\tilde{q}}\| \|\tilde{\xi}\| - \frac{\sigma}{\eta} \|\tilde{\xi}\|^2 \|\dot{\tilde{q}}\|^2 \\ &\leq -\chi^T W \chi + \frac{\varepsilon \rho}{4} + \frac{\eta \sigma}{4} \end{aligned} \quad (26)$$

For  $\|\chi\| \geq \frac{1}{2} \sqrt{\frac{\varepsilon \rho + \eta \sigma}{\lambda_{\min}(W)}}$ , we have  $\dot{V} \leq 0$ .

Therefore, the control law (17) and (20) guarantees the uniformly ultimate boundedness of the closed-loop system.

## 4 Experiments

The performance of the proposed controller is demonstrated by experiments on a two degree of freedom 5-link industrial robot manipulator manufactured by Samsung Electronic, co. as shown in Fig. 1. Experimental systems are composed of a industrial robot manipulator, the main computer with a MMC(multi motion control) board and an image processing board and a CCD camera mounted on the end effector of the manipulator. A MMC board mounted in a main computer is used to execute the control algorithm. The feature signal is acquired by an image processing board(Matrox Meteor II) mounted on a main computer which processes the image obtained from a CCD camera and extracts the image feature.

In these experiments, we assume that a payload of around  $5.5\text{ kg}$  is added to the mass  $m_4$  and the other parameters are not affected by this added payload. The object was located at a distance  $Z = 1[m]$  in front of the camera and parallel to the plane where the manipulator moves. Design parameters are shown in Table 1.

The experimental results are shown in Fig. 2-5. Fig. 2 and 3 show the tracking errors for the proposed method with Jacobian compensation and for the control



**Fig. 1.** Farama-As1 industrial robot manipulator

**Table 1.** Design parameters

$\eta = 0.5$	$\sigma = 50$	$\alpha = 6.941 \times 10^{-6}$	$q_1 = \pi / 2 [rad], q_2 = \pi [rad]$
$\varepsilon = 1.0$	$K = 1.5I$	$\beta = 9.425 \times 10^{-6}$	$\xi = [100 \ 100]^T$
$\rho = 1.0394$	$K_v = 10I$	$f = 0.016 [m]$	$\xi_d = [0 \ 0]^T$

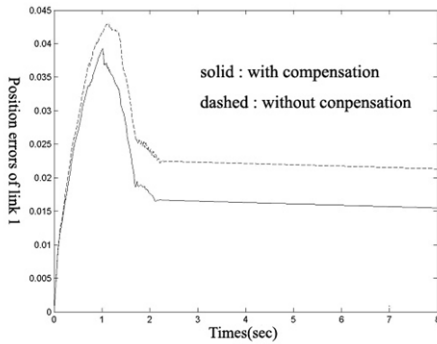


Fig. 2. Position errors of link 1

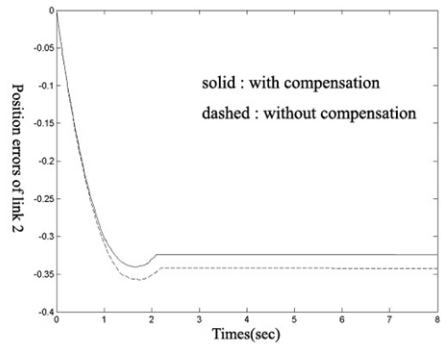


Fig. 3. Position errors of link 2

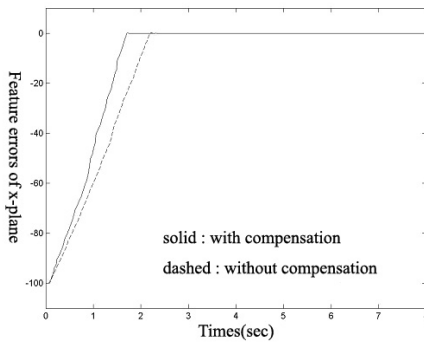


Fig. 4. Feature errors of x-plane

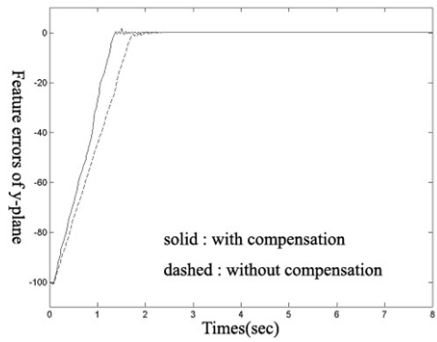


Fig. 5. Feature errors of y-plane

algorithm without Jacobian compensation. The steady-state errors by the proposed method are improved comparing to those by the method without compensation. Fig. 4 and 5 illustrate the trajectory of feature point on the image plane, which show the convergence to the desired feature point by the proposed method is improved. Experimental results show that the implementation of the proposed algorithm can obtain effective control performance.

## 5 Conclusion

In this paper, an image-based robust controller for robot manipulators was proposed. This controller includes the nonlinear control terms to compensate the parametric and Jacobian uncertainty. The ultimate uniform stability of the overall closed-loop system is proved by using the Lyapunov method. Experiment results on a 5-link manipulator with two degree of freedom have shown that the proposed control method has effectiveness to control robot manipulators with parametric and Jacobian uncertainty.



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