A Method for Automatic Membership Function Estimation Based on Fuzzy Measures

Grzegorz Nieradka and Bohdan Butkiewicz

Warsaw University of Technology, Institute of Electronic Systems,
Nowowiejska 15/19, 00 - 665 Warsaw, Poland
nieradka@ise.pw.edu.pl, bb@ise.pw.edu.pl

Abstract. Estimation of membership function is one of the most important problems in the application of fuzzy sets. This paper presents one of approaches to this problem. A method for estimation of membership function is proposed, based on fuzzy measures: fuzzy entropy and fuzzy index. Examples of generating membership function in the field of image processing are shown.The method presented in this paper can be used in other fields of computer sciences, where statistical data are available.

1 Introduction

Proper choosing of membership function's shape and values is usually not an easy task. There are many methods proposed in the literature also in the image processing field [14]. Some methods adopt an approach based on transformation from a probability distribution to a possibility distribution [3]. Sometimes statistical data describing an image are clustered by C-means algorithm. Membership functions of pixels' brightness are chosen based on these clusters [2]. A method based on optimization objective function is presented in [4]. The application of measures of the fuzzy set such as specificity or consistency is also proposed in [7]. The authors of this paper propose objective function related to entropy measure.

The most similar solution this our proposition is described in [5], but there are numerous discrepancies between the two approaches. Whereas both the works pertain the idea of maximization of entropy, only this paper refers to the measure of entropy other than the idea of probability of fuzzy event as described in [5]. Moreover, this work offers another measure, namely fuzzy index, added for the more exact description of available data image.

In this paper the authors proposed a novel method in which calculated membership function is utilized for modeling linguistic commands used for image processing. Some examples, where such commands are applied prove to be successful approach.

2 Membership Function

The shape of S-function is commonly used for the representation of the degree of brightness or whiteness of pixels in grey levels images. This S-function was originally introduced by Zadeh [17].

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For flexibility another definition of S-function was proposed [6]:

$$S\left(x;a,b,c\right)=\begin{cases}0, & x\leq a\\ \frac{(x-a)^2}{(b-a)(c-a)}, & a<x\leq b\\ 1-\frac{(x-c)^2}{(c-b)(c-a)}, & b<x\leq c\\ 1, & x\geq c\end{cases} \tag{1}$$

where x is a variable, and a, b and c are parameters determining the shape of S-function. In this definition b can be any point between a and c. Some examples of possible shapes of this S-function are shown in the Fig. 1

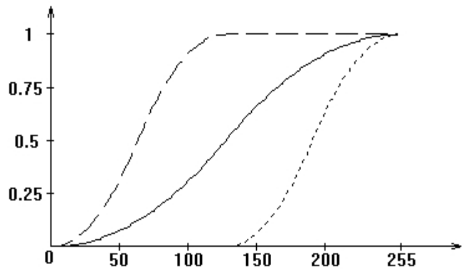

Fig. 1. Different shapes of S-function depend on a, b and c parameters. Lines denoting parameters: dashed line $a = 0, b = 63.5, c = 127.5$; solid line $a = 0, b = 127.5, c = 255$; dotted line $a = 127.5, b = 191, c = 255$

3 Fuzzy Measures

In the literature many fuzzy measures have been proposed [8] [13] as well as measures of fuzzines [15]. This paper incorporates two measures of fuzzy set namely a fuzzy entropy [1] and index of fuzzines introduced by Kaufmann [10].

3.1 Fuzzy Entropy

Many definitions of fuzzy entropy [15] [1] exist in the literature. For the purpose of this work authors employ definition of total entropy [1] [16] which is described as follows.

Let I be a set with randomly occuring events $\{x_1, x_2, \ldots, x_n\}$ in an experiment, and $\{p_1, p_2, \ldots, p_n\}$ are respective probabilities of events. Fuzzyfication of set I induces two kinds of uncertainties. Total entropy of the set F, being fuzzified set I, consists of two parts. The first part of total entropy is a measure deduced from "random" nature of the experiment. Expected value of this uncertainty is computed as Shannon entropy:

$$H\left(p_1, p_2, \ldots, p_n\right) = -\sum_{i=1}^{n} p_i \log\left(p_i\right) \tag{2}$$

The second uncertainty arises from the fuzzines of the fuzzy set F related to the ordinary set. This amount of ambiguity is given by:

$$S(\mu_i) = -\mu_i \log(\mu_i) - (1-\mu_i)\log(1-\mu_i) \tag{3}$$

The statistical average m of the ambiguity for the whole set is given by equation (4):

$$m(\mu, p_1, p_2, \ldots, p_n) = \sum_{i=1}^{n} p_i S(\mu_i) \tag{4}$$

Therefore, the total entropy of the set F is expressed as follows:

$$H_{total} = H(p_1, p_2, \ldots, p_n) + m(\mu, p_1, p_2, \ldots, p_n) \tag{5}$$

3.2 Index of Fuzziness

Let X be universum of discourse and P power set of X. Kaufmann introduced the index of fuzzines γ of fuzzy set $A \subseteq P$:

$$\gamma(A) = \frac{2}{n^k} \times d(A, A^{near}) \tag{6}$$

where d is a suitable metric on the universum X, k is positive number and n number of supporting points. A^{near} is the nearest crisp set to A defined as follows:

$$\mu_A^{near}(x) = \begin{cases} 1\ if\ \mu_A(x) \geq 0.5 \\ 0\ if\ \mu_A(x) \leq 0.5 \end{cases} \tag{7}$$

Using Minkowski's q-norm as a metric, and putting $k = \frac{1}{q}$, the index of fuzzines can be defined as:

$$\gamma(A) = \frac{2}{n^{\frac{1}{q}}} \left\{ \sum_i |\mu_A(x_i) - [1 - \mu_{A^{near}}(x_i)]|^q \right\}^{\frac{1}{q}} \tag{8}$$

In this paper the linear version of this index is used, so the exponential $q = 1$

$$\gamma(A) = \frac{2}{n} \sum_i |\mu_A(x_i) - \mu_{\overline{A}}(x_i)| \tag{9}$$

where $\mu_{\overline{A}}(x_i)$ is complement of set A and $\mu_{\overline{A}}(x_i) = 1 - \mu_A(x_i)$. The difference between the set and nearest ordinal set can be calculated:

$$\gamma(A) = \frac{2}{n} \sum_i [\min\{\mu_A(x_i),\ \ 1 - \mu_A(x_i)\}] \tag{10}$$

Obviously, for an image O of size $M \times N$ with L levels of grey pixels' brightness g, and with the histogram $h(g)$ of the image O, linear index of fuzzines can be given by:

$$\gamma_{linear}(O) = \frac{2}{MN} \sum_{g=0}^{L-1} h(g) \cdot \min[\mu_O(g), \bar{\mu}_{\overline{O}}(g)] \tag{11}$$

where: $\bar{\mu}_{\overline{O}}$ is complement of O and $\mu_{\overline{O}} = 1 - \mu_O(g)$

4 Algorithm of Membership Function Calculation

For estimation of membership function the authors used the measures described above as objective functions for a maximization problem. We would like to find a function which maximizes the information about an image. This condition is reformulated into entropy principle. Taking into consideration that entropy is the measure of information, hence the function which has the maximum entropy is the most informative. However, there still exits need to find a function which describes the fuzzy set in the best way. For this purpose the authors used the index of fuzzines. The result is the function which has the biggest value of index of fuzzines.

Therefore, the problem was reformulated into looking for the function which is optimal with regards to total entropy (5) and index of fuzzines (11).

The objective is find the parameters a, b and c of function (1) describing the shape of function, which fulfils conditions of the maximum entropy as well as maximum value of index of fuzzines. This issue is defined as a two criteria problem. The first criterium is founding the set of parameters $a_{Eopt}, b_{Eopt}, c_{Eopt}$ for which the total entropy (5) has the maximal value.

$$H_{total\_\max}\left[S\left(a_{Eopt}, b_{Eopt}, c_{Eopt}\right)\right] = \max_{a,b,c}\left\{H_{total}\left[S\left(a,b,c\right)\right] \;:\; 0 \le a,b,c \le L\right\} \tag{12}$$

The second criterium is founding the set of parameters $a_{\gamma opt}, b_{\gamma opt}, c_{\gamma opt}$ for which the value of the index of fuzzines (11) has the biggest value.

$$\gamma_{linear\_\max}\left[S\left(a_{\gamma opt}, b_{\gamma opt}, c_{\gamma opt}\right)\right] = \max_{a,b,c}\left\{\gamma\left[S\left(a,b,c\right)\right] \;:\; 0 \le a,b,c \le L\right\} \tag{13}$$

After solving eq. (12) and eq. (13) there are two sets of parameters. The S-function described by the average values of the parameters has been chosen as solution.

$$\begin{aligned} a_{opt} &= \tfrac{a_{Eopt}+a_{\gamma opt}}{2} \\ b_{opt} &= \tfrac{b_{Eopt}+b_{\gamma opt}}{2} \\ c_{opt} &= \tfrac{c_{Eopt}+c_{\gamma opt}}{2} \end{aligned} \tag{14}$$

It is assume that S-function (1) described by set of parameters' values given by (14) is the one which is the most informative and describes the fuzzy set in the best way.

Calculations for finding the set of optimal parameters (14) are performed using well known optimization algorithm Particle Swarm Optimization (in short PSO), which is well described in the literature [12] [11].

5 Examples of the Algorithm Results

Figures Fig. 2 ... Fig. 6 present the shapes of membership functions which were computed by this algorithm. Five different images with different histograms were chosen for illustration.

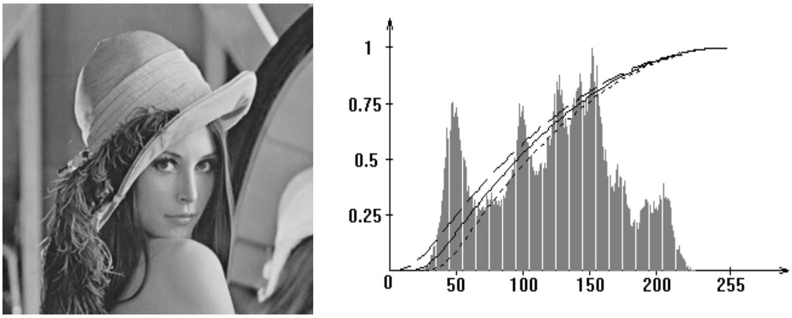

Fig. 2. The original image "Lena" (on the left) and histogram with determined membership function (on the right). Significance of lines: dotted – maximum index of fuzzines; dashed – maximum fuzzy entropy; solid – average between maximum entropy and maximum fuzzy index.

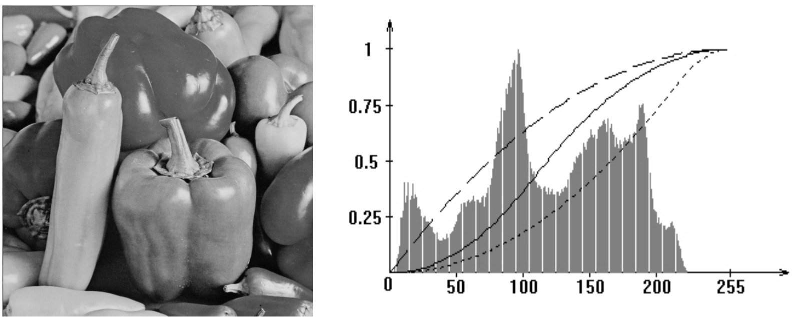

Fig. 3. The image "Peppers" (on the left) and histogram with determined membership functions (on the right). Significance of lines: as in the Fig. 2.

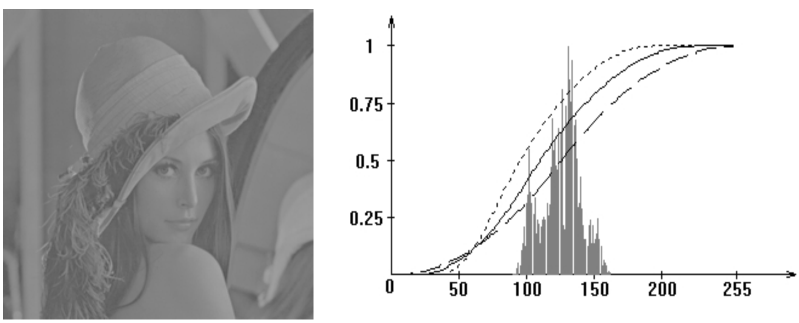

Fig. 4. Image "Lena" with very bad contrast (on the left) and histogram with determined membership functions (on the right). Significance of lines: as in the Fig. 2.

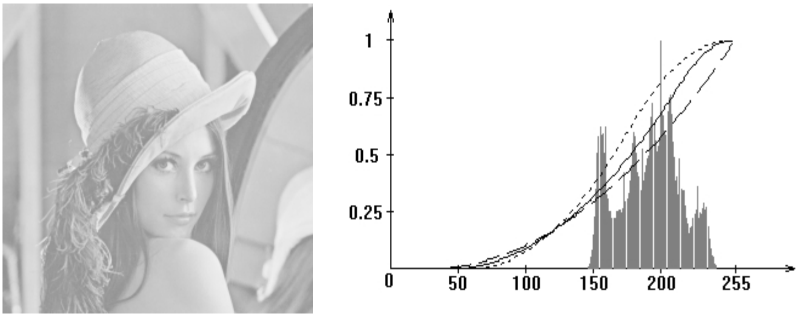

Fig. 5. Image "Lena" which is too bright (on the left) and histogram with determined membership functions (in the right). Significance of lines: as in the Fig. 2.

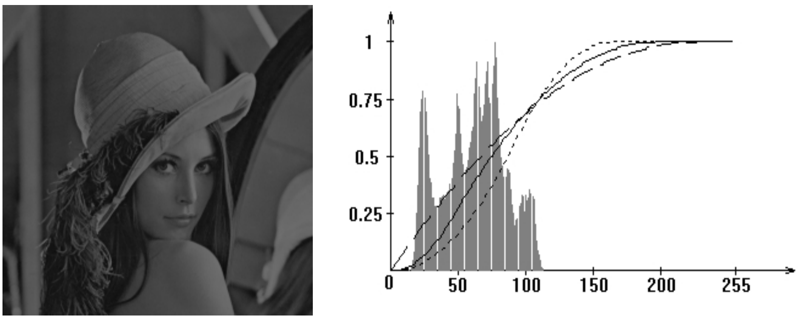

Fig. 6. Image "Lena" which is too dark (on the left) and histogram with determined membership functions (on the right). Significance of lines: as in the Fig. 2.

6 Linguistic Modifiers for Image Enhancement

For image with estimated membership function linguistic modifiers can be used. The effect of application of different modifiers was measured by *Mean Squared Error* (MSE) (15) calculated as follows:

$$MSE = \frac{\sum_{i=1}^{M_1} \sum_{j=1}^{M_2} \left| O(i,j) - O'(i,j) \right|^2}{M_1 \cdot M_2} \tag{15}$$

where $1 \leq i \leq M_1, 1 \leq j \leq M_2$ and M_1 and M_2 are respectively heights and widths of the original image O and modified image O' expressed in pixel values. In the presented figures and the tables, there is shown modifiers influence on image.

6.1 Modifier *"Increase Contrast"*

For modifier "increase contrast" is applied well known operator of intensification $INT(A)$ of fuzzy set [9]:

$$\mu_{INT(A)}(x) = \begin{cases} 2\left[\mu_A(x)\right]^2 & \forall x \; : \; \mu_A(x) < 0.5 \\ 1 - 2\left[1 - \left(\mu_A(x)\right)^2\right] & \forall x \; : \; \mu_A(x) \geq 0.5 \end{cases} \qquad (16)$$

That application of this operator give the good result as is shown in the Fig.7

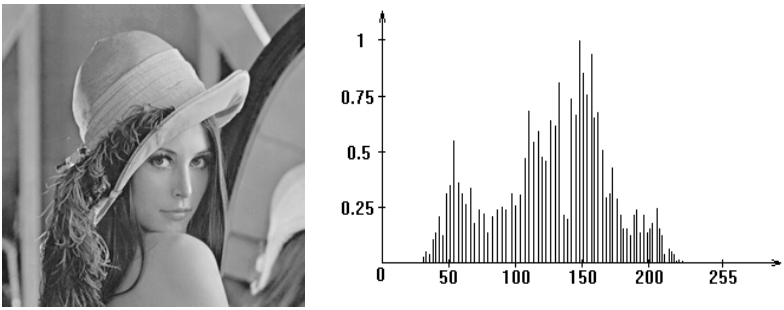

Fig. 7. Illustration of using *"increase contrast"* linguistic modifier. The image "Lena" after modification (on the left) and histogram of the modified image (on the right). The original image presented in the Fig. 4 has bad contrast.

Table 1. Influence of *"increase contrast"* hedge on the MSE error

Image	MSE
"Lena" image with bad contrast	1095.5
After using linguistic hedge	78.7620

6.2 Modifier "Brighter"

As a representation of linguistic modifier "brighter" the operator of dilation of fuzzy set was chosen. Originally dilation $DIL(A)$ of fuzzy set is defined as [9]:

$$\mu_{DIL(A)}(x) = \left[\mu_A(x)\right]^{0.5} \quad \forall x \in X \qquad (17)$$

In this example for better result another power in eq.(17) was chosen. Following equation is used in this example:

$$\mu_{DIL(A)}(x) = \left[\mu_A(x)\right]^{0.25} \quad \forall x \in X \qquad (18)$$

The linguistic modifier *"brighter"* as is given by eq.(18) was used for image with bad dark colors which is presented in the Fig. 6. Application of this modifier gives a very good results. Enhanced image is shown in the Fig. 8. Comparison of MSE error is given in the Tab. 2.

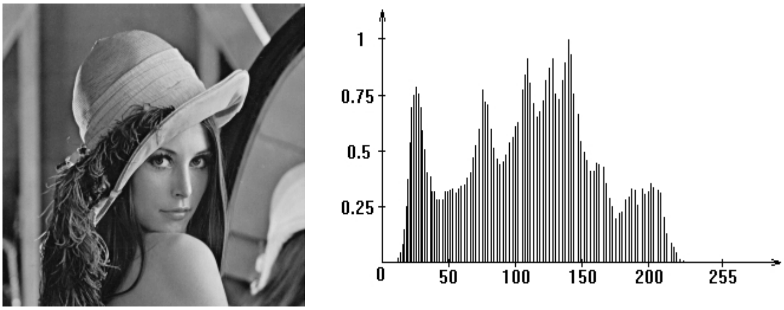

Fig. 8. Illustration of using *"brighter"* linguistic hedge. Image "Lena" after brightening (on the left) and histogram of modified image (on the right). Image with bad bright color before modification is presented in Fig. 6.

Table 2. Influence of *"brighter"* modifier at the MSE error

Image	**MSE**
"Lena" image with bad bright colors	4261.7
After using linguistic hedge *"brighter"*	350.5

6.3 Modifier "Darker"

As a linguistic modifier "darker" concentration of fuzzy set was employed. Originally dilation $CON(A)$ of fuzzy set is defined as [9]:

$$\mu_{CON(A)}(x) = [\mu_A(x)]^2 \quad \forall x \in X \tag{19}$$

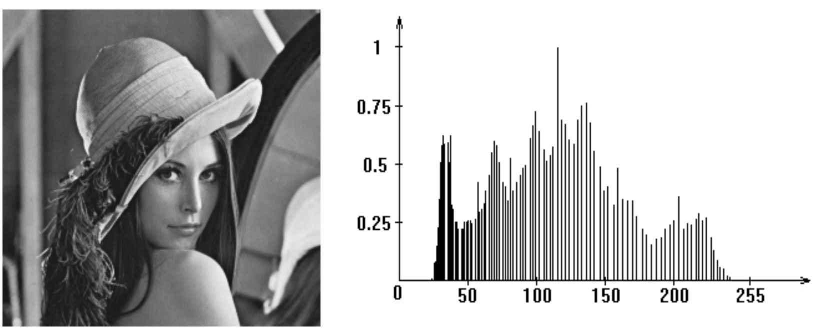

Image "Lena" after darkering   Histogram of image "Lena" after darkering

Fig. 9. Illustration of using *"darker"* linguistic modifier. Image "Lena" after modification (on the left) and histogram (on the right). Image with bad bright colors as presented in Fig. 5.

However, here the power equal 1.5 for better result is applied in this case. The modifier given by eq.(19) with power 1.5, is utilized to image with bad white colors, as is shown in the Fig. 5. The result after application of this modifier is shown in the Fig. 9. Comparison of MSE errors is given in the Tab. 3

Table 3. Influence of *"darker"* hedge at the MSE error

Image	**MSE**
"Lena" image with bad dark colors	5419.0
After using linguistic hedge *"darker"*	505.2

7 Conclusion

Fuzzy set theory has been successful applied to many tasks in image processing as image filtering or pattern recognition. However, for each usage of fuzzy sets not only in image processing but generally, it is needed to know the membership function's shape and values.

This paper presents the method dealing problem of determination membership function. The well known S-function is used for representing pixels' values which belong to the set of bright pixel. The methodology for choosing parameters of S-function in reasonable way is proposed basing on fuzzy measures. The chosen function is one which compromises the conditions of information and good fuzzines.

The new application area namely, the linguistic modifiers of image is outlined. The way of modelling natural language is shown with the good effects of using linguistic modifiers in image enhancement.

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