Chance Constrained Nonlinear Model Predictive Control

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Summary. A novel robust controller, chance constrained nonlinear MPC, is presented. Time-dependent uncertain variables are considered and described with piecewise stochastic variables over the prediction horizon. Restrictions are satisfied with a user-defined probability level. To compute the probability and its derivatives of satisfying process restrictions, the inverse mapping approach is extended to dynamic chance constrained optimization cases. A step of probability maximization is used to address the feasibility problem. A mixing process with both an uncertain inflow rate and an uncertain feed concentration is investigated to demonstrate the effectiveness of the proposed control strategy.

1 Introduction

Model predictive control (MPC) refers to a family of control algorithms which utilize an explicit model to calculate the manipulated variables that optimize the future plant behaviour. The inherent advantages of MPC, including its capability of dealing with multivariate variable problems as well as its capability of handling constraints, make it widely used in the process industry.

Due to the nature of process uncertainty, a robust MPC is desired to obtain satisfactory control performances. Including uncertainty in control system design will enhance the robustness of MPC. Generally speaking, there are three basic approaches to address uncertainty. The constant approach which assumes the model mismatch is unchanged during the prediction horizon [1] leads to an aggressive control strategy. In contrary, the Min-Max approach in which the boundaries of the uncertain variables are taken into account [2] is too conservative. The third one is the stochastic approach, or chance constrained MPC [3], [4], in which uncertain variables in the prediction horizon are described as stochastic variables with known probability distribution functions (PDF). Restrictions are to be satisfied with a user-defined probability level. Due to the fact that using this method a desired compromise between the optimal function

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and the reliability of holding the constraints can be chosen, the derived control strategy can be neither aggressive nor conservative.

Linear chance constrained MPC have been previously studied in ref [10]. In the present study, we extend this approach to nonlinear systems. The major obstacle towards realizing chance constrained nonlinear MPC (CNMPC) lies in the computation of the probability and its deviations of satisfying process restrictions. To address this problem, an inverse mapping approach proposed by Wendt *et al.* [5] is extended to dynamic chance constrained optimization. In addition, a step of maximization is proposed to address the feasibility problem of CNMPC.

The paper is divided into the following sections. Section 2 gives a general formulation of CNMPC considering both parameter and disturbance uncertainties. Section 3 analyzes some computational aspects of CNMPC. The effectiveness of CNMPC is illustrated in Section 4 by controlling a mixing process. Finally, some concluding remarks of this work are given in Section 5.

2 Chance Constrained Nonlinear MPC

It has been recognized that problems in process system engineering (PSE) are almost all confronted with uncertainties [7], [13]. In the industrial practice, uncertainties are usually compensated by using conservative design as well as conservative operating strategies, which may lead to considerably more costs than necessary. To overcome this drawback, the authors have recently developed a chance constrained programming (CCP) framework for process optimization and control [3], [5], [10], [11], [12]. In this framework, the uncertainty properties, obtained from the statistical analysis of historical data, are included in the problem formulation explicitly.

Chance constrained nonlinear MPC (CNMPC) employs a nonlinear model to predict future outputs, based on the current states, past controls as well as uncertain variables. The optimal control sequence is obtained at every sampling instant by optimizing some objective functions and ensuring the chance constraints for the outputs.

The general CNMPC problem to be solved at sampling time k is formulated as follows:

$$
\begin{aligned}
&\text{Min} \quad J = E\{f\} + \omega D\{f\} \\
&s.t. \\
&f = \sum_{i=1}^{P} \|\mathbf{y}(k+i|k) - \mathbf{y}_{ref}\|_{\mathbf{Q}_i} \\
&\qquad\qquad + \sum_{i=0}^{M-1} \{\|\mathbf{u}(k+i|k) - \mathbf{u}_{ref}\|_{\mathbf{R}_i} + \|\Delta\mathbf{u}(k+i|k)\|_{\mathbf{S}_i}\} \qquad (1) \\
&\mathbf{x}(k+i+1|k) = \mathbf{g}_1(\mathbf{x}(k+i|k), \mathbf{u}(k+i|k), \xi(k+i)) \\
&\mathbf{y}(k+i|k) = \mathbf{g}_2(\mathbf{x}(k+i|k), \xi(k+i))
\end{aligned}
$$

$$\Delta \mathbf{u}(k+i|k) = \mathbf{u}(k+i|k) - \mathbf{u}(k+i-1|k)$$
$$\mathbf{u}_{\min} \leq \mathbf{u}(k+i|k) \leq \mathbf{u}_{\max}, i = 0, \ldots, M-1.$$
$$\Delta \mathbf{u}_{\min} \leq \Delta \mathbf{u}(k+i|k) \leq \Delta \mathbf{u}_{\max}, i = 0, \ldots, M-1.$$
$$\mathcal{P}\{\mathbf{y}_{\min} \leq \mathbf{y}(k+i|k) \leq \mathbf{y}_{\max}\} \geq \alpha, i = 1, \ldots, P.$$

where P and M are the length of prediction and control horizon, ξ represents the uncertain variables with known PDF, $\mathcal{P}\{\cdot\}$ represents the probability to satisfy the constraint $\mathbf{y}_{\min} \leq \mathbf{y}(k+i|k) \leq \mathbf{y}_{\max}$ and $0 \leq \alpha \leq 1$ is the predefined confidence level. States $\mathbf{x}$, outputs $\mathbf{y}$ and controls $\mathbf{u}$ are all doubly indexed to indicate values at time $k+i$ given information up to and including time k. Q_i, R_i, and S_i are weighting matrices in the objective function. E and D are the operators of expectation and variation, respectively.

Since the outputs have been confined in the chance constraints, the objective function f in Eq.(1) may exclude the quadratic terms on outputs for the sake of simplicity [10]. The simplified CNMPC objective function can be described as follows:

$$\text{Min} \quad J = \sum_{i=1}^{M-1} \{\|\mathbf{u}(k+i|k) - \mathbf{u}_{ref}\|_{\mathbf{R}_i} + \|\Delta \mathbf{u}(k+i|k)\|_{\mathbf{S}_i}\} \tag{2}$$

This problem can be solved by using a nonlinear programming algorithm. The key obstacle towards solving the CNMPC problem is how to compute $\mathcal{P}\{\cdot\}$ and its gradient with respect to the controls. In the next section, the computational aspects of CNMPC to address this problem as well as the feasibility analysis will be discussed.

3 Computational Aspects of CNMPC

In process engineering practice, uncertain variables are usually assumed to be normally distributed due to the central limit theory. However, a normal distribution means that the uncertain variable is boundless, which is not true for some parameters with physical meanings, e.g. the molar concentration in a flow should be in the range of [0, 1]. In order to describe the physical limits of the uncertainty parameters, it is preferable to employ truncated normal distribution which has been used extensively in the fields of economic theory [9]. The basic definition of truncated normal distribution is given as follows:

Definition 1. *Let z be a normally distributed random variable with the following PDF:*

$$\rho(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{(z-\mu)^2}{2\sigma^2}\} \tag{3}$$

Then the PDF of ξ, the truncated version of z on [a_1, a_2] is given by:

$$\rho(\xi) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}(\Phi(a_2)-\Phi(a_1))} \exp\{-\frac{(\xi-\mu)^2}{2\sigma^2}\}, \ a_1 \leq \xi \leq a_2 \\ 0, \xi \leq a_1 \ or \ a_2 \leq \xi \end{cases} \tag{4}$$

where $\Phi(\cdot)$is the cumulative distribution function of z.

Detailed discussion about the properties of the truncated normal distribution can be found in [9] and it is easy to extend Definition 1 to the multivariate case. In the following, a truncated normally distributed ξ with mean μ, covariance matrix $\mathbf{\Sigma}$ and truncated points $\mathbf{a}_1$, $\mathbf{a}_2$, denoted as $\xi \sim TN(\mu, \mathbf{\Sigma}, \mathbf{a}_1, \mathbf{a}_2)$, is considered.

3.1 Inverse Mapping Approach to Compute the Probability and Gradient

If the joint PDF of the output $\mathbf{y}(k+i|k)$ is available, the calculation of $\mathcal{P}\{\mathbf{y}_{\min} \leq \mathbf{y}(k+i|k) \leq \mathbf{y}_{\max}\}$ and its gradient to $\mathbf{u}$ can be cast as a standard multivariate integration problem [8]. But unfortunately, depending on the form of $\mathbf{g}_2$, the explicit form of the output PDF is not always avaliable. To avoid directly using the output PDF, an inverse mapping method has been recently proposed for situations in which the monotone relation exists between the output and one of the uncertain variables [5].

Without loss of generality, let $y = F(\xi_S)$ denotes the monotone relation between a single output y and one of the uncertain variables ξ_S in ξ=[ξ_1, $\xi_{2,}\dots$, $\xi_S]^T$. Due to the monotony, a point between the interval of $[y_{min}, y_{max}]$ can be inversely mapped to a unique ξ_S through $\xi_S = F^{-1}(y)$:

$$\mathcal{P}\{y_{\min} \leq y \leq y_{\max}\} \Leftrightarrow \mathcal{P}\{\xi_S^{\min} \leq \xi_S \leq \xi_S^{\max}\} \tag{5}$$

It should be noted that the bounds $\xi_S^{\min}$, $\xi_S^{\max}$ depends on the realization of the individual uncertain variables ξ_i, $(i = 1, \cdots, S-1)$ and the value of input u, i.e.

$$[\xi_S^{\min}, \xi_S^{\max}] = F^{-1}(\xi_1, \cdots, \xi_{S-1}, y_{\min}, y_{\max}, u) \tag{6}$$

and this leads to the following representation

$$\mathcal{P}\{y_{\min} \leq y \leq y_{\max}\} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{\xi_S^{\min}}^{\xi_S^{\max}} \rho(\xi_1, \cdots, \xi_{S-1}, \xi_S) d\xi_S d\xi_{S-1} \cdots d\xi_1 \tag{7}$$

From (6) and (7), u has the impact on the integration bound of ξ_S. Thus the following equation can be used to compute the gradient of $\mathcal{P}\{y_{\min} \leq y \leq y_{\max}\}$ with respect to the control variable u:

$$\begin{aligned}\frac{\partial \mathcal{P}\{y_{\min} \leq y \leq y_{\max}\}}{\partial u} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \{\rho(\xi_1, \cdots, \xi_{S-1}, \xi_S^{\max}) \frac{\partial \xi_S^{\max}}{\partial u} - \\ \rho(\xi_1, \cdots, \xi_{S-1}, \xi_S^{\min}) \frac{\partial \xi_S^{\min}}{\partial u}\} d\xi_{S-1} \cdots d\xi_1\end{aligned} \tag{8}$$

A numerical integration of (7) is required when taking a joint distribution function of ξ into account. Note that the integration bound of the last variable in (7) is not fixed. A novel iterative method based on the orthogonal collocation on finite elements was proposed in ref [5] to accomplish the numerical integration in the unfixed-bounded region.

Extending inverse mapping to the dynamic case. If a monotone relation also exists between $\mathbf{y}(k+i|k)$ and $\xi(k+i)$ for $i = 1, \dots, P$ in $\mathbf{g}_2$ of Eq.(1), the inverse mapping method is readily extended to obtain the value and the gradient of $\mathcal{P}\{\mathbf{y}_{\min} \le \mathbf{y}(k+i|k) \le \mathbf{y}_{\max}\}$. For the sake of simplifying notations, a SISO system is considered in the present study, and it is not difficult to generalize the following conclusions. With the monotone relation between $y(k+i|k)$ and $\xi(k+i)$, we have

$$y_{\min} \le y(k+i|k) \le y_{\max} \Leftrightarrow \xi_{k+i}^{\min} \le \xi(k+i) \le \xi_{k+i}^{\max} \tag{9}$$

Due to the propagation of the uncertainty through the dynamic system, $y(k+i|k)$ is influenced not only by $\xi(k+i)$, $u(k+i-1|k)$, but also by previous $\xi(k)$ to $\xi(k+i-1)$ and $u(k|k)$ to $u(k+i-2|k)$. Therefore, the bounds $\xi_{k+i}^{\min}$ and $\xi_{k+i}^{\max}$ are determined based on the realization of the uncertain variables and controls from the time interval k to $k+i$, namely,

$$[\xi_{k+i}^{\min}, \xi_{k+i}^{\max}] = F^{-1}(\xi(k+i-1), \cdots, \xi(k), u(k+i-1|k), \cdots, u(k|k), y_{\min}, y_{\max}) \tag{10}$$

So the joint outputs chance constraint over the prediction horizon can be reformulated as

$$\begin{aligned}
&\mathcal{P}\{y_{\min} \le y(k+i|k) \le y_{\max}, i = 1, 2, \ldots, P\} \\
&= \mathcal{P}\{\xi_{k+i}^{\min} \le \xi(k+i) \le \xi_{k+i}^{\max}, i = 1, 2, \ldots, P\} \\
&= \int_{-\infty}^{\infty} \int_{\xi_{k+1}^{\min}}^{\xi_{k+1}^{\max}} \cdots \int_{\xi_{k+P}^{\min}}^{\xi_{k+P}^{\max}} \rho(\xi(k), \xi(k+1) \cdots, \xi(k+P)) d\xi(k+P) \cdots d\xi(k+1) d\xi(k)
\end{aligned} \tag{11}$$

where ρ is the joint PDF of the future uncertain variables.

The gradient computation of $\mathcal{P}\{\cdot\}$ is more complicated due to the complex relation between the integration bounds and the controls. With the assumption of a same control and prediction horizon, M=P, the gradient with respect to $u(k+i|k)$ can be determined as follows

$$\begin{aligned}
&\partial \mathcal{P}\{y_{\min} \le y(k+i|k) \le y_{\max}, i = 1, 2, \ldots, P\} / \partial u(k+i|k) = \\
&\sum_{j=i+1}^{P} \{ \int_{-\infty}^{\infty} \int_{\xi_{k+1}^{\min}}^{\xi_{k+1}^{\max}} \cdots \int_{\xi_{k+j-1}^{\min}}^{\xi_{k+j-1}^{\max}} \\
&\{ \frac{\partial \xi_{k+j}^{\max}}{\partial u(k+i|k)} \int_{\xi_{k+j+1}^{\min}}^{\xi_{k+j+1}^{\max}} \cdots \int_{\xi_{k+P}^{\min}}^{\xi_{k+P}^{\max}} \rho(\xi(k), \cdots, \xi_{k+j}^{\max}, \cdots \xi(k+P)) d\xi(k+P) \cdots d\xi(k+j+1) \\
&- \frac{\partial \xi_{k+j}^{\min}}{\partial u(k+i|k)} \int_{\xi_{k+j+1}^{\min}}^{\xi_{k+j+1}^{\max}} \cdots \int_{\xi_{k+P}^{\min}}^{\xi_{k+P}^{\max}} \rho(\xi(k), \cdots, \xi_{k+j}^{\min}, \cdots \xi(k+P)) d\xi(k+P) \cdots d\xi(k+j+1) \} \\
&d\xi(k+j-1) \cdots d\xi(k+1) d\xi(k) \}
\end{aligned} \tag{12}$$

Note that if the predictive horizon length P is too large, the integration in (12) will lead to considerable computing time. Thus a value of P less than 10 is suggested in practice.

3.2 Feasibility Analysis

Feasibility analysis concerns the problem of whether the chance constraint $\mathcal{P}\{\mathbf{y}_{\min} \le \mathbf{y}(k+i|k) \le \mathbf{y}_{\max}\} \ge \alpha$ is feasible. This is an important issue for the chance constrained problems, since it is likely that the predefined level α is higher than reachable. In this case the optimization routine can not find a feasible solution. A straightforward way to address this problem is to compute the maximum reachable probability before doing the optimization. As a result, the original objective function in (1) will be replaced with

$$\text{Max}\ \mathcal{P}\{y_{\min} \le y(k+i|k) \le y_{\max}, i = 1, 2, \ldots, P\} \tag{13}$$

The maximum reachable α can be obtained by solving the corresponding optimization problem.

4 Application to a Mixing Process

The discretized model of the tank mixing process under study with unit sampling time interval is:

$$\begin{aligned} V(k+1) &= V(k) + q(k) - u(k) \\ C(k+1) &= C(k) + \tfrac{q(k)}{V(k+1)}[C_0(k) - C(k)] \end{aligned} \tag{14}$$

where V and C are the volume and product mass concentration in the tank, q, u are the feed and outlet flow rates and C_0 is the feed mass concentration, respectively. The control objective is, under the inlet uncertain flow rate and composition, to obtain a possibly flat outlet flow rate while holding the outlet concentration and tank volume in specified intervals. Based on (14), the future process outputs are predicted as

$$V(k+i|k) = V(k) + \sum_{j=0}^{i-1} (q(k+j) - u(k+j|k)) \tag{15}$$

$$\begin{aligned} C(k+i|k) = & \prod_{j=0}^{i-1} \tfrac{V(k+j|k) - u(k+j|k)}{V(k+j+1|k)} C(k) \\ & + \sum_{j=0}^{i-1} \Big(\prod_{s=j+1}^{i-1} \tfrac{V(k+s|k) - u(k+s|k)}{V(k+s+1|k)} \Big) \tfrac{q(k+j)}{V(k+j+1|k)} C_0(k+j) \end{aligned} \tag{16}$$

With the above prediction model, the nonlinear CNMPC problem at sampling instant k can be formulated as:

$$\begin{aligned} &\text{Min}\ \ \Delta\mathbf{u}^T \Delta\mathbf{u} \\ &s.t. \\ &(15) \text{ and } (16) \\ &u_{\min} \le u(k+i|k) \le u_{\max}, i = 0, \ldots, M-1. \\ &\mathcal{P}\{V_{\min} \le V(k+i|k) \le V_{\max}, i = 1, \ldots, P\} \ge \alpha_1 \\ &\mathcal{P}\{C_{\min} \le C(k+i|k) \le C_{\max}, i = 1, \ldots, P\} \ge \alpha_2 \end{aligned} \tag{17}$$

Li *et al.* [3] studied the linear case which only concerns the volume constraints and the outlet flow rate u only affects the mean value of the output V. In contrast, for the nonlinear model in (16), u affects both the mean and covariance of the distribution of the outlet concentration C.

With the assumption that the feed flow rate $q(k+i)$ and feed concentration $C_0(k+i)$ follow a positive truncated normal distribution, namely, the low truncating point a_1 in (4) is positive, the following monotone relation can be found

$$\begin{aligned} q(k+i) \uparrow &\Rightarrow V(k+i+1|k) \uparrow \\ C_0(k+i) \uparrow &\Rightarrow C(k+i+1|k) \uparrow \end{aligned} \tag{18}$$

Thus the chance constraints in (17) can be transformed into

$$\begin{aligned} &\mathcal{P}\{V_{\min} \leq V(k+i|k) \leq V_{\max}, i = 1, \ldots, P\} \\ &\Rightarrow \mathcal{P}\{q_{k+i-1}^{\min} \leq q(k+i-1) \leq q_{k+i-1}^{\max}, i = 1, \ldots, P\} \\ &\mathcal{P}\{C_{\min} \leq C(k+i|k) \leq C_{\max}, i = 1, \ldots, P\} \\ &\Rightarrow \mathcal{P}\{C_{0(k+i-1)}^{\min} \leq C_0(k+i-1) \leq C_{0(k+i-1)}^{\max}, i = 1, \ldots, P\} \end{aligned} \tag{19}$$

Therefore (11) and (12) can be used to compute $\mathcal{P}\{\cdot\}$ and its gradient.

The proposed CNMPC controller is applied to the mixing process. The initial values of tank volume and product concentration are $V(0) = 160\ l$ and $C(0) = 50g/l$, respectively. The inlet flow $q(k)$ and concentration $C_0(k)$ are assumed to be multivariate truncated normal sequences with the truncating intervals of [0, 20] and [46, 56]. The mean profiles of $q(k)$ and $C_0(k)$ within a period of 20 minutes are shown in Fig.1 and 2. In each time interval, they have the stand deviation values of 0.70 and 1.0. In addition, both $q(k)$ and $C_0(k)$ at different intervals are assumed to be independent. The dashed lines in Fig.1 and 2 are 10 realizations of the disturbance from random samples and it is shown that the uncertainty is considerable. The prediction and control horizon of CNMPC is fixed at $P = M = 5$ and the lower and upper bounds of the output variables, V and C, are [130, 170] and [49, 51], respectively. The probability level of α_1 and α_2 are both given as 0.9. The control results are illustrated in Fig.3 to Fig.6. In Fig.3, it can be seen that the control variable $u(k)$ is more flat than the inlet and thus the disturbance to the downstream unit is thus decreased. As shown in

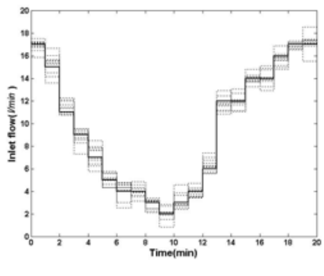

Fig. 1. Inlet flow disturbance profile

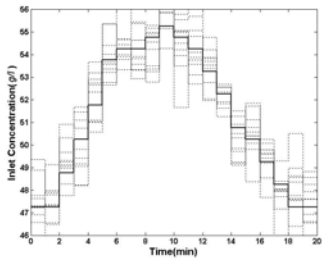

Fig. 2. Inlet concentration disturbance profile

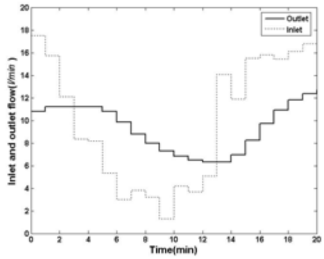

Fig. 3. Inlet $q(k)$ and outlet flow $u(k)$

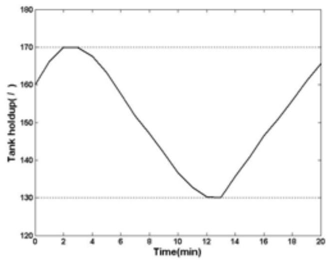

Fig. 4. Tank volume $V(k)$

Fig.4 and Fig.5, the tank volume $V(k)$ and outlet product concentration $C(k)$ are strictly restricted in the predefined bounds. In addition, oscillations also occur in the controlled variables profiles, which means that the controller takes the advantage of the freedom available to keep the control action as flat as possible. The feasibility analysis of production concentration chance constraint is also performed and the maximum reachable possibility of $\mathcal{P}\{C_{\min} \leq C(k+i|k) \leq C_{\max}\}$ in each interval is depicted in Fig.6. It can be seen that the maximum reachable probabilities are all greater than the predefined value (α_2=0.90), which implies that the corresponding CNMPC problem is feasible. Note that at the 4^{th} and 13^{th} minute when the concentration approaches its limits, the maximum probability reaches its minimum value.

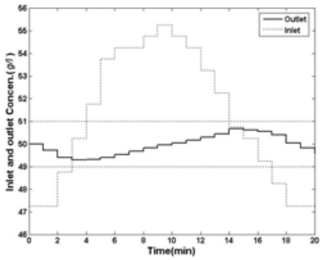

Fig. 5. Inlet and outlet concentration $C(k)$

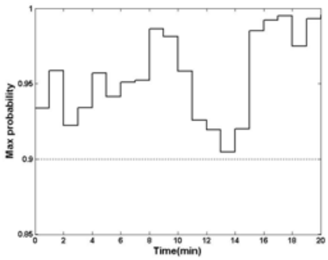

Fig. 6. Max reachable probabilities of the $C(k)$ chance constraint

5 Conclusions

In this work, a robust nonlinear model predictive controller, chance constrained NMPC, is proposed. To compute the probability and derivatives of holding inequality constraints inverse mapping approach is extended to dynamic nonlinear situations. To find a monotone relation between the uncertain variable and the output which is necessary for the inverse mapping approach, truncated normal distribution is considered to describe uncertainty variables. CNMPC is illustrated to be effective by controlling a mixing process with both uncertain feed flow rate and feed concentration.

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