An Evolutionary Approach to Solve a Novel Mechatronic Multiobjective Optimization Problem

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Abstract

In this chapter, we present an evolutionary approach to solve a novel mechatronic design problem of a pinion-rack continuously variable transmission (CVT). This problem is stated as a multiobjective optimization problem, because we concurrently optimize the mechanical structure and the controller performance, in order to produce mechanical, electronic and control flexibility for the designed system. The problem is solved first with a mathematical programming technique called the goal attainment method. Based on some shortcomings found, we propose a differential evolution (DE)-based approach to solve the aforementioned problem. The performance of both approaches (goal attainment and the modified DE) are compared and discussed, based on quality, robustness, computational time and implementation complexity. We also highlight the interpretation of the solutions obtained in the context of the application.

Key words: Parametric Optimal Design, Multiobjective Optimization, Differential Evolution

1 Introduction

The solution of real-world optimization problems poses great challenges, particularly when the problem is relatively unknown, since these uncertainties add an extra complexity layer. Currently, several systems can be considered as mechatronic systems due to the integration of the mechanical and electronical elements in such systems. This is the reason why it is necessary to use new design methodologies that consider integral aspects of the systems.

The traditional approach to the design of mechatronic systems, considers the mechanical behavior and the dynamic performance separately. Therefore, the design of mechanical elements involves kinematic and static behaviors while the design of the control system uses only the dynamic behavior. This design approach from a dynamic point of view cannot produce an optimal system behavior [14, 23]. Recent works on mechatronic systems design propose a concurrent design methodology which considers jointly the mechanical and control performances.

For this concurrent design concept, several approaches have been proposed. However, these concurrent approaches are based on an iterative process. There, the mechanical structure is obtained in a first step and the controller in a second step. If the resulting control structure is very difficult to implement, then the first step must be repeated all over again.

On the other hand, an alternative approach to formulate the system design problem is to consider it as a dynamic optimization problem [2,3]. In order to do this, the parametric optimal design of the mechatronic system needs to be stated as a multiobjective dynamic optimization problem (MDOP). In this approach, both the kinematic and the dynamic models of the mechanical structure and the dynamic model of the controller are considered at the same time, together with system performance criteria. This approach allows us to obtain a set of optimal mechanical and controller parameters in only one step, which could produce a simple system reconfiguration.

In this chapter, we present the parametric optimal design of a pinion-rack continuously variable transmission (CVT). The problem is stated as a multiobjective optimization problem. Two approaches are used to solve it. One is based on a mathematical programming technique called goal attainment [11] and the other is based on an evolutionary algorithm called differential evolution [17]. The remainder of this chapter is organized as follows. In Sect. 2, we detail the transformation of the original problem into a multiobjective optimization problem. In Sect. 3, we present the mathematical programming method, its adaptation to solve the problem and the results obtained. Afterwards, the evolutionary approach is explained and tested in Sect. 4. Later, in Sect. 5, we present a discussion of the behavior of both approaches, based on issues such as quality and robustness of the approach, computation time and implementation complexity. Finally, our conclusions and future paths of research are presented in Sect. 6.

2 Multiobjective Problem

In the concurrent design concept, the mechatronic design problem can be stated as the following general problem:

$$\min \Phi(x,p,t) = [\Phi_1, \Phi_2, \ldots, \Phi_n]^T \tag{1}$$
$$\Phi_i = \int_{t_0}^{t_f} L_i(x,p,t)dt \qquad i = 1,2,\ldots,n$$

under p and subject to:

$$\dot{x} = f(\boldsymbol{x}, \boldsymbol{p}, t) \tag{2}$$

$$g(\boldsymbol{x}, \boldsymbol{p}, t) \leq 0 \tag{3}$$

$$h(\boldsymbol{x}, \boldsymbol{p}, t) = 0 \tag{4}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}_0$$

In the problem stated by (1) to (4): $\boldsymbol{p}$ is a vector of the design variables from the mechanical and control structure, $\boldsymbol{x}$ is the vector of the state variables and t is the time variable. On the other hand, some performance criteria L must be selected for the mechatronic system. The dynamic model (2) describes the state vector $\boldsymbol{x}$ at time t. Also, the design constraints of the mechatronic system must be developed and proposed, respectively. Therefore, the parameter vector $\boldsymbol{p}$ which is a solution of the previous problem will be an optimal set of structure and controller parameters, which minimize the performance criteria selected for the mechatronic system and subject to the constraints imposed by the dynamic model and the design.

Current research efforts in the field of power transmission of rotational propulsion systems, are dedicated to obtaining low energy consumption with high mechanical efficiency. An alternative solution to this problem is the so called continuously variable transmission (CVT), whose transmission ratio can be continuously changed in an established range. There are many CVT configurations built in industrial systems, especially in the automotive industry, due to the requirements to increase fuel economy without decreasing system performance. The mechanical development of CVTs is well known and there is little to modify regarding its basic operating principles. However, research efforts continue on the controller design and the CVT instrumentation side. Different CVT types have been used in different industrial applications; the Van Doorne belt or V-belt CVT is the most widely studied mechanism [19, 20]. This CVT is built with two variable radii pulleys and a chain or metal-rubber belt. Due to its friction-drive operating principle, the speed and torque losses of rubber V-belts are a disadvantage. The Toroidal Traction-drive CVT uses the high shear strength of viscous fluids to transmit torque between an input torus and an output torus. However, the special fluid characteristic used in this CVT makes the manufacturing process expensive. A pinion-rack CVT is a traction-drive mechanism, presented in [21]. This CVT is built-in with conventional mechanical elements as a gear pinion, one cam and two pairs of racks. The conventional CVT manufacture is advantageous over other existing CVTs. However, in the pinion-rack CVT, it has been determined that the teeth size of the gear pinion is an important factor in the performance of the system.

Because the gear pinion is the main mechanical element of the pinion-rack CVT, determining the optimal teeth size of such a mechanical element to obtain an optimal performance is, by no means, easy. On the other hand, an optimal performance system must consider low energy consumption in the controller. Therefore, in order to obtain an optimal performance of the pinion-rack CVT, it is necessary to propose the parametric optimal design of such system.

The goals of the parametric optimal design of the pinion-rack CVT are to obtain a maximum mechanical efficiency as well as a minimum controller energy. Therefore, a MDOP for the pinion-rack CVT will be proposed in this chapter.

2.1 Description and Dynamic CVT Model

In order to adapt the MDOP to the pinion-rack CVT, it is necessary to develop the dynamic model of such a system. The pinion-rack CVT changes its transmission ratio when the distance between the input and output rotation axes is changed. This distance is called "offset" and will be denoted by "e". As was indicated earlier, this CVT is built-in with conventional mechanical elements such as a gear pinion, one cam and two pairs of racks. An offset mechanism is integrated inside the CVT. This mechanism is built-in with a lead screw attached by a nut to the vertical transport cam. Figure 1 depicts the main mechanical CVT components.

The dynamic model of a pinion-rack CVT is presented in [2]. Ordinary differential equations (5), (6) and (7) describe the CVT dynamic behavior. In Eq. (5): T_m is the input torque , J_1 is the mass moment of inertia of the gear pinion, b_1 is the input shaft coefficient viscous damping, r is the gear pinion pitch circle radius, T_L is the CVT load torque, J_2 is the mass moment of inertia of the rotor, R is the planetary gear pitch circle radius, b_2 is the output shaft coefficient viscous damping and θ is the angular displacement of the rotor. In Eqs. (6) and (7): L, R_m, K_b, K_f and n represent the armature circuit inductance, the circuit resistance, the back electromotive force constant, the motor torque constant and the gearbox gear ratio of the

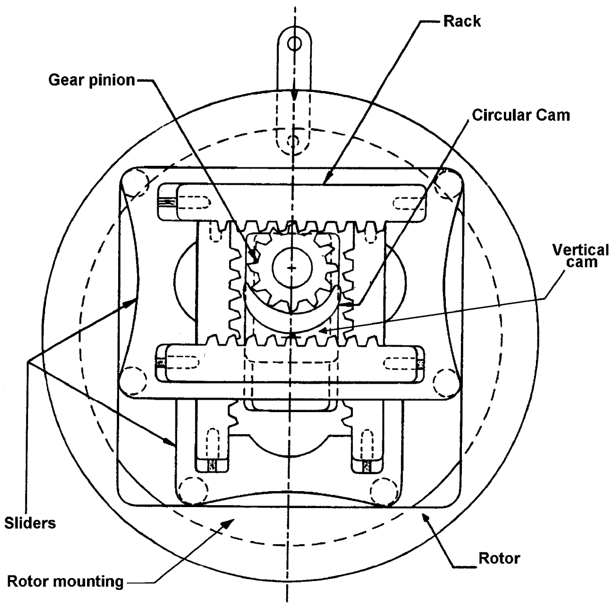

Fig. 1 Main pinion-rack CVT mechanical elements

DC motor, respectively. Parameters r_p, λ_s, b_c and b_l denote the pitch radius, the lead angle, the viscous damping coefficient of the lead screw and the viscous damping coefficient of the offset mechanism, respectively. The control signal $u(t)$ is the input voltage to the DC motor. $J_{eq} = J_{c2} + Mr_p^2 + n^2 J_{c1}$ is the equivalent mass moment of inertia, J_{c1} is the mass moment of inertia of the DC motor shaft, J_{c2} is the mass moment of inertia of the DC motor gearbox and $d = r_p \tan\lambda_s$, is a lead screw function. $\theta_R(t) = \frac{1}{2}\arctan\left[\tan\left(2\Omega t - \frac{\pi}{2}\right)\right]$ is the rack meshing angle. The combined mass to be translated is denoted by M and $P = \frac{T_m}{r_p}\tan\phi\cos\theta_R$ is the load on the gear pinion teeth, where ϕ is the pressure angle.

$$\begin{aligned}\left(\frac{R}{r}\right)T_m - T_L = {}& \left[J_2 + J_1\left(\frac{R}{r}\right)^2\right]\ddot{\theta} \\ & - \left[J_1\left(\frac{R}{r}\right)\frac{e}{r}\sin\theta_R\right]\dot{\theta}^2 \\ & + \begin{bmatrix} b_2 + b_1\left(\frac{R}{r}\right)^2 \\ +J_1\left(\frac{R}{r}\right)\frac{\dot{e}}{r}\cos\theta_R \end{bmatrix}\dot{\theta}\end{aligned} \tag{5}$$

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + R_m i = u(t) - \left[\frac{nK_b}{d}\right]\dot{e} \tag{6}$$

$$\left[\frac{nK_f}{d}\right]i - P = \left[M + \frac{J_{eq}}{d^2}\right]\ddot{e} + \left[b_l + \frac{b_c}{r_p d}\right]\dot{e} \tag{7}$$

In order to fulfill the concurrent design concept, the dynamic model of the pinion-rack CVT must be stated with state variables as it is indicated in the general problem stated by (1) to (4). With the state variables $x_1 = \dot{\theta}$, $x_2 = i$, $x_3 = e$, $x_4 = \dot{e}$, the dynamic model given by (5) to (7) can be written as:

$$\begin{aligned}\dot{x}_1 &= \frac{\begin{matrix} AT_m + \left[J_1 A \frac{2x_3}{p_1 p_2}\sin\theta_R\right]x_1^2 - T_L \\ - \left[b_2 + b_1 A^2 + J_1 A\frac{2x_4}{p_1 p_2}\cos\theta_R\right]x_1 \end{matrix}}{J_2 + J_1 A^2} \\ \dot{x}_2 &= \frac{u(t) - \left(\frac{nK_b}{d}\right)x_4 - Rx_2}{L} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{\left(\frac{nK_f}{d}\right)x_2 - \left(b_l + \frac{b_c}{r_p d}\right)x_4 - \frac{T_m}{r_p}\tan\phi\cos\theta_R}{M + \frac{J_{eq}}{d^2}}\end{aligned} \tag{8}$$

Performance Criteria and Objective Functions

The performance of a system is measured by several criteria. One of the most common is the system efficiency because it reflects the energy loss. In the case of the pinion-rack CVT, the mechanical efficiency criterion of the gear systems is used to state the MDOP. This is because the racks and the gear pinion are the main CVT mechanical elements.

The mathematical equation for mechanical efficiency presented in [22] is used in this work, where μ, N_1, N_2, m, r_1 and r_2 represent the coefficient of sliding friction, the number of gear pinion teeth, the number of spur gear teeth, the gear module, the pitch pinion radius and the pitch spur gear radius, respectively:

$$\eta = 1 - \pi\mu\left(\frac{1}{N_1} + \frac{1}{N_2}\right) = 1 - \frac{\pi\mu}{2m}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{9}$$

In [2], the speed ratio equation is as below, where ω is the input angular speed and Ω is the output angular speed of the CVT:

$$\frac{\omega}{\Omega} = \frac{R}{r} = 1 + \frac{e}{r}\cos\theta_R \tag{10}$$

Considering $r_1 \equiv r$ and $r_2 \equiv R$, the CVT mechanical efficiency is given by

$$\eta(t) = 1 - \frac{\pi\mu}{N_1}\left(1 + \frac{1}{1 + \frac{e\cos\theta_R}{r}}\right) \tag{11}$$

In order to maximize the mechanical CVT efficiency, $F(\cdot)$, which is given below, must be minimized:

$$F(\cdot) = \frac{1}{N_1}\left(1 + \frac{1}{1 + \frac{e\cos\theta_R}{r}}\right) \tag{12}$$

Equation (12) can be written as follows, and is used to state the MDOP:

$$L_1(\cdot) = \frac{1}{N_1}\left(\frac{2r + e\cos\theta_R}{r + e\cos\theta_R}\right) \tag{13}$$

The second objective function of the MDOP must describe the dynamic behavior. In order to fulfill this, a proportional and integral (PI) controller structure is used in the MDOP. This is because, despite the development of many control strategies, the PI controller structure remains one of the most popular approaches in industrial process control because of its good performance. Then, in order to obtain the minimal controller energy, the objective function for the MDOP, given below, is used:

$$L_2(\cdot) = \frac{1}{2}\left[-K_p(x_{\text{ref}} - x_1) - K_I\int_0^t (x_{\text{ref}} - x_1)dt\right]^2 \tag{14}$$

The objective functions previously established fulfill the concurrent design concept, since structural and dynamic behaviors will be considered at the same time in the MDOP.

Constraint Functions

The design constraints for the CVT optimization problem are proposed according to geometric and strength conditions for the gear pinion of the CVT.

To prevent fracture of the annular portion between the axis bore and the teeth root on the gear pinion, the pitch circle diameter of the pinion gear must be greater than the bore diameter by at least 2.5 times the gear module [16]. Then, in order to avoid fracture, the constraint g_1 must be imposed. To achieve a uniform load distribution on the teeth, the face width must be 6 to 12 times the value of the gear module [14]. This is ensured with constraints g_2 and g_3. To maintain the CVT transmission ratio within the range $[2r, 5r]$ constraints g_4, g_5 are imposed. Constraint g_6 ensures the number of teeth on the gear pinion is equal to or greater than 12 [14]. A practical constraint requires that the gear pinion face width is greater than or equal to 20 mm. In order to ensure this, constraint g_7 is imposed. To constrain the distance between the corner edge in the rotor and the edge rotor, constraint g_8 is imposed. Finally, to ensure a practical design for the pinion gear, the pitch circle radius must be equal to or greater than 25.4 mm. For this, constraint g_9 is imposed.

On the other hand, it can be observed that J_1, J_2 are parameters which are a function of the CVT geometry. For these mechanical elements, the mass moments of inertia are defined by

$$J_1 = \frac{1}{32}\rho\pi m^4 \left(N+2\right)^2 N^2 h \tag{15}$$

$$J_2 = \rho h\left[\frac{3}{4}\pi r_c^4 - \frac{16}{6}\left(e_{\max} + mN\right)^4 - \frac{1}{4}\pi r_s^4\right] \tag{16}$$

where ρ, m, N, h, $e_{\max}$, r_c and r_s are the material density, the module, the number of teeth on the gear pinion, the face width, the highest offset distance between axes, the rotor radius and the bearing radius, respectively.

Design Variables

Because the concurrent design concept considers structural and dynamic behaviors at the same time, the vector of the design variables must describe the mechanical and controller structures. In order to fulfill this, design variables of the mechanical structure related to the standard nomenclature for a gear tooth are used. Moreover, the controller gains K_P and K_I which describe the dynamic CVT behavior, are also used.

Equation (17) establishes a parameter called gear module m for metric gears, where d is the pitch diameter and N is the teeth number.

$$m = \frac{d}{N} = \frac{2r}{N} \tag{17}$$

On the other hand, the face width h, which is the distance measured along the axis of the gear and the highest offset distance between axes $e_{\max}$, are parameters which define the CVT size. Therefore, the vector p^i is proposed in order to establish the MDOP of the pinion-rack CVT:

$$\begin{aligned} p^i &= [p_1^i, p_2^i, p_3^i, p_4^i, p_5^i, p_6^i]^T \\ &= [N, m, h, e_{\max}, K_P, K_I]^T \end{aligned} \tag{18}$$

2.2 Optimization Problem

In order to obtain the mechanical CVT parameter optimal values, we propose a MDOP given by Eqs. (19) to (26), where the control signal $u(t)$ is given by (20). As the objective functions must be normalized to the same scale [15], the corresponding factors $W = [0.4397, 563.3585]^T$ were obtained using the algorithm from Sect. 3 by minimizing each objective function subject to constraints given by Eqs. (8) and (20) to (26).

$$\min_{p \in R^6} \Phi(x, p, t) = [\Phi_1, \Phi_2]^T \tag{19}$$

where

$$\Phi_1 = \frac{1}{W_1} \int_0^{10} \left[\frac{1}{p_1} \left(\frac{p_1 p_2 + x_3 \cos\theta_R}{\frac{p_1 p_2}{2} + x_3 \cos\theta_R} \right) \right] dt$$

$$\Phi_2 = \frac{1}{W_2} \int_0^{10} u^2 dt$$

subject to the dynamic model stated by (8) and subject to:

$$u(t) = -p_5(x_{\text{ref}} - x_1) - p_6 \int_0^t (x_{\text{ref}} - x_1) dt \tag{20}$$

$$J_1 = \frac{1}{32} \rho \pi p_2^4 (p_1 + 2)^2 p_1^2 p_3 \tag{21}$$

$$J_2 = \frac{\rho p_3}{4} \left[3\pi r_c^4 - \frac{32}{3} (p_4 + p_1 p_2)^4 - \pi r_s^4 \right] \tag{22}$$

$$A = 1 + \frac{2x_3}{p_1 p_2} \cos\theta_R \tag{23}$$

$$d = r_p \tan\lambda_s \tag{24}$$

$$\theta_R = \frac{1}{2} \arctan\left[\tan\left(2x_1 t - \frac{\pi}{2} \right) \right] \tag{25}$$

$$\begin{aligned}
g_1 &= 0.01 - p_2\left(p_1 - 2.5\right) \leq 0 \\
g_2 &= 6 - \frac{p_3}{p_2} \leq 0 \\
g_3 &= \frac{p_3}{p_2} - 12 \leq 0 \\
g_4 &= p_1 p_2 - p_4 \leq 0 \\
g_5 &= p_4 - \frac{5}{2} p_1 p_2 \leq 0 \\
g_6 &= 12 - p_1 \leq 0 \\
g_7 &= 0.020 - p_3 \leq 0 \\
g_8 &= 0.020 - \left[r_c - \sqrt{2}(p_4 + p_1 p_2)\right] \leq 0 \\
g_9 &= 0.0254 - p_1 p_2 \leq 0
\end{aligned} \tag{26}$$

3 Mathematical Programming Optimization

As we can observe, in a general way, a MDOP is composed by continuous functions given by the dynamic model of the system as well as the objective functions of the problem. In order to find the solution of the MDOP, it must be transformed into a Nonlinear Programming Problem (NLP) [4]. Two transformation approaches exist: the sequential and the simultaneous approach. In the sequential approach, only the control variables are discretized. This approach is also known as control vector parameterization. In the simultaneous approach, the state and control variables are discretized resulting in a large-scale NLP problem which requires special algorithms for its solution [1]. Because of the diversity of mathematical programming algorithms already established, transformation of the MDOP into a NLP problem was done adopting the sequential approach.

The NLP problem which is used to approximate the original problem given by (1) to (4) can be stated as:

$$\min_{\boldsymbol{p}} F(\boldsymbol{p}) \tag{27}$$

subject to:

$$c_i \leq 0 \tag{28}$$

$$c_e = 0 \tag{29}$$

where p is the vector of the design variables, c_i are the inequality constraints and c_e are the equality constraints. In order to obtain the NLP problem given by (27) to (29), the sequential approach requires the value and the gradient calculation of the objective functions. Moreover, the gradient of the constraints with respect to the design variables must be calculated.

3.1 Gradient Calculation and Sensitivity Equations

The gradient calculation for the objective function uses the following equation:

$$\frac{\partial \Phi_i}{\partial p_j} = \int_{t_0}^{t_f} \left(\frac{\partial L_i}{\partial \boldsymbol{x}} \left[\frac{\partial \boldsymbol{x}}{\partial p_j}(t) \right] + \frac{\partial L_i}{\partial p_j} \right) \mathrm{d}t \tag{30}$$

where, it can be seen in the general problem stated by (1) to (4), that L_i is the ith objective function, $\boldsymbol{x}$ is the vector of the state variables, p_j is the jth element of the vector of the design variables and t is the time variable. On the other hand, in order to obtain the partial derivatives $\frac{x}{p_j}$, it is necessary to solve the ordinary differential equations of the sensitivity given by

$$\frac{\partial \dot{x}}{\partial p_j} = \frac{\partial f}{\partial \boldsymbol{x}} \left[\frac{\partial \boldsymbol{x}}{\partial p_j} \right] + \frac{\partial f}{\partial p_j} \tag{31}$$

These sensitivity equations can be obtained by taking the time derivatives with respect to p_j of the dynamic model. Due to the fact that $\dot{x}$ is a function of the time variable t as well as the design variables p_j (we must consider that p_j are independent of t), then:

$$\dot{x} = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \frac{\partial \boldsymbol{x}}{\partial t} \tag{32}$$

moreover

$$\frac{\mathrm{d}\left(\frac{\partial x}{\partial p_j}\right)}{\mathrm{d}t} = \frac{\partial\left(\frac{\partial x}{\partial p_j}\right)}{\partial t} = \frac{\partial\left(\frac{\partial x}{\partial t}\right)}{\partial p_j} = \frac{\partial\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)}{\partial p_j} = \frac{\partial \dot{x}}{\partial p_j} \tag{33}$$

Finally, using the equalities (33) and proposing the following variable:

$$y_j = \frac{\partial \boldsymbol{x}}{\partial p_j} \tag{34}$$

the partial derivatives of $\boldsymbol{x}$ with respect to p_j are now given by the following ordinary differential equations:

$$\dot{y}_j = \frac{\partial f}{\partial \boldsymbol{x}} y_j + \frac{\partial f}{\partial p_j} \tag{35}$$

$$y_j(0) = \frac{\partial \boldsymbol{x}_0}{\partial p_j} \tag{36}$$

3.2 Goal Attainment Method

In order to transform the MDOP into a NLP problem, the sequential approach is used. The resulting problem is solved using the Goal Attainment Method [11]. In the remainder of the chapter, we will refer to it as "MPM" (Mathematical Programming Method). In such a technique, a subproblem is obtained as follows:

$$\min_{p,\lambda} G\left(p,\lambda\right) \triangleq \lambda \tag{37}$$

subject to:

$$\begin{aligned} g(\boldsymbol{p}) &\leq 0 \\ h(\boldsymbol{p}) &= 0 \\ g_{a1}(\boldsymbol{p}) &= \Phi_1\left(p\right) - \boldsymbol{\omega}_1\lambda - \Phi_1^d \leq 0 \\ g_{a2}(\boldsymbol{p}) &= \Phi_2\left(p\right) - \boldsymbol{\omega}_2\lambda - \Phi_2^d \leq 0 \end{aligned} \tag{38}$$

where λ is an artificial variable without sign constraint, and $g(\boldsymbol{p})$ and $h(\boldsymbol{p})$ are the constraints established in the original problem. Moreover, in the last two constraints, $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are the scattering vectors, Φ_1^d and Φ_2^d are the desired goals for each objective function and Φ_1 and Φ_2 are the evaluated functions.

3.3 Numerical Method to Solve the NLP Problem

In order to solve the resulting NLP problem, Eqs. (37) and (38), the Successive Quadratic programming (SQP) method is used. There, a Quadratic Problem (QP) which is a quadratic approximation to the Lagrangian function optimized over a linear approximation to the constraints, is solved. A vector $\boldsymbol{p}^i$ containing the current parameter values is proposed and the NLP problem given by Eqs. (39) and (40) is obtained, where B_i is the Broyden–Fletcher–Goldfarb–Shanno updated (BGFS) positive definite approximation of the Hessian matrix, and the gradient calculation is obtained using sensitivity equations. Hence, if γ solves the subproblem given by (39) and (40) and $\gamma = 0$, then the parameter vector $\boldsymbol{p}^i$ is an original problem optimal solution. Otherwise, we set $\boldsymbol{p}^{i+1} = \boldsymbol{p}^i + \gamma$ and with this new vector the process is repeated all over again.

$$\min_{\gamma} QP(\boldsymbol{p}^i) = G\left(\boldsymbol{p}^i\right) + \nabla G^T\left(\boldsymbol{p}^i\right)\gamma + \frac{1}{2}\gamma^T B_i \gamma \tag{39}$$

subject to

$$\begin{aligned} g(\boldsymbol{p}^i) + \nabla g^T\left(\boldsymbol{p}^i\right)\gamma &\leq 0 \\ h(\boldsymbol{p}^i) + \nabla h^T\left(\boldsymbol{p}^i\right)\gamma &= 0 \\ g_{a1}(\boldsymbol{p}^i) + \nabla g_{a1}^T\left(\boldsymbol{p}^i\right)\gamma &\leq 0 \\ g_{a2}(\boldsymbol{p}^i) + \nabla g_{a2}^T\left(\boldsymbol{p}^i\right)\gamma &\leq 0 \end{aligned} \tag{40}$$

3.4 Experiments and Results of the Mathematical Programming Method

In order to carry out the parametric optimal design of the pinion-rack CVT, we performed 10 independent runs, all of them using a PC with a 2.8 GHz Pentium IV processor with 1 GB of Memory using Matlab 6.5.0 Release 13. The system parameters used in the numerical simulations were: $b_1 = 1.1$ Nms/rad, $b_2 = 0.05$ Nms/rad, $r = 0.0254$ m, $T_m = 8.789$ Nm, $T_L = 0$ Nm, $\lambda_s = 5.4271$, $\phi = 20$, $M = 10$ Kg, $r_p = 4.188 \times 10^{-03}$ m, $K_f = 63.92 \times 10^{-03}$ Nm/A, $K_b = 63.92 \times 10^{-03}$ Vs/rad, $R = 10\ \Omega$, $L = 0.01061$ H, $b_l = 0.015$ Ns/m, $b_c = 0.025$ Nms/rad and $n = ((22{\cdot}40{\cdot}33)/(9{\cdot}8{\cdot}9))$. The initial conditions vector was $[x_1(0), x_2(0), x_3(0), x_4(0)]^T = [7.5, 0, 0, 0]^T$ and the output reference was considered to be $x_{\text{ref}} = 3.2$.

Because the goal attainment method requires a goal for each of the objective functions, further calculations were necessary. The goal for Φ_1 was obtained by minimizing this function subject to Eqs. (8) and (20) to (26). The optimal solution vector $\boldsymbol{p}^1$ is shown in Table 1. The goal for Φ_2 was obtained by minimizing this function subject to Eqs. (8) and (20) to (26). The optimal solution vector $\boldsymbol{p}^2$ for this problem is also shown in Table 1.

Varying the scattering vector can produce different nondominated solutions. In Table 1, two cases are presented: $\boldsymbol{p}_A^*$ is obtained with $\omega = [0.5, 0.5]^T$, and $\boldsymbol{p}_B^*$ is obtained with $\omega = [0.4, 0.6]^T$.

As can be seen in the results in Table 3, 80% of the runs diverged. This behavior shows a high sensitivity of the MPM to the starting point (detailed in Table 2) because it must be carefully chosen in order to allow the approach to obtain a good solution. Information about the time required by the MPM per independent run is summarized in Table 3.

Figure 2 shows the mechanical efficiency and the input control of the pinion-rack CVT with both solutions obtained by the MPM ($\boldsymbol{p}^1$, $\boldsymbol{p}^2$ and $\boldsymbol{p}_A^*$). The solution

Table 1 Details of the solutions obtained by the MPM

$[N^*, m^*, h^*, e^*_{\max}, K_P^*, K_I^*]$	$\Phi_N(\bullet) = [\Phi_1(\bullet), \Phi_2(\bullet)]$	$\Phi(\bullet) = [\Phi_1(\bullet), \Phi_2(\bullet)]$
$p^1 = [38, 0.0017, 0.02, 0.0636, 10.000, 1.00]$	$\Phi_N(p^1) = [1.0000, 4.7938]$	$\Phi(p^1) = [0.4397, 2700.6279]$
$p^2 = [13.4459, 0.0019, 0.02, 0.0826, 5.000, 0.01]$	$\Phi_N(p^2) = [2.8017, 1.0000]$	$\Phi(p^2) = [1.2319, 563.3585]$
$p_A^* = [26.7805, 0.0017, 0.02, 0.0826, 5.000, 0.01]$	$\Phi_N(p_A^*) = [1.4696, 1.4696]$	$\Phi(p_A^*) = [0.6461, 827.9116]$
$p_B^* = [29.0171, 0.0017, 0.02, 0.0789, 5.000, 0.01]$	$\Phi_N(p_B^*) = [1.3646, 1.5469]$	$\Phi(p_B^*) = [0.6000, 871.4592]$

Table 2 Initial points used for the MPM. Also shown is the corresponding scattering vector

Initial search point	Scattering vector
[13.4459, 0.0019, 0.02, 0.0826, 5.000, 0.01]	[0.5, 0.5]
[38, 0.0017, 0.02, 0.0636, 10.000, 1.00]	[0.5, 0.5]
[38, 0.0017, 0.02, 0.0636, 10.000, 1.00]	[0.4, 0.6]
[38, 0.0017, 0.02, 0.0636, 10.000, 1.00]	[0.6, 0.4]
[28.8432, 0.0017, 0.02, 0.0550, 5.024, 0.017]	[0.5, 0.5]
[13.4459, 0.0019, 0.02, 0.0826, 5.000, 0.01]	[0.4, 0.6]
[28.8432, 0.0017, 0.02, 0.0550, 5.024, 0.017]	[0.4, 0.6]
[28.8432, 0.0017, 0.02, 0.0550, 5.024, 0.017]	[0.6, 0.4]
[30.77, 0.0017, 0.02, 0.0694, 5.121, 0.010]	[0.5, 0.5]
[30.77, 0.0017, 0.02, 0.0694, 5.121, 0.010]	[0.4, 0.6]

Table 3 Time required by each run of the MPM. Note that only two runs could converge to a solution. The remaining 8 runs could not provide any result

Run	Time required
1	Diverged
2	23.78 Min
3	Diverged
4	Diverged
5	Diverged
6	Diverged
7	Diverged
8	Diverged
9	Diverged
10	48.5 Min
Average	**36.365 Min**

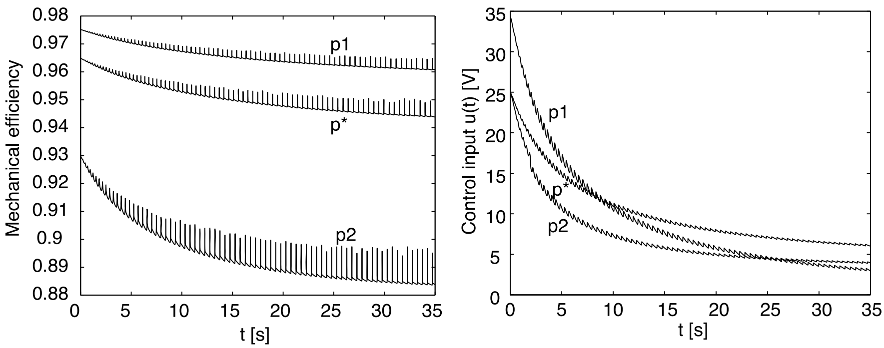

Fig. 2 Mechanical efficiency and control input for the pinion-rack CVT obtained by the MPM

$\boldsymbol{p}_A^*$ was selected because it has the same overachievement of the proposed goal for each objective function.

As can be observed in Fig. 2, when the number of teeth is increased ($\boldsymbol{p}_1^*$) and their size is decreased ($\boldsymbol{p}_2^*$), a higher CVT mechanical efficiency is obtained. Also, we can observe perturbations in the mechanical efficiency, which are produced because of tip-to-tip momentary contact prior to full engagement between teeth. With the optimal solution, this tip-to-tip contact is reduced because a better CVT planetary gear is obtained when the tooth size is decreased. Summarizing, the optimal solution implies a lower sensitivity of the mechanical efficiency with respect to reference changes. On the other hand, a more compact CVT size is obtained since ($\boldsymbol{p}_3^*$) is decreased. Furthermore, a minimal controller energy is obtained when the controller gains ($\boldsymbol{p}_5^*$) and ($\boldsymbol{p}_6^*$) are decreased. In Fig. 2, it can be observed that the optimal vector minimizes the initial overshoot of the control input.

Despite the sensitivity of the NLP method, the optimal solutions obtained are good from the mechanical and controller point of view.

4 Evolutionary Optimization

The high sensitivity of the MPM to its initial conditions, and its implementation complexity motivated us to solve the problem using an evolutionary algorithm (EA). This is because one of the main advantages of an EA is that competitive results are obtained regardless of its initial conditions (i.e. a set of solutions is randomly generated). We selected Differential Evolution [17] for several reasons: (1) it is an EA which has provided very competitive results when compared with traditional EAs such as genetic algorithms and evolution strategies in real-world problems [6]; (2) it is very simple to implement [17]; and (3) its parameters for the crossover and mutation operators generally do not require a careful fine-tuning [13].

DE is an evolutionary direct-search algorithm to solve optimization problems. DE shares similarities with traditional EAs, however, it does not use binary encoding as a simple genetic algorithm [8] and it does not use a probability density function to self-adapt its parameters as an Evolution Strategy [18]. Instead, DE performs mutation based on the distribution of the solutions in the current population. In this way, search directions and possible stepsizes depend on the location of the individuals selected to calculate the mutation values.

Several DE variants have been proposed [17]. The most popular is called *"DE/-rand/1/bin"*, where "DE" means Differential Evolution, the word "rand" indicates that the individuals selected to compute the mutation values are chosen at random, "1" is the number of pairs of solutions chosen to calculate the differences for the mutation operator and finally "bin" means that a binomial recombination is used. A detailed pseudocode of this variant is presented in Fig. 3.

Four parameters must be defined in DE: (1) the population size, (2) the number of generations, (3) the factor $F \in [0.0, 1.0]$, which scales the value of the differences computed from randomly selected individuals (typically three, where two are used to compute the difference and the other is only added) from the population (row 11 in Fig. 3). A value of $F = 1.0$ indicates that the complete difference value is used; and finally, (4) the $CR \in [0.0, 1.0]$ parameter, which controls the influence of the parent on its corresponding offspring; a value of $CR = 0.0$ means that the offspring will take its values from its parent instead of taking its values from the mutation values generated by the combination of the differences of the individuals chosen at random (rows 9–15 in Fig. 3).

DE was originally proposed to solve global optimization problems. Moreover, like other EAs, DE lacks a mechanism to handle the constraints of a given optimization problem. Hence, we decided to modify the DE algorithm in order to solve constrained multiobjective optimization problems. It is worth remarking that the goal when performing these modifications was to maintain the simplicity of DE as much as possible.

Three modifications were made to the original DE:

1. The selection criterion between a parent and its corresponding offspring was modified in order to handle multiobjective optimization problems.
2. A constraint-handling technique to guide the approach to the feasible region of the search space was added.

```
1  Begin
2      G=0
3      Create a random initial population X_{i,G}  ∀i, i = 1,...,NP
4      Evaluate f(X_{i,G}) ∀i, i = 1,...,NP
5      For G=1 to MAX_GEN Do
6          For i=1 to NP Do
7              Select randomly r_1 ≠ r_2 ≠ r_3 :
8              j_rand = randint(1, D)
9              For j=1 to D Do
10                 If (rand_j[0, 1) < CR or j = j_rand) Then
11                     u_{i,j,G+1} = x_{r_3,j,G} + F(x_{r_1,j,G} − x_{r_2,j,G})
12                 Else
13                     u_{i,j,G+1} = x_{i,j,G}
14                 End If
15             End For
16             If (f(U_{i,G+1}) ≤ f(X_{i,G})) Then
17                 X_{i,G+1} = U_{i,G+1}
18             Else
19                 X_{i,G+1} = X_{i,G}
20             End If
21         End For
22         G = G + 1
23     End For
24 End
```

Fig. 3 "DE/rand/1/bin" algorithm. randint(min,max) is a function that returns an integer number between min and max. rand$[0,1)$ is a function that returns a real number between 0 and 1. Both are based on a uniform probability distribution. "NP", "MAX_GEN", "CR" and "F" are user-defined parameters. "D" is the dimensionality of the problem

3. A simple external archive to save the nondominated solutions found during the process was added.

4.1 Selection Criterion

We changed the original criterion to select between parent and offspring (rows 16–20 in Fig. 3) based only on the objective function value. As in multiobjective optimization we are looking for a set of trade-off solutions, we used, as traditionally adopted in Evolutionary Multiobjective Optimization [5], Pareto Dominance as the criterion to select between the parent and its corresponding offspring. The aim is to keep the nondominated solutions from the current population.

A vector $\mathbf{U} = (u_1, \ldots, u_k)$ is said to dominate $\mathbf{V} = (v_1, \ldots, v_k)$ (denoted by $\mathbf{U} \preceq \mathbf{V}$) if and only if $\mathbf{U}$ is partially less than $\mathbf{V}$, i.e. $\forall i \in \{1, \ldots, k\}$, $u_i \leq v_i \wedge \exists i \in \{1, \ldots, k\} : u_i < v_i$. If we denote the feasible region of the search space as $\mathcal{F}$, the evolutionary multiobjective algorithm will look for the Pareto optimal set ($\mathcal{P}^*$) defined as:

$$\mathcal{P}^* := \{x \in \mathcal{F} \mid \neg\exists\, x' \in \mathcal{F}\;\; \mathbf{F}(x') \preceq \mathbf{F}(x)\}. \tag{41}$$

In our case, $k = 2$, as we are optimizing two objectives.

4.2 Constraint Handling

The most popular approach to incorporate the feasibility information into the fitness function of an EA is the use of a penalty function. The aim is to decrease the fitness value of the infeasible individuals (i.e., those that do not satisfy the constraints of the problem). In this way, feasible solutions will have a higher probability of being selected and the EA will eventually reach the feasible region of the search space. However, the main drawback of penalty functions is that they require the definition of penalty factors. These factors determine the severity of the penalty. If the penalty value is very high, the feasible region will be approached mostly at random and the feasible global optimum will be hard to find. On the other hand, if the penalty is too low, the probability of not reaching the feasible region will be high. Based on the aforementioned disadvantage, we decided to avoid the use of a penalty function. Instead, we incorporated a set of criteria based on feasibility, originally proposed by Deb [7] and further extended by other researchers [9,10,12]:

- Between two feasible solutions, the one which dominates the other wins.
- If one solution is feasible and the other one is infeasible, the feasible solution wins.
- If both solutions are infeasible, the one with the lowest sum of constraint violation is preferred.

We combine Pareto dominance and the set of feasibility rules into one selection criterion, which substitutes rows 16–20 in Fig. 3 as presented in Fig. 4.

4.3 External Archive

One of the features that distinguishes a modern evolutionary multiobjective optimization algorithm is the concept of elitism [5]. In our modified DE, we added an external archive, which stores the set of nondominated solutions found during the evolutionary process. This archive is updated at each generation in such a way that all nondominated solutions from the population will be included in the archive. After that, nondominance checking is performed with respect to all the solutions (the newcomers and also the solutions in the archive). The solutions that are nondominated with respect to everybody else will remain in the archive. When the search ends, the set of nondominated solutions in the archive will be reported as the final set of solutions obtained by the approach.

If ($\mathbf{U}^i_{G+1}$ is better than $\boldsymbol{X}^i_G$ (based on the three *selection criteria))* **Then**
    $\mathbf{X}^i_{G+1} = \mathbf{U}^i_{G+1}$
Else
    $\mathbf{X}^i_{G+1} = \mathbf{X}^i_G$
End If

Fig. 4 Modified selection mechanism added to the DE algorithm in order to solve the multi-objective optimization problem

4.4 Results of the EA Approach

In our experiments, we performed 10 independent runs. A fixed set of values for the parameters was used in all runs and they were defined as follows: Population size $NP = 200$, $MAX\_GENERATIONS = 100$; the parameters F and CR were randomly generated within an interval. The parameter F was generated per generation in the range $[0.3, 0.9]$ (the differences can be scaled in different proportions without affecting the performance of the approach) and CR was generated per run in the range $[0.8, 1.0]$ (greater influence of the mutation operator instead of having such influence from the parent when generating the offspring). These values were empirically derived. This way of defining the values for F and CR makes it evident that they do not require to be fine-tuned. We will refer to the evolutionary approach as "EA" (Evolutionary Algorithm).

The experiments were performed on the same platform on which the goal attainment experiments were carried out. This was done to have a common point of comparison to measure the computational time required by each approach.

In Table 4 we present the number of nondominated solutions and also the time required per run.

The 10 different Pareto fronts obtained are presented in Fig. 5.

In order to help the decision maker, we filtered the 10 different set of solutions in order to obtain the final set of nondominated solutions. The final Pareto front obtained from the 10 runs contains 28 nondominated points and is presented in Fig. 6. Finally, the details of the 28 solutions are presented in Table 5.

Figure 7 shows the mechanical efficiency and the input control of the pinion-rack CVT with the optimal solution obtained with the MPM and the solution ([30.185435, 0.017, 0.020075, 0.059569, 5.133269,0.019914]) in the middle of the filtered Pareto front obtained with the EA (Fig. 6). We can observe that the mechanical efficiency found by the EA is better than that of the MPM solution. We can also see a smooth behavior of

Table 4 Time required and number of nondominated solutions found at each independent run by the EA

Run	Time required	Nondominated solutions
1	18.53 Hrs.	17
2	20.54 Hrs.	15
3	18.52 Hrs.	25
4	18.63 Hrs.	16
5	18.55 Hrs.	17
6	17.57 Hrs.	19
7	18.15 Hrs.	18
8	18.47 Hrs.	24
9	18.67 Hrs.	16
10	20.24 Hrs.	18
Average	**18.78 Hrs**	**18.5 solutions**

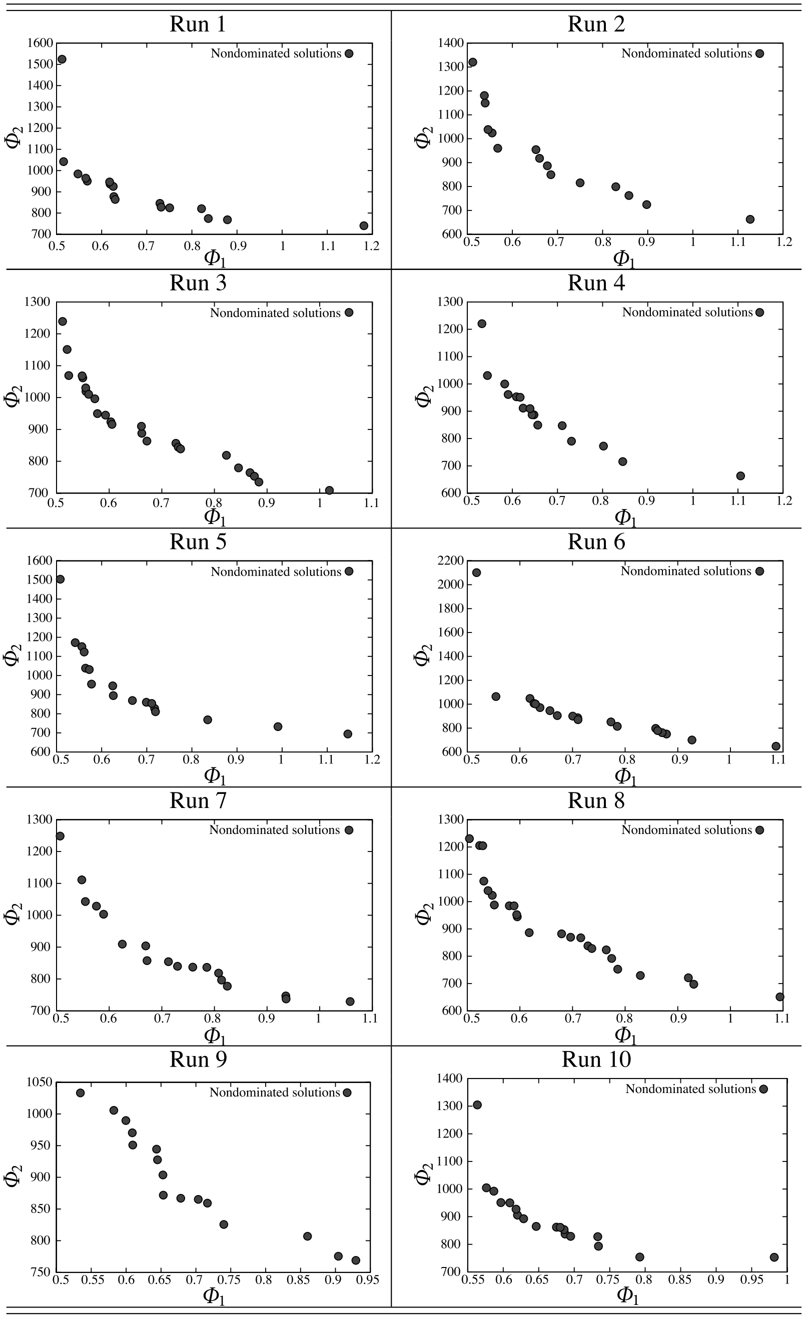

Fig. 5 Different Pareto fronts obtained by the EA in 10 independent runs

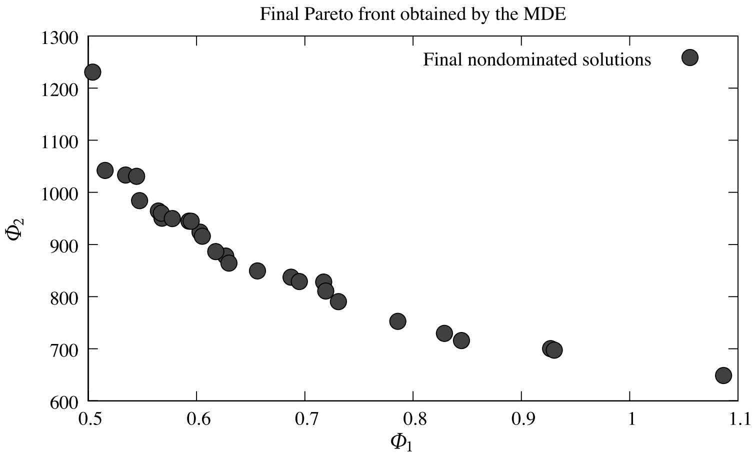

Fig. 6 Final set of solutions obtained by the EA in 10 independent runs

Table 5 Details of the trade-off solutions found by the EA. All solutions are feasible

$[N^*, m^*, h^*, e^*_{max}, K^*_P, K^*_I]$	$[\Phi_1(\bullet), \Phi_2(\bullet)]$
[32.949617, 0.001780, 0.020413, 0.063497, 5.131464, 0.022851]	[0.534496, 1033.243548]
[25.022005, 0.001699, 0.020103, 0.052385, 5.087026, 0.024991]	[0.687214, 837.167059]
[24.764331, 0.001723, 0.020662, 0.048119, 5.104801, 0.011072]	[0.694969, 828.856396]
[32.203853, 0.001793, 0.021356, 0.066703, 5.033164, 0.012833]	[0.547385, 984.149814]
[30.774167, 0.001710, 0.020092, 0.069459, 5.129618, 0.010260]	[0.568131, 950.480089]
[34.231339, 0.001756, 0.020974, 0.065426, 5.104461, 0.023469]	[0.515604, 1042.009590]
[31.072336, 0.001760, 0.020295, 0.072332, 5.018621, 0.024963]	[0.564775, 964.310541]
[27.647589, 0.001685, 0.020151, 0.069264, 5.001687, 0.031805]	[0.627021, 877.670407]
[27.548056, 0.001696, 0.020083, 0.067970, 5.006868, 0.017859]	[0.629913, 864.206663]
[30.866972, 0.001735, 0.020305, 0.058766, 5.002777, 0.032694]	[0.567519, 960.120458]
[28.913492, 0.001747, 0.020478, 0.058322, 5.021887, 0.027174]	[0.603222, 923.771423]
[28.843277, 0.001764, 0.020282, 0.055027, 5.024443, 0.017157]	[0.605340, 915.753294]
[30.185435, 0.001700, 0.020075, 0.059569, 5.133269, 0.019914]	[0.577733, 949.842309]
[29.448640, 0.001755, 0.020601, 0.063276, 5.019318, 0.033931]	[0.593085, 944.906551]
[20.002905, 0.001697, 0.020098, 0.053235, 5.114809, 0.018447]	[0.844657, 715.605541]
[26.373053, 0.001718, 0.020176, 0.068410, 5.031773, 0.014986]	[0.656264, 849.215816]
[32.227085, 0.001764, 0.020567, 0.070369, 5.178989, 0.026127]	[0.544721, 1030.722785]
[23.476167, 0.001731, 0.020618, 0.057264, 5.050345, 0.010533]	[0.730990, 790.412654]
[23.853314, 0.001696, 0.020054, 0.063646, 5.097374, 0.040464]	[0.717403, 827.978369]
[23.936736, 0.001767, 0.020179, 0.054081, 5.026456, 0.013965]	[0.719347, 810.685134]
[18.094865, 0.001754, 0.020097, 0.033930, 5.263513, 0.012051]	[0.926890, 700.251032]
[15.287561, 0.001836, 0.020539, 0.065247, 5.001634, 0.077960]	[1.086582, 648.563140]
[20.410186, 0.001689, 0.020082, 0.067889, 5.005502, 0.046545]	[0.828891, 729.481066]
[29.319668, 0.001754, 0.020557, 0.057790, 5.140154, 0.012875]	[0.595073, 944.511281]
[28.165197, 0.001722, 0.020449, 0.069922, 5.035457, 0.013965]	[0.617721, 886.468167]
[34.733111, 0.001738, 0.020849, 0.064827, 5.470063, 0.078838]	[0.504179, 1230.655492]
[18.028162, 0.001753, 0.021026, 0.075356, 5.185506, 0.027797]	[0.930299, 697.362827]
[21.642511, 0.001694, 0.020196, 0.061009, 5.040619, 0.029378]	[0.785859, 752.464167]

the mechanical efficiency for the EA, maintaining a more compact CVT size for the EA solution. However, the initial overshoot of the input control is greater than that of the MPM solution. These behaviors are observed with all the solutions lying on the middle of the Pareto front, because a higher number of teeth and a corresponding smaller size are obtained (p_1^* was increased and p_2^* was decreased) whereas the input

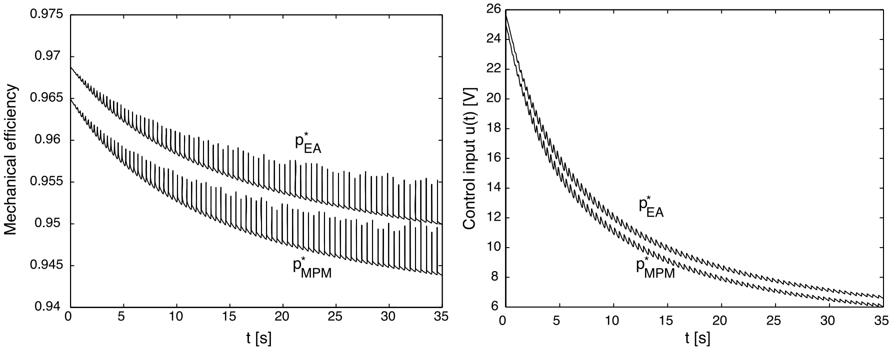

Fig. 7 Mechanical efficiency and Control input for the pinion-rack CVT. obtained by the EA approach

energy controller is greater (p_5^* and p_6^* were increased) in these optimal solutions. In conclusion, from a mechanical point of view, the solutions in the middle of the Pareto front, offer many possible system reconfigurations of the pinion-rack CVT.

5 Advantages and Disadvantages of Both Approaches

5.1 Quality and Robustness

As we can see, the results provided by the EA were as good as those obtained by the MPM method because the latter solutions were also nondominated with respect to those found by the EA. However, the EA was not sensitive to the initial conditions (a randomly generated set of solutions was adopted at all times). The EA approach provided a more robust behavior than that shown by the MPM. Despite the fact that the results obtained by both approaches are considered similar (from a mechanical and from a control point of view), as the EA obtains several solutions from a single run, it gives the designer the chance to select from them, the best choice based on his preferences.

5.2 Computation Cost

It is clear, based on the results shown in Tables 3 and 4 for the MPM and the EA approaches respectively, that the EA is the most expensive, computationally speaking. However, as pointed out in Table 4, the EA obtains a set of nondominated solutions per single run. In contrast, the MPM always returns a single solution on each run. Therefore, based on the average time (18.78 h) and the average number of solutions obtained (18.5 solutions), approximately, one solution per hour is obtained. On the other hand, the MPM obtained a solution after approximately 36 minutes computation time. Therefore, we can conclude that the EA requires, roughly, twice the time used by the MPM to find a single solution.

5.3 Implementation Issues

As mentioned in Sect. 3.3, in order to solve the multiobjective optimization problem using the MPM, a sequential quadratic programming method was used. There, a quadratic programming problem, which is an approximation to the original CVT problem, was solved, and some difficulties detected:

- This method requires gradient calculation, sensitivity equations and gradient equations of the constraints. In general, the number of sensitivity equations is the product of the number of state variables and the number of design variables. Gradient equations are related to the number of design variables. Summarizing, we must calculate two objective functions equations, 24 sensitivity equations, six gradient equations and 54 constraint gradient equations. On the other hand, with the EA only two objective functions equations must be calculated. Therefore, reconfiguration of the EA is simple.
- Due to the fact that the QP problem is an approximation to the original problem and that the constraints are a linear approximation, this problem might be unbounded or infeasible, whereas the original problem is not. With the EA, the original problem is solved. Therefore, the search for the optimal solution is performed in the feasible region of the search space, directly. In this way, in the case of the EA, new structural parameters can be obtained when additional mechanical constraints to the design problem are added. These mechanical constraints could be considered directly in the constraint-handling mechanism of the algorithm without the need for any further changes.

It is worth recalling that another additional step related to the use of the MPM is that it requires minimizing each objective function considered, separately. This is because the goal attainment method requires a goal for each function to be optimized. This step is not required by the EA. Finally, the EA showed no significant sensitivity to its parameters.

5.4 Goal Attainment to Refine Solutions

It is important to mention that we carried out a set of runs of the MPM using a nondominated solution obtained by the EA, as a starting point. However, the approach was unable to improve the solution in all cases.

6 Conclusions and Future Work

We have presented the multiobjective optimization of a pinion-rack continuously variable transmission (CVT). The aim is to maximize the mechanical efficiency and to mininize the corresponding control. The problem is subject to geometric and strength conditions for the gear pinion of the CVT. Two different approaches were used to solve the problem: A mathematical programming method called Goal Attainment and also an evolutionary algorithm. The first one was very sensitive to the initial start point of the search (the point must be given by the user and must be carefully selected), but the computation time required was of about 30 minutes to obtain a solution. On the other hand, the evolutionary algorithm, which in our case was differential evolution, showed no sensitivity to the initial conditions, i.e. a set of randomly generated solutions was used in all experiments. Also, the approach did not show any sensitivity to the values of the parameters related to the crossover and mutation operators. Furthermore, the EA returned a set of solutions on each single run, which gave the designer more options to select the best solutions, based on his preferences. The computational time required for the EA was about 60 minutes to find a solution. The results obtained with the two approaches were similar based on quality, but the EA was more robust (in each single run it obtained feasible results). Finally, the EA was easier to implement, which is one clear advantage of the approach.

Future work will include designing a preferences-handling mechanism in order to let the EA concentrate the search on those regions of the Pareto front where the most convenient solutions are located. Furthermore, we plan to solve other mechatronic problems using the proposed approach.

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