Fuzzy Ontology, Fuzzy Description Logics and Fuzzy-OWL

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Abstract. The conceptual formalism supported by an ontology is not sufficient for handling vague information that is commonly found in many application domains. We describe how to introduce fuzziness in an ontology. To this aim we define a framework consisting of a fuzzy ontology based on Fuzzy Description Logic and Fuzzy–Owl.

1 Introduction

In recent years ontologies played a major role in knowledge representation. For example, applications of the Semantic Web [1] (i.e., e-commerce, knowledge management, web portals, etc.) are based on ontologies. In the Semantic Web an ontology is a formal conceptualization of a domain of interest, shared among heterogeneous applications. It consists of *entities, attributes, relationships* and *axioms* to provide a common understanding of the real world [2, 3]. With the support of ontologies users and systems can communicate through an easy information exchange and integration. Unfortunately, the conceptual formalism supported by the ontology structure is not sufficient for handling imprecise information that is commonly found in many application domains. Indeed, humans use linguistic adverbs and adjectives to describe their requests. For instance, a user can be interested in finding topics about "an expensive item" or "a fun holiday" using web portals. The problem that emerges is how to represent these non-crisp data within the ontology definition.

Fuzzy sets theory, introduced by L. A. Zadeh [4], allows to deal with imprecise and vague data, so that a possible solution is to incorporate fuzzy logic into ontologies. In [5] we gave a first definition of fuzzy ontology. Here we present a better formalization which can be mapped to a suitable Fuzzy Description Logic. Let us note that also [6] gives a formalization of a fuzzy ontology, but it does not investigate its relationship to Fuzzy DL and Fuzzy–OWL. This is of great importance due to the central role that Description Logic and OWL play in the Semantic Web.

Further, $\mathcal{SHOIN}(\mathcal{D})$ is the theoretical counterpart of the OWL Description Logic. Thus, in the current paper, we define a fuzzy extension of the OWL language considering fuzzy $\mathcal{SHOIN}(\mathcal{D})$ [7]. We have extended the syntax and semantic of fuzzy $\mathcal{SHOIN}(\mathcal{D})$ with the possibility to add a concept modifier to a relation and introducing a new constructor which enable us to define a subset of objects belonging to a given concept with a membership value greater or lower that a fixed value.

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Our idea is to map the fuzzy ontology definition (presented in this paper) into the corresponding Fuzzy-OWL language through the syntax and semantic of fuzzy $\mathcal{SHOIN}(\mathcal{D})$. Finally, we propose an extension of the KAON project [8] in order to directly define some axioms of the fuzzy ontology through graph-based and tree-based metaphors.

2 Fuzzy Ontology

In this section, we formally introduce the notion of Fuzzy Ontology. Our definition is based on the vision of an ontology for the Semantic Web where knowledge is expressed in a DL-based ontology. Thus, a fuzzy ontology is defined in order to correspond to a DL knowledge base as we will give in Section 3 [9].

Definition 1. *A Fuzzy Ontology is defined as the tuple* $\mathbf{O_F} = \{\mathbf{I}, \mathbf{C}, \mathbf{R}, \mathbf{F}, \mathbf{A}\}$ *where:*
- $\mathbf{I}$ *is the set of individuals, also called instances of the concepts.*
- $\mathbf{C}$ *is the set of concepts. Each concept* $C \in \mathbf{C}$ *is a fuzzy set on the domain of instances* $C : \mathbf{I} \mapsto [0, 1]$. *The set of entities of the fuzzy ontology will be indicated by* $\mathbf{E}$, *i.e.,* $\mathbf{E} = \mathbf{C} \cup \mathbf{I}$.
-$\mathbf{R}$ *is the set of relations. Each* $R \in \mathbf{R}$ *is a n-ary fuzzy relation on the domain of entities,* $R : \mathbf{E}^n \mapsto [0, 1]$. *A special role is held by the taxonomic relation* $\mathcal{T} : \mathbf{E}^2 \mapsto [0, 1]$ *which identifies the fuzzy subsumption relation among the entities.*
- $\mathbf{F}$ *is the set of the fuzzy relations on the set of entities* $\mathbf{E}$ *and a specific domain contained in* $\mathcal{D} = \{integer, string, ...\}$. *In detail, they are n-ary functions such that each element* $F \in \mathbf{F}$ *is a relation* $F : \mathbf{E}^{(n-1)} \times P \mapsto [0, 1]$ *where* $P \in \mathcal{D}$.
- $\mathbf{A}$ *is the set of axioms expressed in a proper logical language, i.e., predicates that constrain the meaning of concepts, individuals, relationships and functions.*

Let us note that any concept and any relation is fuzzy. In particular the taxonomic relationship $\mathcal{T}(i, j)$ indicates that the child j is a conceptual specification of the parent i with a certain degree. For example, in an ontology of the "animals" an expert can have some problems on how to insert the "platypus" instance, since it is in part a "mammal" and in part an "oviparous". Using the fuzzy subsumption relationships $\mathcal{T}(mammal, platypus) = x$ and $\mathcal{T}(oviparous, platypus) = y$, where x, y are two arbitrary fuzzy values, it is possible to declare partial relations in order to better specify the ontology knowledge. The same holds for non-taxonomic relationships. For instance, a way to describe the fact "Paul lives sometimes in London and sometimes in Rome" could be $Lives(Paul, London) = 0.6$, $Lives(Paul, Rome) = 0.5$.

Of course, since fuzzy sets are a sound extension of classical boolean sets, it is always possible to define crisp (i.e, non-fuzzy) concepts (resp., relations) by using only values in the set $\{0, 1\}$.

A particular interest in our work is held by the non-taxonomic fuzzy relationship "correlation" defined as $Corr : \mathbf{E}^2 \mapsto [0, 1]$ (see [10, 11]). The correlation is a binary and symmetric fuzzy relationship that allows to specify the semantic link among the entities of the fuzzy ontology. The values of correlation between two objects can be assigned not only by the expert of the domain, but also considering the knowledge based on how the two objects are used together (for instance, in the queries or in the documents definition). For example, it is possible to state that "sun and yellow" are semantically correlated with value 0.8, i.e., $Corr(sun, yellow) = 0.8$. Furthermore, it is possible to have the special case where an entity x is itself correlated. For instance, we can affirm that $Corr(sun, sun) = 0.3$. In the implementation phase, for the fuzzy relationship $Corr$ is necessary to define the attribute "count" that allows to storage how many times the entities are searched together.

Properties of relations. In the fuzzy ontology the properties on the relations we are interested in are symmetry and transitivity. Given a fuzzy ontology $\mathbf{O_F}$, a binary relation $R : \mathbf{E} \times \mathbf{E} \mapsto [0,1]$ is *Symmetric* if $\forall i, j \in \mathbf{E}$, $R(i,j) = R(j,i)$ and *Transitive* if $\forall i, j \in \mathbf{E}$, $\sup_{k \in \mathbf{E}}\{t(R(i,k), R(k,j))\} \leq R(i,j)$, where t is a t-norm. Further, given a binary relation $R : \mathbf{E} \times \mathbf{E} \mapsto [0,1]$, its *inverse* relation is defined as $R^-(i,j) := R(j,i)$. Thus, we have that a relation is symmetric if and only if $\forall i, j \in \mathbf{E}$, $R(i,j) = R^-(i,j)$.

3 Fuzzy Description Logic

Our next step in the description of a complete framework for a fuzzy ontology is the definition of a fuzzy description logic. Let us note that in literature there are several approaches to this topic. The most complete and coherent one is [7]. Stoilos et. al [12] have also presented a Fuzzy-OWL language version based only on $\mathcal{SHOIN}$ discarding datatypes and concept modifiers. We take inspiration mainly from Straccia's work [7], adding a complete formalization of fuzzy axioms and introducing some differences:

- we add the possibility to have fuzzy relations with modifiers, and not only modified fuzzy concepts. This can be helpful to express a sentence as "there is a *strong* correlation between sun and yellow" where strong is a modifier and "correlation" a fuzzy relation;
- we give a different semantic of cardinality restriction;
- we add a new possibility to define a concept: $\leq_\alpha C$ (and similarly $\geq_\alpha$, $<_\alpha$, $>_\alpha$) which enable us to define, for instance, the fuzzy set of "people which are tall with value lower than 0.3" or the "wines which have a dry taste with a value at least of 0.6".

Decidability and computability issues of these modifications will be investigated in a forthcoming paper.

3.1 Syntax

The alphabet of the logic is (C, R_a, R_c, I_a, I_c) where C is the set of concept names, R_a (resp., R_c) is the set of abstract (resp., concrete) role names, I_a (resp., I_c) the set of abstract (resp., concrete) individual names. All these sets are non-empty and they are pair-wise disjoint. A concrete domain is a pair $\langle \Delta_D, \Phi_D \rangle$ where Δ_D is an interpretation domain and Φ_D the set of concrete fuzzy relations p on the domain Δ_D with interpretation $p^D : \Delta_D^n \mapsto [0,1]$. The set of modifier names is denoted as M and to each element $m \in M$ is associated its interpretation $f_m : [0,1] \mapsto [0,1]$.

Finally, using the following notation: $A \in C$ is a concept, $R \in R_a$ an abstract relation name, $T \in R_c$ a concrete relation name, $S \in R_a$ an abstract simple relation name (a relation is simple if it is not transitive and it has not transitive sub-relations), $m \in M$ a modifier name, $p \in \Phi_D$ a concrete predicate name, $a \in I_a$ an abstract instance name,

$c \in I_c$ a concrete instance name, $n \in \mathbf{N}$, we can define a fuzzy-$\mathcal{SHOIN}(D)$ concept according to the following rules.

$$\begin{aligned} C &\rightarrow \top|\bot|A|C_1 \sqcup C_2|C_1 \sqcap C_2|\neg C|\forall P.C|\exists P.C|(\leq n\,S)|(\geq n\,S)|\{a_1,\ldots,a_n\}|mC| \\ &\quad |(\leq n\,T)|(\geq n\,T)|<_\alpha C| \leq_\alpha C| >_\alpha C| \geq_\alpha C|\forall T_1 \ldots T_n.D|\exists T_1 \ldots T_n.D| \\ D &\rightarrow p|\{c_1,\ldots,c_n\} \qquad P \rightarrow R|R^-|mR \end{aligned}$$

Now, we introduce the axioms, which, as usual, are divided in three categories. From now on, by $\star$ we mean a symbol in $\{<,\leq,>,\geq,=,\neq\}$ and by α a value in $[0,1]$.

TBox. Let A, B be concepts. A fuzzy inclusion axiom is $(A \sqsubseteq B) \star \alpha$. Let us note that non-fuzzy inclusion axioms can be obtained as $(A \sqsubseteq B) = 1$
RBox. Let $R_1, R_2 \in R_a$ and $T_1, T_2 \in R_a$. Fuzzy role inclusion axioms are $(R_1 \sqsubseteq R_2) \star \alpha$ and $(T_1 \sqsubseteq T_2) \star \alpha$. Further, we can have transitivity axioms $\textsc{trans}(R)$.
ABox. Let $a, b \in I_a$, $c \in I_c$ and C a concept. Then, ABox axioms are $\langle a : C\rangle \star \alpha$, $\langle (a,b) : R\rangle \star \alpha$, $\langle (a,c) : T\rangle \star \alpha$, $a = b$ and $a \neq b$.

A *Knowledge Base* is a triple $\langle \mathcal{T}, \mathcal{R}, \mathcal{A}\rangle$ with $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$ respectively a TBox, RBox and ABox.

3.2 Semantics

The interpretation is given by a pair $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$ where $\Delta^{\mathcal{I}}$ is a set of objects with empty intersection with the concrete domain Δ_D: $\Delta^{\mathcal{I}} \cap \Delta_D = \emptyset$. An individual $a \in I_a$ is mapped to an object in $\Delta^{\mathcal{I}}$: $a^I \in \Delta^{\mathcal{I}}$. An individual $c \in I_c$ is mapped to an object $c^{\mathcal{I}} \in \Delta_D$. A concept $A \in C$ is interpreted as a fuzzy set on the domain $\Delta^{\mathcal{I}}$, $A^{\mathcal{I}} : \Delta^{\mathcal{I}} \mapsto [0,1]$. Abstract roles $R \in R_a$ and concrete roles $T \in R_c$ are interpreted as fuzzy binary relations, respectively: $R : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mapsto [0,1]$ and $T : \Delta^{\mathcal{I}} \times \Delta_D \mapsto [0,1]$. The interpretation of the concepts is given according to table 1, where t is t-norm, s a t-conorm, $\rightarrow$ a (residual) implication, N a negation, $m \in M$ a modifier, $x, y \in \Delta^{\mathcal{I}}$ and $v \in \Delta_D$. For the sake of simplicity we omit the semantic of $\leq nS$ which is dual to $\geq nS$ and of $<_\alpha, \leq_\alpha, >_\alpha$ which are similar to $\geq_\alpha$.

Let us note that the semantic of cardinality restrictions $\geq nS$ and $\leq nS$ is different from both [7] and [12]. Indeed, we do not fuzzify them, since, in our opinion, the property $\forall x$ "there are at least n distinct elements that satisfy to some degree" [7] $S(x,y)$, i.e, the semantic of $\geq nS$, is satisfied or not, in a Boolean fashion. For instance, a "Tortoiseshell cat" is characterized by having three colours. This can be expressed in fuzzy DL as $\geq 3HasColor \sqcap \leq 3HasColor$ which is a crisp concept. That is, a cat is a "Tortoiseshell" if it has exactly three colours, each of them to some (fuzzy) degree. Further, the *classical* relationship $\leq nS \equiv \neg(\geq (n+1)S)$ is satisfied and, as showed below, the semantic of a function is coeherent with the idea that a function assigns to any instance only one value, in this fuzzy environment with a certain degree.

In Table 2 the interpretation of axioms is given. Further important axioms derivable in fuzzy $\mathcal{SHOIN}(D)$ from the primitive ones are the requirement that a relation is symmetric and that a relation is a function. They can be respectively expressed as $R \equiv R^-$ and $(\top \sqsubseteq \leq 1S) = 1$ whose semantic, according to Table 2, is $\forall a, b \in \Delta^{\mathcal{I}}$, $R^{\mathcal{I}}(a,b) = (R^-)^{\mathcal{I}}(a,b)$ and $\forall x \in \Delta^{\mathcal{I}}$, $|\{y \in \Delta^{\mathcal{I}} : S(x,y) \geq 0\}| \leq 1$.

Table 1. Interpretation of concepts in fuzzy $\mathcal{SHOIN}(D)$

$\bot^{\mathcal{I}}(x)$	0
$\top^{\mathcal{I}}(x)$	1
$(C_1 \sqcap C_2)^{\mathcal{I}}(x)$	$t(C_1^{\mathcal{I}}(x), C_2^{\mathcal{I}}(x))$
$(C_1 \sqcup C_2)^{\mathcal{I}}(x)$	$s(C_1^{\mathcal{I}}(x), C_2^{\mathcal{I}}(x))$
$(\neg C)^{\mathcal{I}}(x)$	$(N(C^{\mathcal{I}}))(x)$
$(mC)^{\mathcal{I}}(x)$	$f_m(C^{\mathcal{I}}(x))$
$(R^-)^{\mathcal{I}}(x,y)$	$R^{\mathcal{I}}(y,x)$
$(mR)^{\mathcal{I}}(x,y)$	$f_m(R^{\mathcal{I}}(x,y))$
$(\forall P.C)^{\mathcal{I}}(x)$	$\inf_{y \in \Delta^{\mathcal{I}}}\{P^{\mathcal{I}}(x,y) \rightarrow C^{\mathcal{I}}(y)\}$
$(\exists P.C)^{\mathcal{I}}(x)$	$\sup_{y \in \Delta^{\mathcal{I}}}\{t(P^{\mathcal{I}}(x,y), C^{\mathcal{I}}(y))\}$
$(\geq nS)^{\mathcal{I}}(x)$	$\begin{cases} 1 & \text{if} \quad \lvert\{y \in \Delta^{\mathcal{I}} : S(x,y) > 0\}\rvert \geq n \\ 0 & \text{otherwise} \end{cases}$
$(\geq_\alpha C)^I(x)$	$\begin{cases} C(x) & \text{if} \quad C(x) \geq \alpha \\ 0 & \text{otherwise} \end{cases}$
$\{a_1,\ldots,a_n\}^{\mathcal{I}}(x)$	$\begin{cases} 1 & \text{if } x \in \{a_1,\ldots,a_n\} \\ 0 & \text{otherwise} \end{cases}$
$\{c_1,\ldots,c_n\}^{\mathcal{I}}(v)$	$\begin{cases} 1 & \text{if } v \in \{c_1,\ldots,c_n\} \\ 0 & \text{otherwise} \end{cases}$
$(\forall T_1 \ldots T_n.D)^{\mathcal{I}}(x)$	$\inf_{y_i \in \Delta_D}\{t_{i=1}^n T_i^{\mathcal{I}}(x,y_i) \rightarrow D^{\mathcal{I}}(y_1,\ldots,y_n)\}$
$(\exists T_1 \ldots T_n.D)^{\mathcal{I}}(x)$	$\sup_{y_i \in \Delta_D}\{t(t_{i=1}^n T_i^{\mathcal{I}}(x,y_i), D^{\mathcal{I}}(y_1,\ldots,y_n))\}$

Table 2. Interpretation of axioms in fuzzy $\mathcal{SHOIN}(D)$

$(C \equiv D)^{\mathcal{I}}$	$\forall x \in \Delta^{\mathcal{I}}\ C^{\mathcal{I}}(x) = D^{\mathcal{I}}(x)$
$((C \sqsubseteq D) \star \alpha)^{\mathcal{I}}$	$(\inf_{x \in \Delta^{\mathcal{I}}}\{C^{\mathcal{I}}(x) \rightarrow D^{\mathcal{I}}(x)\}) \star \alpha$
$(R_1 \equiv R_2)^{\mathcal{I}}$	$\forall x,y \in \Delta^{\mathcal{I}}\ R_1^{\mathcal{I}}(x,y) = R_2^{\mathcal{I}}(x,y)$
$((R_1 \sqsubseteq R_2) \star \alpha)^{\mathcal{I}}$	$(\inf_{x,y \in \Delta^{\mathcal{I}}}\{R_1^{\mathcal{I}}(x,y) \rightarrow R_2^{\mathcal{I}}(x,y)\}) \star \alpha$
$((T_1 \sqsubseteq T_2) \star \alpha)^{\mathcal{I}}$	$(\inf_{x \in \Delta^{\mathcal{I}}, v \in \Delta_D}\{T_1^{\mathcal{I}}(x,v) \rightarrow T_2^{\mathcal{I}}(x,v)\}) \star \alpha$
$(\langle a : C\rangle \star \alpha)^{\mathcal{I}}$	$C^{\mathcal{I}}(a^{\mathcal{I}}) \star \alpha$
$(\langle (a,b) : R\rangle \star \alpha)^{\mathcal{I}}$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \star \alpha$
$Trans(R)$	$\forall a,b,c \in \Delta^{\mathcal{I}}$ $\sup_{b \in \Delta^{\mathcal{I}}} t(R^{\mathcal{I}}(a,b), R^{\mathcal{I}}(b,c)) \leq R^{\mathcal{I}}(a,c)$
$(a = b)^{\mathcal{I}}$	$a^{\mathcal{I}} = b^{\mathcal{I}}$
$(a \neq b)^{\mathcal{I}}$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$

As an example let us consider the property HasColor with value white, In fuzzy $\mathcal{SHOIN}(D)$ it can be expressed as $(\exists HasColor.\{white\})$ which, once applied to the individual Silvester, becomes $(\exists HasColor.\{white\})(Silvester) = \sup_y\{t(HasColor(Silvester, y), \{white\}(y))\}$. According to the given semantics, $\{white\}(y)$ is different from 0 (and in particular equal to 1) only when $y = white$. Thus, the above statement $(\exists HasColor.\{white\})(Silvester)$ is equivalent to $HasColor(Silvester, white)$. Finally, it is possible to define the set of "white cats which are white with a degree at least of 0.3" as the axiom $(White - Cat \sqsubseteq \geq_{0.3} (\exists HasColor.\{white\})) = 1$. Indeed, the semantics of the last statement is

$$\left(\inf_{x \in \Delta^{\mathcal{I}}} \left\{ White - Cat(x) \rightarrow \begin{cases} HC(x, white) & HC(x, white) \geq 0.3 \\ 0 & \text{otherwise} \end{cases} \right\}\right) = 1$$

and considering that $\rightarrow$ is a residual implication, we must have that

$$\forall x\, White - Cat(x) \leq \begin{cases} HC(x, white) & HC(x, white) \geq 0.3 \\ 0 & \text{otherwise} \end{cases}$$

Thus, if for a cat x, $HasColor(x, white) = 0.2$, it must be $White - Cat(x) = 0$, i.e., it does not belong to the set of white cats.

4 Fuzzy-OWL

Once we have defined a fuzzy ontology and after to have showed how to extend $\mathcal{SHOIN}(D)$, the next step is to define the new fuzzy language suitable to implement the fuzzy ontology. In order to achieve this goal, the logical framework of the KAON project has been extended .

4.1 Defining Fuzzy Ontology in KAON

The KAON project is a meta-project carried out at the Institute AIFB, University of Karlsruhe and at the Research Center for Information Technologies (FZI) [13]. KAON includes a comprehensive tool suite allowing easy creation, maintenance and management of ontologies. An important user-level application supplied by KAON is an ontology editor called OI-modeler whose most important features are its support for manipulation of large ontologies and for user-directed evolution of ontologies. In the last years, KAON has been applied to the Semantic Web [14].

An ontology in KAON consists of concepts (sets of elements), properties (specifications of how objects may be connected) and instances grouped in reusable units called OI-models (ontology-instance models) [13]. The conceptual model proposed allows to define an entity in different ways, depending on the point of view of the observer. That is, an entity can be interpreted as a concept, as well as an instance.

Fuzzy ontologies in KAON. Our aim is to enrich KAON language adding the proposed fuzzy-sets approach. In order to integrate our framework in the KAON project we have developed a suited "Fuzzy Inspector". The Fuzzy Inspector is composed by a table representing fuzzy entity, a membership degree and a number of updates Q. This new panel allows to the expert an easy fuzzy logic integration.

Furthermore, the Fuzzy Inspector allows to assign the fuzzy values in two ways in order to handle the trade off between understandability and precision [15]. In the first case, he/she can assign a precise value (between 0 and 1) to define an high degree of accuracy according to his/her experience. Whereas, in the second case, he/she can assign a linguistic value defining an high degree of interpretability. He/she can choose a linguistic value by a combo-list where automatically a numerical value is assigned. The choice of this list has been made arbitrarily, an expert can choose the linguistic values suitable to the context, and the numerical values relative to the labels can be calculated by the Khang et al.'s algorithm [16].

Fuzzy-OWL language. KAON's ontology language is based on RDFS [17] with proprietary extensions for algebraic property characteristics (symmetric, transitive and

Table 3. Constructor

Fuzzy constr.	Example for Fuzzy-OWL
$\geq_\alpha \exists R.\{x\}$	<fowl:Restriction> <fowl:onProperty rdf:resource="♯R"/ > <fowl:hasValue rdf:resource="♯x"/ > <fowl:moreOrEquivalent fowl:degree=α/ > < /fowl:Restriction>

Table 4. Axioms and fuzzy constraint between concepts

Fuzzy axioms	Example for Fuzzy-OWL
$A \sqsubseteq B \star \alpha$	<fowl:Class rdf:ID="A"> <fowl:subClassOf rdf:resource=♯B / > <fowl:ineqType fowl:degree=α / > < /fowl:Class>
$(\top \sqsubseteq (\leq 1S)) = 1$	<fowl:ObjectProperty rdf:ID="S"> <rdf:type rdf:resource="FunctionalProperty"/ > <rdfs:domain rdf:resource="♯A"/ > <rdfs:range rdf:resource="♯B"/ > <fowl:ObjectProperty/ >
A(a) $\star \alpha$	<fowl:Thing rdf:ID="a"/ > <fowl:Thing rdf:about="♯a"/ > <rdf:type rdf:resource="♯A"/ > <fowl:ineqType fowl:degree=α / > < /fowl:Thing>
R(a,b) $\star \alpha$	<fowl:Thing rdf:ID="a"> <R rdf:resource="♯b"/ > <fowl:ineqType fowl:degree=α / > < /fowl:Thing>

Fuzzy constraints	Example for Fuzzy-OWL
R(c,d) $\star \alpha$	<rdf:Description rdf:about"c"> <R rdf:resource="♯d" fowl:ineqType fowl:degree=α / > < /rdf:Description>

inverse), cardinality, modularization, meta-modelling and explicit representation of lexical information. But it is possible to export the fuzzy ontology file in the OWL [18] format. OWL DL is the language chosen by the major ontology editors because it supports those users who want the maximum expressiveness without losing computational completeness (all conclusions are guaranteed to be computed) and decidability of reasoning systems (all computations will finish in finite time) [18]. However, OWL DL does not allow to handle the information represented with a not precise definition. Our aim is to present an extension of OWL DL, named Fuzzy-OWL, by adding a fuzzy value to the entities and relationships of the ontology following the fuzzy $\mathcal{SHOIN}(\mathcal{D})$ syntax of Section 3 and the fuzzy ontology definition given in Section 2. In Table 3 is reported only the new constructor $\geq_\alpha$ defined in Section 3 (here the namespace is "fowl"), where $\alpha \in [0, 1]$ allows to state the fuzzy values into the two ways previously described (i.e. by a combo-list or directly editing the value). Other constructors are defined analogously (see also [19]).

Table 4 reports the major axioms of Fuzzy-OWL language, where "a,b" are two individuals and "ineqType"= "moreOrEquivalent | lessOrEquivalent | moreThan | lessThan | Exactly". The fuzzy constraint between concepts "c" and "d" is useful for defining the non-taxonomic relationship "Corr" (see Section 2). In order to represent this, we adopt the solution proposed in [20] using RDF/XML syntax in Fuzzy OWL's DL language.

5 Conclusions

We outlined a complete framework for building a fuzzy ontology for the Semantic Web. Apart from defining a fuzzy ontology this required a coherent definition of Fuzzy

Description Logic and Fuzzy-OWL. With this new framework is possible to introduce and handle vagueness, an intrinsic characteristic of the web (and of human reasoning in general). From the applicative point of view some work is still needed. Indeed KAON is based on RDF(S) and all its limits with respect to OWL DL are well-known [21]. Although KAON language is based on own extension, this is not sufficient for representing all the constructors and axioms of OWL DL. For example, it is not possible to define the union, intersection and complement constructor between classes. A possible solution is to export the KAON file in the Fuzzy-OWL language. This will also enable the use of the fuzzy ontology in a fuzzy inference engine (for example KAON2 [22]).

References

[1] Berners-Lee, T., Hendler, T., Lassila, J.: The semantic web. Scientific American 284, 34–43 (2001)

[2] Gruber, T.: A Translation Approach to Portable Ontology Specifications. Knowledge Acquisition 5, 199–220 (1993)

[3] Guarino, N., Giaretta, P.: Ontologies and Knowledge Bases: Towards a Terminological Clarification. In: Mars, N. (ed.) Towards Very Large Knowledge Bases: Knowledge Building and Knowledge Sharing, pp. 25–32. IOS Press, Amsterdam (1995)

[4] Zadeh, L.A.: Fuzzy sets. Inform. and Control 8, 338–353 (1965)

[5] Calegari, S., Ciucci, D.: Integrating Fuzzy Logic in Ontologies. In: Manolopoulos, Y., Filipe, J., Constantopoulos, P., Cordeiro, J. (eds.) ICEIS, pp. 66–73. INSTICC press (2006)

[6] Sanchez, E., Yamanoi, T.: Fuzzy ontologies for the semantic web. In: Larsen, H.L., Pasi, G., Ortiz-Arroyo, D., Andreasen, T., Christiansen, H. (eds.) FQAS 2006. LNCS (LNAI), vol. 4027, pp. 691–699. Springer, Heidelberg (2006)

[7] Straccia, U.: A fuzzy description logic for the semantic web. In: Sanchez, E., ed.: Fuzzy Logic and the Semantic Web. Capturing Intelligence. Elsevier, pp. 73–90 (2006)

[8] KAON: Karlsruhe Ontology and Semantic Web Tool Suite (2005), http://kaon.semanticweb.org

[9] Baader, F., Calvanese, D., McGuinness, D.L., Nardi, D., Patel-Schneider, P.F. (eds.): The Description Logic Handbook: Theory, Implementation, and Applications. Cambridge University Press, Cambridge (2003)

[10] Calegari, S., Loregian, M.: Using dynamic fuzzy ontologies to understand creative environments. In: Larsen, H.L., Pasi, G., Ortiz-Arroyo, D., Andreasen, T., Christiansen, H. (eds.) FQAS 2006. LNCS (LNAI), vol. 4027, pp. 404–415. Springer, Heidelberg (2006)

[11] Calegari, S., Farina, F.: Fuzzy ontologies and scale-free networks analysis. In: RCIS, IEEE (2007) (in printing)

[12] Stoilos, G., Stamou, G., Tzouvaras, V., Pan, J.Z., Horrocks, I.: Fuzzy OWL: Uncertainty and the Semantic Web. In: International Workshop of OWL: Experiences and Directions (OWL-ED2005), Galway, Ireland (2005)

[13] AA.VV.: Developer's Guide for KAON 1.2.7. Technical report, FZI Research Center for Information and WBS Knowledge Management Group (2004)

[14] Oberle, D., Staab, S., Studer, R., Volz, R.: Supporting application development in the semantic web. ACM Trans. Inter. Tech. 5, 328–358 (2005)

[15] Casillas, J., Cordon, O., Herrera, F., Magdalena, L.: Accuracy improvements to find the balance interpretability–accuracy in linguistic fuzzy modeling:an overview. In: Accuracy Improvements in Linguistic Fuzzy Modeling, pp. 3–24. Physica-Verlag, Heidelberg (2003)

[16] Khang, T.D., Störr, H., Hölldobler, S.: A fuzzy description logic with hedges as concept modifiers. In: Third International Conference on Intelligent Technologies and Third Vietnam-Japan Symposium on Fuzzy Systems and Applications, pp. 25–34 (2002)
[17] RDFS: Resource Description Framework Schema (2004) http://www.w3.org/TR/PR-rdf-schema
[18] OWL: Ontology Web Language (2004) http://www.w3.org/2004/OWL/
[19] Gao, M., Liu, C.: Extending OWL by Fuzzy Description Logic. In: IEEE-ICTAI05 (2005)
[20] Stoilos, G., Nikos Simou, G.S., Kollias, S.: Uncertainty and the Semantic Web. IEEE Intelligent System 21, 84–87 (2006)
[21] Jeff, Z., Pan, I.H.: RDFS(FA):Connecting RDF(S) and OWL DL. IEEE Transactions on Knowledge and Data Engineering 19, 192–2006 (2007)
[22] KAON2: Karlsruhe Ontology and Semantic Web Tool Suite 2 (2005), http://kaon2.semanticweb.org