Evaluation of Spectral-Based Methods for Median Graph Computation*

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Abstract. The median graph is a useful tool to cluster a set of graphs and obtain a prototype of them. The spectral graph theory is another approach to represent graphs and find "good" approximate solutions for the graph-matching problem. Recently, both approaches have been put together and a new representation has emerged, which is called Spectral-Median Graphs. In this paper, we summarize and compare two techniques to synthesize a Spectral-Median Graph: one is based on the correlation of the modal matrices and the other one is based on the averaging of the spectral modes. Results show that, although both approaches obtain good prototypes of the clusters, the first one is slightly more robust against the noise than the second one.

1 Introduction

Graphs, specially labelled or attributed relational graphs, are general and powerful data structures for object representation in structural pattern recognition and computer vision applications. When objects are represented by graphs, *graph matching* is used to compare such objects. Algorithms for graph matching include graph and subgraph isomorphism [1]. However, due to errors and noise in the input data, many times it is not possible to find a perfect match between two elements and then, algorithms for approximate or error-tolerant matching must be considered. These algorithms compute a similarity measure between two given graphs. An excellent survey on graph matching algorithms and applications to pattern recognition is [2].

In some of these applications it may be necessary to obtain the prototype of a set of objects. Given a set of noisy samples of a certain object, error-tolerant graph matching can be useful to infer a representative model that captures the essential information of the class while rejecting small distortions due to noise. In this context the concept of median graph [3] can be very useful and it has already been applied to the synthesis of a prototype of a set of graphical symbols [4].

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It is well-known that one of the drawbacks of graph matching is its computational complexity. However, in the last years, spectral graph theory have been applied to graph matching as an alternative way to obtain approximate solutions in a reasonable time [5]. In this paper, we perform an evaluation and comparison between two spectral-based methods for the computation of the median graph. The first one [6] is based on the correlation of the modal matrices and the latter [7] is based on the averaging of the spectral modes. Concretely, we have applied such methods to compute the representative prototype of a set of graphical symbols. Thus, we first define a graph-based representation of symbols that is suitable for applying spectral techniques. Then we evaluate both methods performing two experiments: 1) the similarity of the approximate solutions to the ideal median graph 2) the recognition rate. Finally, such methods are compared based on these experiments. The results show that with both methods good prototypes are obtained. However the method based on the correlation of the modal matrices is slightly more robust against the distortions than the method based on the spectral modes average.

The rest of the paper is organized as follows. In section 2, we present the methods for the spectral-median graph computation. In section 3 we introduce the representation of graphical symbols used to perform the tests. Section 4 presents the experiments and the results obtained. We terminate with some conclusions and possible future research lines.

2 Synthesis of Spectral-Median Graphs

Given a set of graphs, the generalized median is defined as the graph that has the smallest sum of distances to all graphs in the set. Formally speaking, median graph can be defined as follows:

Let Z be the set of graphs that can be constructed using labels from L_V and L_E. Given $S = \{G_1, G_2, ..., G_n\} \subset Z$, the generalized median graph $\bar{g}$ of S are defined as follows:

$$\bar{g} = arg\left(\underset{G \in Z}{min} \sum_{G_i \in S} d(G, G_i)\right) \tag{1}$$

In the following lines we present two methods for the synthesis of spectral-median graphs. The first one is based on the correlation of the modal matrices while the latter is based on the averaging of the spectral modes.

2.1 Modal Matrix Correlation Method (*C-Method*)

The first method is that presented in [6]. Merging the concepts of median graph [3] and spectral graph theory and using the Umeyama's method [5] to solve the weighted graph matching problem, they presented the concept of spectral median graph. Concretely, they used an incremental algorithm to compute

the generalized spectral median graph. Let $\{\Phi_1, \Phi_2, \ldots, \Phi_n\}$ be the set of modal matrices which represents the spectral counterpart of S in definition 1. If $\Phi = \left(\phi_1|\phi_2|\ldots|\phi_{|V|}\right)$ is the modal matrix of a graph $G = (V, E)$ with ordered eigenvectors and $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_{|V|})$ the diagonal matrix containing the ordered eigenvalues, they first perform a maximization of the correlation between the modal matrices of two graphs in the set using the procedure explained in [5]. In this step they obtain an intermediate median graph. Then, the modal matrix of this intermediate median graph is used to maximize the correlation to the next graph in the set, and the process is repeated iteratively until the last graph in the set is processed, giving the final spectral-median graph. The median graph is obtained in each iteration computing the adjacency matrix by means of the eigenvalues and eigenvectors applying $G = \Phi\Lambda\Phi^T$. The reader is referred to [6] for more details.

2.2 Spectral Modes Averaging Method (*M-Method*)

The second approach has been presented in [7]. They propose the direct mixing or averaging of spectral modes. If $\Phi = (\phi_1|\phi_2|\ldots|\phi_{|V|})$ is the modal matrix of a graph $G = (V, E)$ with ordered eigenvectors and the diagonal matrix of ordered eigenvalues is $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_{|V|})$, the first step before mixing the two representations of two graphs is to align the rows of Φ. This can be done using the methods proposed in [5,6]. Once aligned a sign correction must be done on each modal matrix in order to mix correctly the eigenmodes. First they must find the largest magnitude component for each mode. Then, they have to correct the sign of the eigenvectors by ensuring that the largest component is positive for each mode in all modal matrices. Once aligned the spectral matrices may be merged by simply taking the average of the matrices, $\Phi_m = (\widehat{\Phi}_1 + \widehat{\Phi}_2)/2$ and $\Lambda_m = (\widehat{\Lambda}_1 + \widehat{\Lambda}_2)/2$. The reconstruction of the graph can be done performing a reverse eigendecomposition $X_m = \Phi_m\Lambda_m\Phi_m^T$ as in the previous described method. As we are working with adjacency matrices, we will obtain in general consistent results and the last step (projection of the obtained graph onto the nearest graph) described in [7] is not necessary.

3 Representation of Graphical Symbols and the Dataset

In this paper we have applied such methods to the computation of the prototype of a given set of graphical symbols. We have chosen a subset of the symbols used in the *Sixth IAPR International Workshop on Graphics Recognition - GREC 2005* [8]. This subset contains 80 different symbols (classes), extracted from architectural, electric and other technical fields. Some representative symbols of such subset are shown in figure 1. Notice that all of them are composed of a set of straight lines. Each segment terminates either with a terminal point or a junction point (confluence point between two or more segments). For convenience, from now to the end of this work, we will refer to these kinds of points as TP and JP respectively.

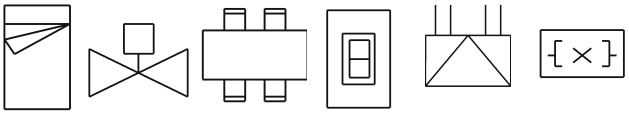

Fig. 1. Six symbols corresponding to *GREC 2005* database

Graph-based representation: In order to compute the prototypes a graph-based representation of the symbols must be defined. We have defined two different representations, namely *node-based* representation and *edge-based* representation. In both of them a symbol is represented as an undirected labeled graph, where the TPs and JPs are represented as nodes. Edges correspond to the segments connecting those points. The information associated to nodes or edges are their coordinates (x, y). As labels can only be real numbers we have created two adjacency matrices for each symbol, one of them containing x-coordinates and the other containing y-coordinates. In the *edge-based* representation, information associated to nodes is always 0 while edge labels are the coordinates (x, y) of the mid point of the segment. In the *node-based* representation, labels of nodes are the coordinates (x, y) of the point while labels of edges are always 1. In both cases we store a 0 when no edge exists between two nodes. The distance between two symbols will be the mean between the x and y distances. Figure 2 shows the two representations of a symbol.

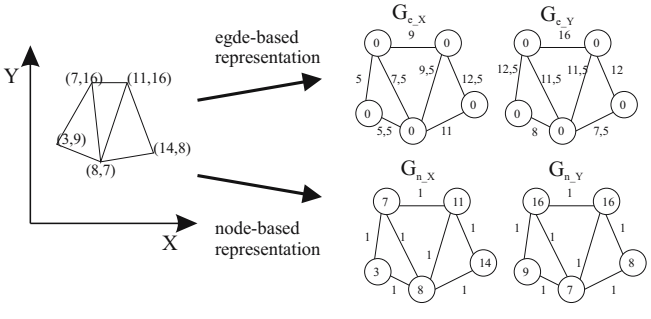

Fig. 2. Two graph-based representations of a graphical symbol

Generation of the dataset: In order to prove the robustness of the prototypes against noise, 7 different levels of distortion have been introduced. Distortion is generated moving each TP or JP randomly within a circle of radius r, given as a parameter for each level, centered at original coordinates of the point. If a JP is randomly moved, all the segments connected to it are also moved. With such distortion, gaps in line segments, missing line segments and wrong line segments are not allowed. But the number of nodes of each symbol is not changed. Figure 3 shows an example of such distortions. In addition we have generated another set of symbols using the same distortion and adding structural variations by

randomly dropping an edge in each symbol. For each class, for each distortion, and for each structural variation level we have created 100 images. Thus for each class we have 1400 elements (100 for each distortion and structural variation). Therefore, we have 11200 (80*700*2) images to perform the experiments.

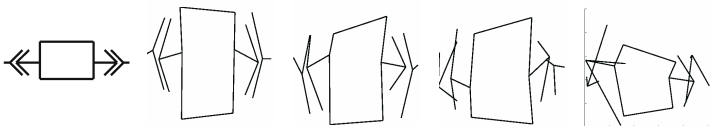

Fig. 3. Original model (a) and distorted models (levels: 1 (b), 3 (c), 5 (d) and 7 (e))

4 Experiments and Results

In this section the experiments we have performed will be further explained. Concretely, we propose two measures in order to test the accuracy and the robustness of the two methods explained in section 2 to compute the prototype of a set of a given models. This measures are *Intra-class Median Accuracy* and *Recognition Rate*. Recall that for each class and distortion we have generated 100 elements. For the experiments mentioned before we have defined, for each class and distortion, a training set composed of 25 symbols used to compute the medians and a test set composed of the remaining 75 symbols. Both methods explained in section 2 have been tested using the two representations explained above. In addition, we have introduced some structural variations in the node-based representation by randomly dropping one edge in each symbol. Due to space constraints we will refer to these combinations in the experiments as Edge, Node-0 and Node-1 respectively.

Intra-class Median Accuracy: In this measure, the sum of distances (SOD) of the median to all the other elements in the class is computed and compared to the SOD of all the elements in the class. According to its definition, the median graph would always have the minimum SOD. So, if we rank the median and all the other elements according to SOD, the lower is the position of the median, the better is the representation for the median. Two variants of this experiment were performed. In both cases the Spectral-median graphs were computed using 1, 10 and 25 symbols from the training set. While in the first variant, the *Intra-class Median Accuracy* was computed using the 25 symbols in the training set, in the second variant, the *Intra-class Median Accuracy* was computed using the 75 symbols in the test set. These two variants were designed in order to prove the goodness of the obtained Spectral-median graphs. The results for these two variants are shown in figures 4 and 5 respectively.

Regarding the first variant, the results show that when the median is computed with only one model, the position of the SOD of the median with respect to the rest of the elements in the class is distributed randomly. As a consequence, the accumulative frequency tends to be linear from 0 to 100. For the

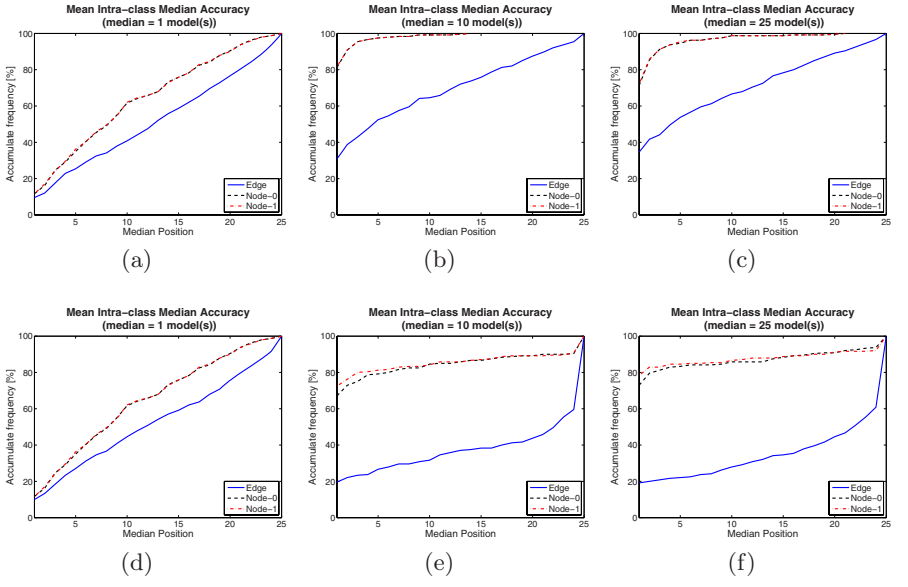

Fig. 4. Intra-class median accuracy of C-method ((a)-(c)) and M-method ((d)-(f)) for the training set

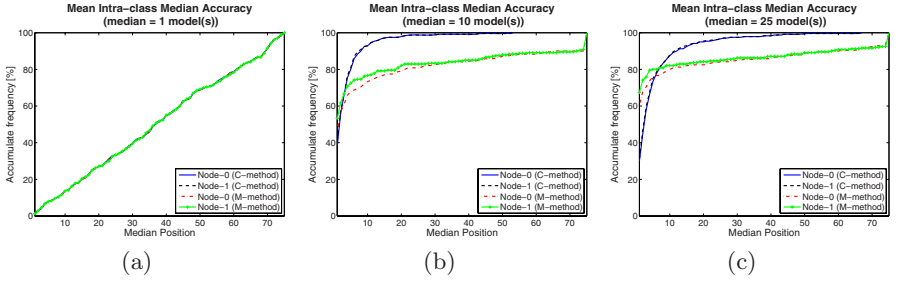

Fig. 5. Intra-class median accuracy for the test set

other cases we can see that both methods have better behavior. However these results show that the representation of the median obtained with the C-method (figure 4 (b)-(c)) outperforms the results of M-method (figure 4 (e)-(f)). In addition, the results obtained in both methods for the node-based representations are better with respect to those obtained in the edge-based representation. For this reason we performed the second variant only taking into account the node-based representation. Results for this variant (figure 5) show similar behavior of the methods as in the first variant. It is interesting to notice that for the M-method the curves for the training and the test set are more similar than in the case of the C-method. This fact means that the median obtained using the M-method is more general and therefore, it represents better the class. However,

the curve of the C-method is more abrupt. This could mean more stability in the generalization of the solution.

Recognition Rate: In this case, one median was computed for each class and distortion level using the training set. Then, all the models in the database (test set) were matched against the computed medians and classified according to the median with minimum distance. It must be noticed, that, as spectral graph matching requires the two graphs to have the same number of nodes, only 20 classes, those with the same number of nodes in their elements, have been used. Tables 1 and 2 show the mean recognition rates as a function of distortion level and number of symbols used to compute the median respectively. In both tables the results are the mean values over all 20 classes.

Table 1. *Experiment 2:* Recognition Rate [%] vs Distortion Level

	C-method			**M-method**		
Dist. Level	**Edge**	**Node-0**	**Node-1**	**Edge**	**Node-0**	**Node-1**
1	95.90	99.67	99.68	89.83	99.62	99.57
2	93.18	97.96	97.97	92.21	97.97	97.95
3	93.63	97.12	97.11	88.18	97.12	97.15
4	93.11	95.93	95.92	90.01	95.63	95.91
5	91.81	95.55	95.56	91.68	95.01	95.01
6	93.54	94.14	94.14	90.15	93.05	93.32
7	92.81	93.40	93.40	80.77	90.88	90.64

Table 2. *Experiment 2:* Recognition Rate [%] vs Number of symbols

	C-method			**M-method**		
Num. of Symbols	**Edge**	**Node-0**	**Node-1**	**Edge**	**Node-0**	**Node-1**
1	91.49	94.40	94.40	92.26	94.40	94.40
5	93.44	96.45	96.43	88.95	96.15	96.13
10	92.80	96.27	96.28	87.98	95.01	95.27
15	93.91	96.66	96.67	87.77	96.10	96.12
20	95.05	96.85	96.85	88.20	95.77	95.86
25	93.85	96.88	96.88	88.70	96.24	96.11

Results show that both methods have similar recognition rates. Nevertheless, concerning the results obtained regarding the distortion levels the C-method is slightly better than M-method specially in high levels of distortion and using the edge-based representation. The results regarding the number of symbols used to compute the median are very similar for both methods, except in the case of the edge-based representation. It is to be noted too that the node-based representation obtains better results in all cases.

5 Conclusion

The median graph concept as an alternative to represent prototypes of a set of graphs has been turned out very useful, but the computation of both exact and approximate solutions has been shown very hard.

In this paper we have applied two different schemes for the computation of spectral-median graphs. In particular they have been applied to the computation of approximate solutions for the mean graph to the graphical symbol recognition problem. Intra-class median accuracy experiment shows some differences between the methods regarding the generalization and the stability of the solutions they provide. The results for the recognition rate experiment show that the two methods have similar results, but some differences have been detected regarding the level of distortion. In particular we have shown that the C-method outperforms the M-method when high distortion is introduced in the symbols. We have defined two graph representations of graphical symbols, obtaining better results with the node-based representation in both algorithms. These results suggest that a deep study of the influence of the representation and the structure of the adjacency matrix should be done in order to characterize as well as possible the behavior of spectral techniques.

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