Toward Rough-Granular Computing
Extended Abstract

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Developing methods for approximation of compound concepts expressing the result of perception belongs to the main challenges of Perception Based Computing (PBC) [70]. The perceived concepts are expressed in natural language. We discuss the rough-granular approach to approximation of such concepts from sensory data and domain knowledge. This additional knowledge, represented by ontology of concepts, is used to make it feasible searching for features (condition attributes) relevant for the approximation of concepts on different levels of the concept hierarchy defined by a given ontology. We report several experiments of the proposed methodology for approximation of compound concepts from sensory data and domain knowledge. The approach is illustrated by examples relative to interactions of agents, ontology approximation, adaptive hierarchical learning of compound concepts and skills, behavioral pattern identification, planning, conflict analysis and negotiations, and perception-based reasoning. The presented results seem to justify the following claim of Lotfi A. Zadeh: “In coming years, granular computing is likely to play an increasingly important role in scientific theories-especially in human-centric theories in which human judgement, perception and emotions are of pivotal importance”. The question of how ontologies of concepts can be discovered from sensory data remains as one of the greatest challenges for many interdisciplinary projects on learning of concepts.

The concept approximation problem is the basic problem investigated in machine learning, pattern recognition and data mining [24]. It is necessary to induce approximations of concepts (models of concepts) consistent (or almost consistent) with some constraints. In the most typical case, constraints are defined by a training sample. For more compound concepts, we consider constraints defined by domain ontology consisting of vague concepts and dependencies between them. Information about the classified objects and concepts is partial. In the most general case, the adaptive approximation of concepts is performed under interaction with dynamically changing environment. In all these cases, searching for sub-optimal models relative to the minimal length principle (MLP) is

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performed. Notice that in adaptive concept approximation one of the components of the model should be the adaptation strategy. Components involved in construction of concept approximation which are tuned in searching for sub-optimal models relative to MLP are called information granules. In rough granular computing (RGC), information granule calculi are used for construction of components of classifiers and classifiers themselves (see, e.g., [60]) satisfying given constraints. An important mechanism in RGC is related to generalization schemes making it possible to construct more compound patterns from less compound patters. Generalization degrees of schemes are tuned using, e.g., some evolutionary strategies.

Rough set theory due to Zdzisław Pawlak [43,44,45,46,17] is a mathematical approach to imperfect knowledge. The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. Recently it became also a crucial issue for computer scientists, particularly in the area of artificial intelligence. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful one is, no doubt, the fuzzy set theory proposed by Lotfi A. Zadeh [69]. Rough set theory presents still another attempt to solve this problem. It is based on an assumption that objects and concepts are perceived by partial information about them. Due to this some objects can be indiscernible. From this fact it follows that some sets can not be exactly described by available information about objects; they are rough not crisp. Any rough set is characterized by its (lower and upper) approximations. The difference between the upper and lower approximation of a given set is called its boundary. Rough set theory expresses vagueness relative to the boundary region of a set. If the boundary region of a set is empty, it means that the set is crisp; otherwise, the set is rough (inexact). A nonempty boundary region of a set indicates that our knowledge about the set is not sufficient to define the set precisely. One can recognize that rough set theory is, in a sense, a formalization of the idea presented by Gotlob Frege [23].

One of the consequences of perceiving objects using only available information about them is that for some objects one cannot decide if they belong to a given set or not. However, one can estimate the degree to which objects belong to sets. This is another crucial observation in building the foundations for approximate reasoning. In dealing with imperfect knowledge, one can only characterize satisfiability of relations between objects to a degree, not precisely. Among relations on objects, the rough inclusion relation plays a special role in describing to what degree objects are parts of other objects. A rough mereological approach (see, e.g., [52,59,42]) is an extension of the Leśniewski mereology [31] and is based on the relation *to be a part to a degree*. It will be interesting to note here that Jan Łukasiewicz was the first who started to investigate the inclusion to a degree of concepts in his discussion on relationships between probability and logical calculi [35].

The very successful technique for rough set methods has been Boolean reasoning [12]. The idea of Boolean reasoning is based on construction for a given problem P a corresponding Boolean function f_P with the following property:

the solutions for the problem P can be decoded from prime implicants of the Boolean function f_P . It is worth while to mention that to solve real-life problems, it is necessary to deal with Boolean functions having a large number of variables.

A successful methodology based on the discernibility of objects and Boolean reasoning has been developed in rough set theory for computing of many key constructs like reducts and their approximations, decision rules, association rules, discretization of real valued attributes, symbolic value grouping, searching for new features defined by oblique hyperplanes or higher order surfaces, pattern extraction from data as well as conflict resolution or negotiation [55,38,46]. Most of the problems involving the computation of these entities are NP-complete or NP-hard. However, we have been successful in developing efficient heuristics yielding sub-optimal solutions for these problems. The results of experiments on many data sets are very promising. They show very good quality solutions generated by the heuristics in comparison with other methods reported in literature (e.g., with respect to the classification quality of unseen objects). Moreover, they are very time-efficient. It is important to note that the methodology makes it possible to construct heuristics having a very important approximation property. Namely, *expressions generated by heuristics (i.e., implicants) close to prime implicants define approximate solutions for the problem* (see, e.g., [1]).

The rough set approach offers tools for approximate reasoning in multiagent systems (MAS). The typical example is the approximation by one agent of concepts of another agent. The approximation of a concept is based on a decision table representing information about objects perceived by both agents.

The strategies for inducing data models developed so far are often not satisfactory for approximation of compound concepts that occur in the perception process. Researchers from the different areas have recognized the necessity to work on new methods for concept approximation (see, e.g., [11,68]). The main reason for this is that these compound concepts are, in a sense, too far from measurements which makes the searching for relevant features infeasible in a very huge space. There are several research directions aiming at overcoming this difficulty. One of them is based on the interdisciplinary research where the knowledge pertaining to perception in psychology or neuroscience is used to help to deal with compound concepts (see, e.g., [37,22,21]). There is a great effort in neuroscience towards understanding the hierarchical structures of neural networks in living organisms [20,51,37]. Also mathematicians are recognizing problems of learning as the main problem of the current century [51]. These problems are closely related to complex system modeling as well. In such systems again the problem of concept approximation and its role in reasoning about perceptions is one of the challenges nowadays. One should take into account that modeling complex phenomena entails the use of local models (captured by local agents, if one would like to use the multi-agent terminology [34,65,19]) that should be fused afterwards. This process involves negotiations between agents [34,65,19] to resolve contradictions and conflicts in local modeling. This kind of modeling is becoming more and more important in dealing with complex real-life phenomena which we are unable to model using traditional analytical approaches. The latter approaches lead to exact models. However, the necessary assumptions used to develop them result in solutions that are too far from reality to be accepted. New methods or even a new science therefore should be developed for such modeling [25].

One of the possible approaches in developing methods for compound concept approximations can be based on the layered (hierarchical) learning [62,9]. Inducing concept approximation should be developed hierarchically starting from concepts that can be directly approximated using sensor measurements toward compound target concepts related to perception. This general idea can be realized using additional domain knowledge represented in natural language. For example, one can use some rules of behavior on the roads, expressed in natural language, to assess from recordings (made, e.g., by camera and other sensors) of actual traffic situations, if a particular situation is safe or not (see, e.g., [39,8,7,17]). Hierarchical learning has been also used for identification of risk patterns in medical data and extended for therapy planning (see, e.g. [5,4]). Another application of hierarchical learning for sunspot classification is reported in [40]. To deal with such problems, one should develop methods for concept approximations together with methods aiming at approximation of reasoning schemes (over such concepts) expressed in natural language. The foundations of such an approach, creating a core of perception logic, are based on rough set theory [43,44,45,46,17] and its extension called rough mereology [52,59,42]. Approximate Boolean reasoning methods can be scaled to the case of compound concept approximation.

Let us consider more examples.

The prediction of behavioral patterns of a compound object evaluated over time is usually based on some historical knowledge representation used to store information about changes in relevant features or parameters. This information is usually represented as a data set and has to be collected during long-term observation of a complex dynamic system. For example, in case of road traffic, we associate the object-vehicle parameters with the readouts of different measuring devices or technical equipment placed inside the vehicle or in the outside environment (e.g., alongside the road, in a helicopter observing the situation on the road, in a traffic patrol vehicle). Many monitoring devices serve as informative sensors such as GPS, laser scanners, thermometers, range finders, digital cameras, radar, image and sound converters (see, e.g. [66]). Hence, many vehicle features serve as models of physical sensors. Here are some exemplary sensors: location, speed, current acceleration or deceleration, visibility, humidity (slipperiness) of the road. By analogy to this example, many features of compound objects are often dubbed sensors. In the lecture, we discuss (see also [7]) some rough set tools for perception modelling that make it possible to recognize behavioral patterns of objects and their parts changing over time. More complex behavior of compound objects or groups of compound objects can be presented in the form of *behavioral graphs*. Any behavioral graph can be interpreted as a *behavioral pattern* and can be used as a complex classifier for recognition of complex behaviours. The complete approach to the perception of behavioral patterns, that is based on behavioral graphs and the dynamic elimination of behavioral patterns, is presented in [7]. The tools for dynamic elimination of behavioral patterns are used for switching-off in the *system attention* procedures searching for identification of some behavioral patterns. The developed rough set tools for perception modeling are used to model networks of classifiers. Such networks make it possible to recognize behavioral patterns of objects changing over time. They are constructed using an ontology of concepts provided by experts that engage in approximate reasoning on concepts embedded in such an ontology. Experiments on data from a vehicular traffic simulator [3] show that the developed methods are useful in the identification of behavioral patterns.

The following example concerns human computer-interfaces that allow for a dialog with experts to transfer to the system their knowledge about structurally compound objects. For pattern recognition systems [18], e.g., for Optical Character Recognition (OCR) systems it will be helpful to transfer to the system a certain knowledge about the expert view on border line cases. The central issue in such pattern recognition systems is the construction of classifiers within vast and poorly understood search spaces, which is a very difficult task. Nonetheless, this process can be greatly enhanced with knowledge about the investigated objects provided by an human expert. We developed a framework for the transfer of such knowledge from the expert and for incorporating it into the learning process of a recognition system using methods based on rough mereology (see, e.g., [41]). Is is also demonstrated how this knowledge acquisition can be conducted in an interactive manner, with a large dataset of handwritten digits as an example.

The next two examples are related to approximation of compound concepts in reinforcement learning and planning.

In temporal difference reinforcement learning [63,16,36,28,60,47,48,50,49,71,72], the main task is to learn the approximation of the function $Q(s,a)$, where s,a denotes a global state of the system and an action performed by an agent ag and, respectively and the real value of $Q(s,a)$ describes the reward for executing the action a in the state s. In approximation of the function $Q(s,a)$, probabilistic methods are used. However, for compound real-life problems it may be hard to build such models for such a compound concept as $Q(s,a)$ [68]. We propose another approach to the approximation of $Q(s,a)$ based on ontology approximation. The approach is based on the assumption that in a dialog with experts an additional knowledge can be acquired making it possible to create a ranking of values $Q(s,a)$ for different actions a in a given state s. In the explanation given by expert about possible values of $Q(s,a)$ concepts from a special ontology are used. Then, using this ontology one can follow hierarchical learning methods to learn approximations of concepts from ontology. Such concepts can have a temporal character too. This means that the ranking of actions may depend not only on the actual action and the state but also on actions performed in the past and changes caused by these actions.

In [5,4] a computer tool based on rough sets for supporting automated planning of the medical treatment (see, e.g., [26,67]) is discussed. In this approach, a given patient is treated as an investigated complex dynamical system, whilst diseases of this patient (RDS, PDA, sepsis, Ureaplasma and respiratory failure) are treated as compound objects changing and interacting over time. As a measure of planning success (or failure) in experiments, we use a special hierarchical classifier that can predict the similarity between two plans as a number between 0.0 and 1.0. This classifier has been constructed on the basis of the special ontology specified by human experts and data sets. It is important to mention that besides the ontology, experts provided the exemplary data (values of attributes) for the purpose of concepts approximation from the ontology. The methods of construction such classifiers are based on approximate reasoning schemes (AR schemes, for short) and were described, e.g., in [8,39,8,7]. We applied this method for approximation of similarity between plans generated in automated planning and plans proposed be human experts during the realistic clinical treatment.

Further radical changes in the design of intelligent systems depend on the advancement of technology to acquire, represent, store, process, discover, communicate and learn wisdom. We call this technology *wisdom technology* (or **wistech**, for short) [27]. The term *wisdom* commonly means "judging rightly". This common notion can be refined. By *wisdom*, we understand an adaptive ability to make judgements correctly to a satisfactory degree (in particular, correct decisions) having in mind real-life constraints.

One of the basic objectives is to indicate the methods for potential directions for the design and implementation of wistech computation models. An important aspect of wistech is that the complexity and uncertainty of real-life constraints mean that in practise we must reconcile ourselves to the fact that our judgements are based on non-crisp concepts and which do not take into account all the knowledge accumulated and available to us. This is also why consequences of our judgements are usually imperfect. But as a consolation, we also learn to improve the quality of our judgements via observation and analysis of our experience during interaction with the environment. Satisfactory decision-making levels can be achieved as a result of improved judgements.

The intuitive nature of wisdom understood in this way can be expressed metaphorically as shown in *wisdom equation* (1)

$$wisdom = KSN + AJ + IP, \tag{1}$$

where *KSN, AJ, IP* denote *knowledge sources network, adaptive judgement*, and *interactive processes*, respectively. The combination of the technologies represented in (1) offers an intuitive starting point for a variety of approaches to designing and implementing computational models for wistech. We focus in the research on an adaptive RGC approach.

The issues we discuss on wistech are relevant for the other reported current research directions (see, e.g., [14,13,21,22,30,54,64] and the literature cited in these articles).

Wistech can be perceived as the integration of three technologies (corresponding to three components in the wisdom equation (1)). At the current stage the following two of them seem to be conceptually relatively clear: (i) *knowledge sources network* – by knowledge we traditionally understand every organized set of information along with the inference rules; (ii) *interactive processes* – interaction is understood here as a sequence of stimuli and reactions over time. Far more difficult conceptually seems to be the concept of (iii) *adaptive judgement* distinguishing wisdom from the general concept of problem solving. Adaptive judgement is understood here as mechanisms in a metalanguage (meta-reasoning) which on the basis of selection of available sources of knowledge and on the basis of understanding of history of interactive processes and their current status are enable to perform the following activities under real life constraints: (i) identification and judgement of importance (for future judgement) of phenomena available for observation in the surrounding environment; (ii) planning current priorities for actions to be taken (in particular, on the basis of understanding of history of interactive processes and their current status) toward making optimal judgements; (iii) selection of fragments of ordered knowledge (hierarchies of information and judgement strategies) satisfactory for making decision at the planned time (a decision here is understood as a commencing interaction with the environment or as selecting the future course to make judgements); (iv) prediction of important consequences of the planned interaction of processes; (v) learning and, in particular, reaching conclusions from experience leading to adaptive improvement in the adaptive judgement process.

One of the main barriers hindering an acceleration in the development of wistech applications lies in developing satisfactory computation models implementing the functioning of "adaptive judgement". This difficulty primarily consists of overcoming the complexity of the process of integrating the local assimilation and processing of changing non-crisp and incomplete concepts necessary to make correct judgements. In other words, we are only able to model tested phenomena using local (subjective) models and interactions between them. In practical applications, usually, we are not able to give global models of analyzed phenomena (give quotes from MAS and complex adaptive systems (CAS); see, e.g., [65,32,33,19,15]). However, we one can approximate global models by integrating the various incomplete perspectives of problem perception. One of the potential computation models for "adaptive judgement" might be the RGC approach.

The research on the foundations on wistech is based on a continuation of approaches to computational models of approximate reasoning developed by Rasiowa (see [53]), Pawlak (see [43]) and their students. In some sense, it is a continuation of ideas initiated by Leibniz, Boole and currently continued in a variety of forms. Of course, the Rasiowa - Pawlak school is also some kind continuation of the Polish School of Mathematics and Logics which led to the development of the modern understanding of the basic computational aspects of logic, epistemology, ontology, foundations of mathematics and natural deduction. The two fundamental tools of the Rasiowa - Pawlak school are the following: (i) *Computation models of logical concept (especially such concepts as deduction or algebraic many-valued models for classical, modal, and constructive mathematics)* - based on the method of treating the sets of logically equivalent statements (or formulas) as abstract algebras known as Lindebaum - Tarski algebras; (ii) *Computation models of vague concept*- originally Łukasiewicz has proposed to treat uncertainty (or vague concepts) as concepts of many valued logic. The rough set concept, due to Pawlak [43], developed in the Rasiowa-Pawlak school is based on classical two valued logic. The rough set approach has been developed to deal with uncertainty and vagueness. The approach makes it possible to reason precisely about approximations of vague concepts. These approximations are temporary, subjective, and are adaptively changing with changes in environments [6,57,60].

Solving complex problems by multi-agent systems requires new approximate reasoning methods based on new computing paradigms. One such recently emerging computing paradigm is RGC. Computations in RGC are performed on information granules representing often vague, partially specified, and compound concepts delivered by agents engaged in tasks such as knowledge representation, communication with other agents, and reasoning.

One of the RGC challenges is to develop approximate reasoning techniques for reasoning about dynamics of distributed systems of judges, i.e., agents judging rightly. These techniques should be based on systems of evolving local perception logics rather than on a global logic [56,58]. The approximate reasoning about global behavior of judge's system is infeasible without methods for approximation of compound vague concepts and approximate reasoning about them. One can observe here an analogy to phenomena related to the emergent patters in complex adaptive systems [15]. Let us observe that judges can be organized into a hierarchical structure, i.e., one judge can represent a coalition of judges in interaction with other agents existing in the environment [2,29,32]. Such judges representing coalitions play an important role in hierarchical reasoning about behavior of judges populations. Strategies for coalition formation and cooperation [2,32,33] are of critical importance in designing systems of judges with dynamics satisfying to a satisfactory degree the given specification. Developing strategies for discovery of information granules representing relevant coalitions and cooperation protocols is another challenge for RGC.

All these problems can be treated as problems of searching for information granules satisfying vague requirements. The strategies for construction of information granules should be adaptive. It means that the adaptive strategies should make it possible to construct information granules satisfying constraints under dynamically changing environment. This requires reconstruction or tuning of already constructed information granules which are used as components of data models, e.g., classifiers. In the adaptive process, the construction of information granules generalizing some constructed so far information granules plays a special role. The mechanism for relevant generalization here is crucial. One can imagine for this task many different strategies, e.g., based on adaptive feedback control for tuning the generalization. Cooperation with specialists from different areas such as neuroscience (see, e.g., [37] for visual objects recognition), psychology (see, e.g., [51] for discovery of mechanisms for hierarchical perception), biology (see, e.g., [10] for cooperation based on swarm intelligence), adaptive learning based on ethology and approximation spaces [48,50] or social science (see, e.g., [32] for modeling of agents behavior) can help to discover such adaptive strategies for extracting sub-optimal (relative to the minimal length principle) data models satisfying soft constraints. This research may also help us to develop strategies for discovery of ontologies relevant for compound concept approximation.

In the current projects, we are developing rough set based methods in combination with other soft computing and statistical methods for RGC on which wistech can be based. The developed methods are used to construct wisdom engines. By wisdom engine we understand a system which implements the concept of wisdom. We plan to design specific systems for some tasks such as (1) Intelligent Document Manager; (2) Job Market Search; (3) Brand Monitoring; (4) Decision Support for global management systems (e.g., World Forex, Stock Market, World Tourist); (5) Intelligent Assistant (e.g., Physician, Lawyer); (6) Discovery of Processes from Data (e.g., Gene Expression Networks); (7) Rescue System (for more details see [27,19]).

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References

1. *Rough Set Exploration System (RSES).* Available at: `logic.mimuw.edu.pl/~rses`.
2. R. M. Axelrod. *The Complexity of Cooperation.* Princeton University Press, Princeton, NJ, 1997.
3. J. Bazan. *The Road simulator*
   Available at: `logic.mimuw.edu.pl/~bazan/simulator`.
4. J. Bazan, P. Kruczek, S. Bazan-Socha, A. Skowron, and J. J. Pietrzyk. Automatic planning of treatment of infants with respiratory failure through rough set modeling. In *Proceedings of RSCTC'2006*, LNAI. Springer, Heidelberg, 2006. to be published.
5. J. Bazan, P. Kruczek, S. Bazan-Socha, A. Skowron, and J. J. Pietrzyk. Risk pattern identification in the treatment of infants with respiratory failure through rough set modeling. In *Proceedings of IPMU'2006, Paris, France, July 2-7, 2006*, pages 2650–2657. Éditions E.D.K., Paris, 2006.
6. J. Bazan, A. Skowron, and R. Swiniarski. Rough sets and vague concept approximation: From sample approximation to adaptive learning. *Transactions on Rough Sets V: LNCS Journal Subline, Springer, Heidleberg*, LNCS 4100:39–62, 2006.
7. J. G. Bazan, J. F. Peters, and A. Skowron. Behavioral pattern identification through rough set modelling. In Ślęzak et al. [61], pages 688–697.
8. J. G. Bazan and A. Skowron. Classifiers based on approximate reasoning schemes. In Dunin-Kęplicz et al. [19], pages 191–202.
9. S. Behnke. *Hierarchical Neural Networks for Image Interpretation*, volume 2766 of *LNCS.* Springer, Heidelberg, 2003.

10. E. Bonabeau, M. Dorigo, and G. Theraulaz. *Swarm Intelligence. From Natural to Artificial Systems.* Oxford University Press, Oxford, UK, 1999.
11. L. Breiman. Statistical modeling: The two cultures. *Statistical Science*, 16(3):199–231, 2001.
12. F. Brown. *Boolean Reasoning.* Kluwer Academic Publishers, Dordrecht, 1990.
13. N. L. Cassimatis. A cognitive substrate for achievinbg human-level intelligence. *AI Magazine*, 27:45–56, 2006.
14. N. L. Cassimatis, E. T. Mueller, and P. H. Winston. Achieving human-level intelligence through integrated systems and research. *AI Magazine*, 27:12–14, 2006.
15. A. Desai. Adaptive complex enterprices. *Comm. ACM*, 48:32–35, 2005.
16. T. G. Dietterich. Hierarchical reinforcement learning with the MAXQ value function decomposition. *Artificial Intelligence*, 13(5):227–303, 2000.
17. P. Doherty, W. Łukaszewicz, A. Skowron, and A. Szałas. *Knowledge Representation Techniques : A Rough Set Approach*, volume 202 of *Studies in Fuzziness and Soft Computing.* Springer, Heidelberg, Germany, 2006.
18. R. Duda, P. Hart, and R. Stork. *Pattern Classification.* John Wiley & Sons, New York, NY, 2002.
19. B. Dunin-Kęplicz, A. Jankowski, A. Skowron, and M. Szczuka, editors. *Monitoring, Security, and Rescue Tasks in Multiagent Systems (MSRAS'2004).* Advances in Soft Computing. Springer, Heidelberg, 2005.
20. M. Fahle and T. Poggio. *Perceptual Learning.* The MIT Press, Cambridge, MA, 2002.
21. K. D. Forbus and T. R. Hinrisch. Companion congnitive systems: A step toward human-level ai. *AI Magazine*, 27:83–95, 2006.
22. K. D. Forbus and T. R. Hinrisch. Engines of the brain: The computational instruction set of human cognition. *AI Magazine*, 27:15–31, 2006.
23. G. Frege. *Grundgesetzen der Arithmetik, 2.* Verlag von Hermann Pohle, Jena, 1903.
24. J. Friedman, T. Hastie, and R. Tibshirani. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction.* Springer, Heidelberg, 2001.
25. M. Gell-Mann. *The Quark and the Jaguar - Adventures in the Simple and the Complex.* Brown and Co., London, 1994.
26. M. Ghallab, D. Nau, and P. Traverso. *Automated Planning: Theory and Practice.* Elsevier, Morgan Kaufmann, CA, 2004.
27. A. Jankowski and A. Skowron. A wistech paradigm for intelligent systems. *Transactions on Rough Sets VI: LNCS Journal Subline, Springer, Heidleberg*, LNCS 4374:94-132, 2007.
28. L. P. Kaelbling, M. L. Littman, and A. W. Moore. Reinforcement learning: A survey. *Journal of Artificial Intelligence Research*, 4:227–303, 1996.
29. S. Kraus. *Strategic Negotiations in Multiagent Environments.* The MIT Press, Massachusetts, 2001.
30. P. Langley. Cognitive architectures and general intelligent systems. *AI Magazine*, 27:33–44, 2006.
31. S. Leśniewski. Grungzüge eines neuen Systems der Grundlagen der Mathematik. *Fundamenta Mathematicae*, 14:1–81, 1929.
32. J. Liu. *Autonomous Agents and Multi-Agent Systems: Explorations in Learning, Self-Organization and Adaptive Computation.* World Scientific Publishing, Singapore, 2001.
33. J. Liu, X. Jin, and K. C. Tsui. *Autonomy Oriented Computing: From Problem Solving to Complex Systems Modeling.* Kluwer/Springer, Heidelberg, 2005.
34. M. Luck, P. McBurney, and C. Preist. *Agent Technology. Enabling Next Generation Computing: A Roadmap for Agent Based Computing.* AgentLink, 2003.

35. J. Łukasiewicz. Die logischen Grundlagen der Wahrscheinlichkeitsrechnung, Kraków1913. In L. Borkowski, editor, *Jan Łukasiewicz - Selected Works*, pages 16–63. North Holland & Polish Scientific Publishers, Amsterdam, London, Warsaw, 1970.
36. A. McGovern. *Autonomous Discovery of Temporal Abstractions from Interaction with an Environment.* PhD thesis, University of Massachusetts, Amherst, 2002.
37. R. Miikkulainen, J. A. Bednar, Y. Choe, and J. Sirosh. *Computational Maps in the Visual Cortex.* Springer, Hiedelberg, 2005.
38. H. S. Nguyen. Approximate boolean reasoning: Foundations and applications in data mining. In J. F. Peters and A. Skowron, editors, *Transactions on Rough Sets V: Journal Subline, Springer, Heidelberg*, LNCS 4100:344–523, 2006.
39. H. S. Nguyen, J. Bazan, A. Skowron, and S. H. Nguyen. Layered learning for concept synthesis. *Transactions on Rough Sets I: LNCS Journal Subline, Springer, Heidleberg*, LNCS 3100:187–208, 2004.
40. S. H. Nguyen, T. T. Nguyen, and H. S. Nguyen. Rough set approach to sunspot classification. In Ślęzak et al. [61], pages 263–272.
41. T. T. Nguyen and A. Skowron. Rough set approach to domain knowledge approximation. In G. Wang, Q. Liu, Y. Yao, and A. Skowron, editors, *Proceedings of the 9-th International Conference on Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing (RSFDGrC'2003), Chongqing, China, Oct 19-22, 2003*, volume 2639 of *LNCS*, pages 221–228. Springer, Heidelberg.
42. S. K. Pal, L. Polkowski, and A. Skowron, editors. *Rough-Neural Computing: Techniques for Computing with Words.* Cognitive Technologies. Springer, Heidelberg, 2004.
43. Z. Pawlak. *Rough Sets: Theoretical Aspects of Reasoning about Data*, volume 9 of *System Theory, Knowledge Engineering and Problem Solving.* Kluwer Academic Publishers, Dordrecht, The Netherlands, 1991.
44. Z. Pawlak and A. Skowron. Rudiments of rough sets. *Information Sciences* 177(1) (2007) 3-27.
45. Z. Pawlak and A. Skowron. Rough sets: Some extensions. *Information Sciences* 177(1) (2007) 28-40.
46. Z. Pawlak and A. Skowron. Rough sets and boolean reasoning. *Information Sciences* 177(1) (2007) 41-73.
47. J. F. Peters. Approximation spaces for hierarchical intelligent behavioural system models, In: B.D.-Kepliçz, A. Jankowski, A. Skowron, M. Szczuka (Eds.), *Monitoring, Security and Rescue Techniques in Multiagent Systems*, Advances in Soft Computing, pages 13–30, Physica-Verlag, Heidelberg, 2004.
48. J. F. Peters. Rough ethology: Towards a biologically-inspired study of collective behaviour in intelligent systems with approximation spaces. *Transactions on Rough Sets III: LNCS Journal Subline, Springer, Heidleberg*, LNCS 3400:153–174, 2005.
49. J.F. Peters, C. Henry, S. Ramanna. Rough ethograms: A study of intelligent system behavior. In: Mieczyslaw A. Kłopotek, Sławomir Wierzchoń, Krzystof Trojanowski(Eds.), Intelligent Information Systems. *Advances in Soft Computing.* Springer-Verlag, Heidelberg, (2005) 117-126
50. J. F. Peters, C. Henry. Reinforcement learning with approximation spaces. *Fundamenta Informaticae* **71** (2-3) (2006) 323-349.
51. T. Poggio and S. Smale. The mathematics of learning: Dealing with data. *Notices of the AMS*, 50(5):537–544, 2003.
52. L. Polkowski and A. Skowron. Rough mereology: A new paradigm for approximate reasoning. *International Journal of Approximate Reasoning*, 15(4):333–365, 1996.

53. H. Rasiowa. *Algebraic models of logics.* Warsaw University, Warsaw, 2001.
54. C. Schlenoff, J. Albus, E. Messina, A. J. Barbera, R. Madhavan, and S. Balakirsky. Using 4d/rcs to address ai knowledge integration. *AI Magazine*, 27:71–81, 2006.
55. A. Skowron. Rough sets in KDD (plenary talk). In Z. Shi, B. Faltings, and M. Musen, editors, *16-th World Computer Congress (IFIP'2000): Proceedings of Conference on Intelligent Information Processing (IIP'2000)*, pages 1–14. Publishing House of Electronic Industry, Beijing, 2000.
56. A. Skowron. Perception logic in intelligent systems. In S. Blair et al, editor, *Proceedings of the 8th Joint Conference on Information Sciences (JCIS 2005), July 21-26, 2005, Salt Lake City, Utah, USA*, pages 1–5. X-CD Technologies: A Conference & Management Company, ISBN 0-9707890-3-3, Toronto, Ontario, Canada, 2005.
57. A. Skowron. Rough sets and vague concepts. *Fundamenta Informaticae*, 64(1-4):417–431, 2005.
58. A. Skowron. Rough sets in perception-based computing (keynote talk). In S. K. Pal, S. Bandoyopadhay, and S. Biswas, editors, *First International Conference on Pattern Recognition and Machine Intelligence (PReMI'05) December 18-22, 2005, Indian Statistical Institute, Kolkata*, volume 3776 of *LNCS*, pages 21–29, Heidelberg, 2005. Springer.
59. A. Skowron and J. Stepaniuk. Information granules and rough-neural computing. In Pal et al. [42], pages 43–84.
60. A. Skowron, J. Stepaniuk, J. F. Peters, and R. Swiniarski. Calculi of approximation spaces. *Fundamenta Informaticae*, 72(1-3):363–378, 2006.
61. D. Ślęzak, J. T. Yao, J. F. Peters, W. Ziarko, and X. Hu, editors. *Proceedings of the 10th International Conference on Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing (RSFDGrC'2005), Regina, Canada, August 31-September 3, 2005, Part II*, volume 3642 of *LNAI*. Springer, Heidelberg, 2005.
62. P. Stone. *Layered Learning in Multi-Agent Systems: A Winning Approach to Robotic Soccer*. The MIT Press, Cambridge, MA, 2000.
63. R. S. Sutton and A. G. Barto. *Reinforcement Learning: An Introduction*. The MIT Press, Cambridge, MA, 1998.
64. W. Swartout, J. Gratch, R. W. Hill, E. Hovy, S. Marsella, J. Rickel, and D. Traum. Towards virtual humans. *AI Magazine*, 27:96–108, 2006.
65. K. Sycara. Multiagent systems. *AI Magazine*, pages 79–92, Summer 1998.
66. C. Urmson, J. Anhalt, M. Clark, T. Galatali, J. P. Gonzalez, J. Gowdy, A. Gutierrez, S. Harbaugh, M. Johnson-Roberson, H. Kato, P. L. Koon, K. Peterson, B. K. Smith, S. Spiker, E. Tryzelaar, and W. R. L. Whittaker. High speed navigation of unrehearsed terrain: Red team technology for grand challenge 2004. Technical Report CMU-RI-TR-04-37, Robotics Institute, Carnegie Mellon University, Pittsburgh, PA, June 2004.
67. W. Van Wezel, R. Jorna, and A. Meystel. *Planning in Intelligent Systems: Aspects, Motivations, and Methods*. John Wiley & Sons, Hoboken, New Jersey, 2006.
68. V. Vapnik. *Statistical Learning Theory*. John Wiley & Sons, New York, NY, 1998.
69. L. A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.
70. L. A. Zadeh, A new direction in AI: Toward a computational theory of perceptions, AI Magazine 22 (1) (2001) 73-84.
71. C.J.C.H. Watkins. *Learning from Delayed Rewards*, Ph.D. Thesis, supervisor: Richard Young, King's College, University of Cambridge, UK, May, 1989.
72. P. Wawrzyński. *Intensive Reinforcement Learning*, Ph.D. dissertation, supervisor: Andrzej Pacut, Institute of Control and Computational Engineering, Warsaw University of Technology, May 2005.