Homomorphisms Between Relation Information Systems

Changzhong Wang[1], Congxin Wu[1], and Degang Chen[2]

[1] Department of Mathematics, Harbin Institute of Technology
Harbin, Heilongjiang, 151000, P.R.China
[2] School of Mathematics & Physics, North China Electric Power University
Beijing, 102206, P.R.China
changzhongwang@126.com

Abstract. Information system is one of the important mathematical models in the field of artificial intelligence. The concept of homomorphism is very useful to study the communication between two information systems. In this paper, some properties of relation information systems under homomorphisms are investigated. The concept of a relation mapping between two universes is proposed in order to construct a binary relation on one universe according to the given binary relation on the other universe. The main properties of the mapping are studied. Furthermore, the notion of homomorphism of information systems based on arbitrary binary relations is proposed, and it is proved that the reductions of the original system and image system are equivalent to each other under the condition of homomorphism.

Keywords: Consistent functions, relation mappings, relation information systems, homomorphism, reduction.

1 Introduction

Rough set theory [7], proposed by Pawlak, is an excellent tool for data analysis with important applications in data mining and knowledge discovery. A concept related to rough set is information system. In fact, most applications based on rough set theory, such as classification, decision support and knowledge discovery problems, can fall into the knowledge representation model, i.e. an information system. In recent years, many topics on information systems have been widely investigated by many scholars [1-6,9-11].

The theory of rough sets deals with the approximation of an arbitrary subset of a universe by two definable or observable subset called lower and upper approximations. However, lower and upper approximations are not primitive notions. They are constructed from other concepts, such as binary relations on a universe, partitions and coverings of a universe, and approximation space. For an information system, it can be seen as a composition of some approximation spaces on the same universe. The communication between two information systems is a very important topic in the field of artificial intelligence. In mathematics, it can be explained as a mapping between two information systems.

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The notion of homomorphism on information systems as a kind of tool to study the relationship between two information systems was introduced by Grzymala Busse in [1,2]. A homomorphism can be regarded as a special communication between two information systems. Image system is seen as an explanation system of the original system. A homomorphism on information systems is very useful for aggregating sets of objects, attributes, and descriptors of the original system. The notions of superfluousness and reducts of an information system are central notions in decision making, data analysis, reasoning about data and other subfields of artificial intelligence[2-4,6,8,10,11]. In [2], the authors depicted the conditions which make an information system to be selective in terms of endomorphism of the system. In [4], with algebraic approach, the authors discussed the features of superfuousness and reducts of an information system under some homomorphisms. .

However, the requirement of an indiscernibilty relation or a partition in rough set theory is a condition that limits the application domain of rough set theory. So several important generalizations were proposed to solve this problem. One of these generalizations is to relax an equivalence relation to general binary relation [5]. The work in our paper represents a new contribution to the development of the theory of homomorphism between information systems. We develop a method for defining an arbitrary binary on a universe according to a relation on another universe. In this sense, our method is a mechanism for communicating between two information systems. We define the concept of homomorphism between two information systems based on arbitrary binary relations. Under the condition of the homomorphism, some characters of relation operations in the original system and some structure features of the original system are guaranteed in explanation system.

2 Consistent Function and Its Properties

Let U and V be finite and nonempty universes. The class of all binary relations on U (respectively, on V) will be denoted by $\Re(U)$ (respectively, by $\Re(V)$). Let $R \in \Re(U)$, the successor neighborhood of $x \in U$ with respect to R will be denoted by $R_s(x)$, that is, $R_s(x) = \{y \in U : xRy\}$. In this section, we introduce the concepts of consistent functions and investigate their main properties which will be used in the following sections.

Definition 2.1. Let U and V be finite and nonempty universes, $f : U \to V$ a mapping from U to V, and R a binary relation on U. Let

$$[x]_f = \{y \in U : f(y) = f(x)\}, [x]_R = \{y \in U : R_s(y) = R_s(x)\}.$$

Then both of $\left\{[x]_f : x \in U\right\}$ and $\{[x]_R : x \in U\}$ are partitions on U. If $[x]_f \subseteq R_s(y)$ or $[x]_f \cap R_s(y) = \emptyset$ for any $x, y \in U$, then f is called a type-1 consistent function with respect to R on U. If $[x]_f \subseteq [x]_R$ for any $x \in U$, then f is called a type-2 consistent function with respect to R on U.

From definition 2.1, an injection is trivially both a type-1 and a type-2 consistent function.

Theorem 2.2. Let $f : U \to V, R \in \Re(U)$. If f is a type-1 consistent function with respect to R on U, then $\forall x \in U, f^{-1}(f(R_s(x))) = R_s(x)$.

Proof. Because $f^{-1}(f(R_s(x))) \supseteq R_s(x)$ for any $x \in U$ is always true, we only need to prove that $f^{-1}(f(R_s(x))) \subseteq R_s(x), \forall x \in U$.

For any $x_1 \in f^{-1}(f(R_s(x)))$, we have $f(x_1) \in f(R_s(x))$, which implies $\exists x_2 \in R_s(x)$ such that $f(x_1) = f(x_2)$. Since $[x_2]_f = \{y \in U : f(y) = f(x_2)\}$, thus, $x_2 \in [x_1]_f$, which implies $[x_1]_f \cap R_s(x) \neq \emptyset$. Since f is a type-1 consistent function with respect to R on U, we must have $x_1 \in [x_2]_f \subseteq R_s(x)$. Thus $f^{-1}(f(R_s(x))) \subseteq R_s(x)$. Therefore $f^{-1}(f(R_s(x))) = R_s(x), \forall x \in U$.

Corollary 2.3. Let $f : U \to V, R_1, R_2, \cdots, R_n \in \Re(U)$. If f is a type-1 consistent function with respect to each relation $R_i\,(i \le n)$ on U, then $\forall x \in U, f^{-1}\left(f\left(\bigcap_{i=1}^{n}(R_i)_s(x)\right)\right) = \bigcap_{i=1}^{n}(R_i)_s(x)$.

Proof. It is similar to the proof Theorem 2.2.

Theorem 2.4. Let $f : U \to V, R_1, R_2 \in \Re(U)$. If f is a type-1 consistent function with respect to R_1 and R_2 on U, then $\forall x \in U, f((R_1 \cap R_2)_s(x)) = f((R_1)_s(x)) \cap f((R_2)_s(x))$.

Proof. Since $f((R_1 \cap R_2)_s(x)) \subseteq f((R_1)_s(x)) \cap f((R_2)_s(x))$ for any $x \in U$ is always true, we only need to prove the inverse inclusion for any $x \in U$.

For any $y \in f((R_1)_s(x)) \cap f((R_2)_s(x))$, we have $y \in f((R_1)_s(x))$ and $y \in f((R_2)_s(x))$. Thus $f^{-1}(y) \subseteq f^{-1}(f((R_1)_s(x)))$ and $f^{-1}(y) \subseteq f^{-1}(f((R_2)_s(x)))$. Since f is a type-1 consistent function with respect to R_1 and R_2 on U, by Theorem 2.2, $f^{-1}(y) \subseteq (R_1)_s(x)$ and $f^{-1}(y) \subseteq (R_2)_s(x)$. Hence

$$f^{-1}(y) \subseteq (R_1)_s(x) \cap (R_2)_s(x) = (R_1 \cap R_2)_s(x)$$

This implies $y \in f((R_1 \cap R_2)_s(x))$. Therefore $f((R_1 \cap R_2)_s(x)) \supseteq f((R_1)_s(x)) \cap f((R_2)_s(x))$. It follows that $\forall x \in U, f((R_1 \cap R_2)_s(x)) = f((R_1)_s(x)) \cap f((R_2)_s(x))$.

By Corollary 2.3 and Theorem 2.4, we directly get the following corollary.

Corollary 2.5. Let $f : U \to V, R_1, R_2, \cdots, R_n \in \Re(U)$. If f is a type-1 consistent function with respect to each relation R_i on U, then $\forall x \in U, f\left(\bigcap_{i=1}^{n}(R_i)_s(x)\right) = \bigcap_{i=1}^{n} f((R_i)_s(x))$.

3 Relation Mapping and Its Properties

In this section, we define the notions of relation mappings and study their main properties.

Definition 3.1. Let $f : U \to V, x| \to f(x) \in V, x \in U$. f can induce a mapping from $\Re(U)$ to $\Re(V)$ and a mapping from $\Re(V)$ to $\Re(U)$, that is,

$$\hat{f} : \Re(U) \to \Re(V), R| \to \hat{f}(R) \in \Re(V), \forall R \in \Re(U);$$

$$\hat{f}(R) = \bigcup_{x \in U} \{f(x) \times f(R_s(x))\}.$$

$$\hat{f}^{-1} : \Re(V) \to \Re(U), T| \to \hat{f}^{-1}(T) \in \Re(U), \forall T \in \Re(V);$$

$$\hat{f}^{-1}(T) = \bigcup_{y \in V} \left\{f^{-1}(y) \times f^{-1}(T_s(y))\right\}.$$

Then $\hat{f}$ and $\hat{f}^{-1}$ are called relation mapping and inverse relation mapping induced by f respectively; $\hat{f}(R)$ and $\hat{f}^{-1}(T)$ are called binary relations induced by f on V and U respectively. In the subsequent discussion, we simply denote $\hat{f}$ and $\hat{f}^{-1}$ by f and f^{-1} respectively.

Theorem 3.2. Let $f : U \to V, R_1, R_2 \in \Re(U)$. If f is both type-1 and type-2 consistent with respect to R_1 and R_2, then $f(R_1 \cap R_2) = f(R_1) \cap f(R_2)$.

Proof.

$$\begin{aligned}
& f(R_1 \cap R_2) = \bigcup_{x \in U} \{f(x) \times f((R_1 \cap R_2)_s(x))\} \\
& \subseteq \bigcup_{x \in U} \{f(x) \times (f((R_1)_s(x)) \cap f((R_2)_s(x)))\} \\
& = \bigcup_{x \in U} \{f(x) \times f((R_1)_s(x)) \cap f(x) \times f((R_2)_s(x))\} \\
& \subseteq \left(\bigcup_{x \in U} \{f(x) \times f((R_1)_s(x))\}\right) \cap \left(\bigcup_{x \in U} \{f(x) \times f((R_2)_s(x))\}\right) \\
& = f(R_1) \cap f(R_2).
\end{aligned}$$

Next, we are to prove the inverse inclusion.

Let $(y_1, y_2) \in f(R_1) \cap f(R_2)$. Then $(y_1, y_2) \in f(R_1)$ and $(y_1, y_2) \in f(R_2)$. By the definition of $f(R_1)$, there exists $x_1 \in U$ such that $(y_1, y_2) \in f(x_1) \times f((R_1)_s(x_1))$, which implies $y_1 = f(x_1)$ and $y_2 \in f((R_1)_s(x_1))$. Similarly, there exists $x_2 \in U$ such that $(y_1, y_2) \in f(x_2) \times f((R_2)_s(x_2))$, which implies $y_1 = f(x_2)$ and $y_2 \in f((R_2)_s(x_2))$. Thus $f(x_1) = f(x_2)$ and $y_2 \in f((R_1)_s(x_1)) \cap f((R_2)_s(x_2))$. Since f is a type-2 consistent function with respect to R_1 and R_2, we have $(R_1)_s(x_1) = (R_1)_s(x_2)$ and $(R_2)_s(x_1) = (R_2)_s(x_2)$. Hence by Theorem 2.4, $y_2 \in f((R_1)_s(x_2)) \cap f((R_2)_s(x_2)) = f((R_1)_s(x_2) \cap (R_2)_s(x_2)) = f((R_1 \cap R_2)_s(x_2))$. Then we can conclude that $(y_1, y_2) = (f(x_2), y_2) \in f(x_2) \times f((R_1 \cap R_2)_s(x_2)) \subseteq f(R_1 \cap R_2)$. Thus $f(R_1 \cap R_2) \supseteq f(R_1) \cap f(R_2)$. Therefore $f(R_1 \cap R_2) = f(R_1) \cap f(R_2)$.

Corollary 3.3. Let $f : U \to V, R_1, R_2, \cdots, R_n \in \Re(U)$. If f is both type-1 and type-2 consistent with respect to each relation $R_i\,(i \le n)$, then $f\left(\bigcap_{i=1}^{n} R_i\right) = \bigcap_{i=1}^{n} f(R_i)$.

Proof. It is similar to the proof of Theorem 3.2.

Theorem 3.4. Let $f : U \to V, R \in \Re(U)$. If f is both type-1 and type-2 consistent with respect to R, then $f^{-1}(f(R)) = R$.

Proof. Let $(x_1, x_2) \in R$, namely, $x_2 \in R_s(x_1)$. Thus $f(x_2) \in f(R_s(x_1))$. By the definition of $f(R)$, we have that $(f(x_1), f(x_2)) \in f(R)$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$, then $y_2 \in f(R)_s(y_1)$. Thus $f^{-1}(y_2) \subseteq f^{-1}(f(R)_s(y_1))$. It follows that $f^{-1}(y_1) \times f^{-1}(y_2) \subseteq f^{-1}(y_1) \times f^{-1}(f(R)_s(y_1)) \subseteq f^{-1}(f(R))$, which implies $(x_1, x_2) \in f^{-1}(f(R))$. Therefore $f^{-1}(f(R)) \supseteq R$. Next, we are to prove that $f^{-1}(f(R)) \subseteq R$.

Let $(x_1, x_2) \in f^{-1}(f(R))$, then there exists $y_1 \in V$ such that $(x_1, x_2) \in f^{-1}(y_1) \times f^{-1}((f(R))_s(y_1))$. This implies $x_1 \in f^{-1}(y_1)$ and $x_2 \in f^{-1}((f(R))_s(y_1))$. Hence $y_1 = f(x_1)$ and $f(x_2) \in (f(R))_s(y_1)$. Let $y_2 = f(x_2)$, then $(y_1, y_2) \in f(R)$. By the definition of $f(R)$, there exists $x_3 \in U$ such that $(y_1, y_2) \in f(x_3) \times f(R_s(x_3))$. This implies $y_1 = f(x_3)$ and $y_2 \in f(R_s(x_3))$. Hence $f(x_1) = f(x_3)$ and $f(x_2) \in f(R_s(x_3))$. Since f is type-2 consistent with respect to R, we have $R_s(x_3) = R_s(x_1)$ and $f(x_2) \in f(R_s(x_1))$. Hence $x_2 \in f^{-1}(f(R_s(x_1)))$. Again, since f is type-1 consistent with respect to R, by Theorem 2.2, $x_2 \in f^{-1}(f(R_s(x_1))) = R_s(x_1)$. Thus $(x_1, x_2) \in R$. It follows that $f^{-1}(f(R)) \subseteq R$. Therefore $f^{-1}(f(R)) = R$.

Corollary 3.5. Let $f : U \to V, R_1, R_2, \cdots, R_n \in \Re(U)$. If f is both type-1 and type-2 consistent with respect to each relation $R_i\,(i \le n)$, then
$$f^{-1}\left(f\left(\bigcap_{i=1}^{n} R_i\right)\right) = \left(\bigcap_{i=1}^{n} R_i\right).$$

Proof. It is similar to the proof of Theorem 3.4.

4 Homomorphism Between Relation Information Systems and Its Properties

By means of the results of the above sections, we introduce the notion of a homomorphism between two information systems and show that reductions of the original system and image system are equivalent to each other.

Definition 4.1. Let U and V be finite universes, $f : U \to V$ a mapping from U to V, and $\mathbf{R} = \{R_1, R_2, \cdots, R_n\}$ a family of binary relations on U, let $f(\mathbf{R}) = \{f(R_1), f(R_2), \cdots, f(R_n)\}$. Then the pair $(U, \mathbf{R})$ is referred to as a relation information system, and the pair $(V, f(\mathbf{R}))$ is referred to as a $f-$ induced relation information system of $(U, \mathbf{R})$.

By Theorem 3.2, we can introduce the following concept.

Definition 4.2. Let $(U,\mathbf{R})$ be a relation information system and $(V,f(\mathbf{R}))$ a $f-$ induced relation information system of $(U,\mathbf{R})$. If $\forall R_i \in \mathbf{R}$, f is both type-1 and type-2 consistent with respect to R_i on U, then f is referred to as a homomorphism from $(U,\mathbf{R})$ to $(V,f(\mathbf{R}))$.

Remark. After the notion of homomorphism is introduced, all the theorems and corollaries in the above sections may be seen as the properties of homomorphism.

Definition 4.3. Let $(U,\mathbf{R})$ be a relation information system. The subset $\mathbf{P} \subseteq \mathbf{R}$ is referred to as a reduct of $\mathbf{R}$ if $\mathbf{P}$ satisfies the following conditions:

(1) $\cap\,\mathbf{P} = \cap\mathbf{R}$; (2) $\forall R_i \in \mathbf{P}, \cap\mathbf{P} \subset \cap(\mathbf{P}-R_i)$.

Theorem 4.4. Let $(U,\mathbf{R})$ be a relation information system, $(V,f(\mathbf{R}))$ a $f-$ induced relation information system of $(U,\mathbf{R})$, and f a homomorphism from $(U,\mathbf{R})$ to $(V,f(\mathbf{R}))$. Then $\mathbf{P} \subseteq \mathbf{R}$ is a reduct of $\mathbf{R}$ if and only if $f(\mathbf{P})$ is a reduct of $f(\mathbf{R})$.

Proof. $\Rightarrow$ Since $\mathbf{P}$ is a reduct of $\mathbf{R}$, we have $\cap\mathbf{P} = \cap\mathbf{R}$. Hence $f(\cap\mathbf{P}) = f(\cap\mathbf{R})$. Since f is a homomorphism from $(U,\mathbf{R})$ to $(V,f(\mathbf{R}))$, by Definition 4.2 and Corollary 3.3, we have $\cap f(\mathbf{P}) = \cap f(\mathbf{R})$. Assume that $\exists R_i \in \mathbf{P}$ such that $\cap(f(\mathbf{P})-f(R_i)) = \cap f(\mathbf{P})$. Because $f(\mathbf{P})-f(R_i) = f(\mathbf{P}-R_i)$, we have that $\cap(f(\mathbf{P})-f(R_i)) = \cap f(\mathbf{P}-R_i) = \cap f(\mathbf{P}) = \cap f(\mathbf{R})$. Similarly, by Definition 4.2 and Corollary 3.3, it follows that $f(\cap(\mathbf{P}-R_i)) = f(\cap\mathbf{R})$. Thus $f^{-1}(f(\cap(\mathbf{P}-R_i))) = f^{-1}(f(\cap\mathbf{R}))$. By Definition 4.2 and Corollary 3.5, $\cap(\mathbf{P}-R_i) = \cap\mathbf{R}$. This is a contradiction to that $\mathbf{P}$ is a reduct of $\mathbf{R}$.

$\Leftarrow$ Let $f(\mathbf{P}) \subseteq f(\mathbf{R})$ be a reduct of $f(\mathbf{R})$, then $\cap f(\mathbf{P}) = \cap f(\mathbf{R})$. Since f a homomorphism from $(U,\mathbf{R})$ to $(V,f(\mathbf{R}))$, by Definition 4.2 and Corollary 3.3, we have $f(\cap\mathbf{P}) = f(\cap\mathbf{R})$. Hence $f^{-1}(f(\cap\mathbf{P})) = f^{-1}(f(\cap\mathbf{R}))$. By Definition 4.2 and Corollary 3.5, $\cap\mathbf{P} = \cap\mathbf{R}$. Assume that $\exists R_i \in \mathbf{P}$ such that $\cap(\mathbf{P}-R_i) = \cap\mathbf{R}$, then $f(\cap(\mathbf{P}-R_i)) = f(\cap\mathbf{R})$. Again, by Definition 4.2 and Corollary 3.3, we have $\cap f(\mathbf{P}-R_i) = \cap f(\mathbf{R})$. Hence $\cap(f(\mathbf{P})-f(R_i)) = \cap f(\mathbf{R})$. This is a contradiction to that $f(\mathbf{P})$ is a reduct of $f(\mathbf{R})$. This completes the proof of this theorem.

By Theorem 4.4, we immediately get the following corollary.

Corollary 4.5. Let $(U,\mathbf{R})$ be a relation information system, $(V,f(\mathbf{R}))$ a $f-$ induced relation information system of $(U,\mathbf{R})$, and f a homomorphism from $(U,\mathbf{R})$ to $(V,f(\mathbf{R}))$. Then $\mathbf{P} \subseteq \mathbf{R}$ is is superfluous in $\mathbf{R}$ if and only if $f(\mathbf{P})$ is superfluous in $f(\mathbf{R})$.

The following example is employed to illustrate our idea in this paper.

Example 4.6. Let $(U,\mathbf{R})$ be a relation information system, where $U = \{x_1, x_2, \cdots, x_{10}\}$, $\mathbf{R} = \{R_1, R_2, R_3\}$,

$$R_1 = \{(x_2,x_3),(x_2,x_6),(x_5,x_2),(x_5,x_3),(x_5,x_6),(x_5,x_8),(x_7,x_{12}),\\ (x_7,x_{13}),(x_7,x_{14}),(x_7,x_{15}),(x_8,x_3),(x_8,x_6)(x_9,x_{12}),(x_9,x_{13}),\\ (x_9,x_{14}),(x_9,x_{15}),(x_{10},x_{12}),(x_{10},x_{13}),(x_{10},x_{14}),(x_{10},x_{15})\},$$

$R_2 = (x_1, x_{12}), (x_1, x_{13}), (x_1, x_{14}), (x_1, x_{15}), (x_2, x_3), (x_2, x_6), (x_4, x_{12}),$
$(x_4, x_{13}), (x_4, x_{14}), (x_4, x_{15}), (x_5, x_2), (x_5, x_8), (x_8, x_3), (x_8, x_6),$
$(x_{11}, x_{12}), (x_{11}, x_{13}), (x_{11}, x_{14}), (x_{11}, x_{15})\},$
$R_3 = (x_1, x_7), (x_1, x_9), (x_1, x_{10}), (x_2, x_3), (x_2, x_6), (x_4, x_7), (x_4, x_9),$
$(x_4, x_{10}), (x_5, x_3), (x_5, x_6), (x_8, x_3), (x_8, x_6), (x_{11}, x_7), (x_{11}, x_9),$
$(x_{11}, x_{10}), (x_{12}, x_5), (x_{13}, x_5), (x_{14}, x_5), (x_{15}, x_5)\}$
$R_1 \cap R_2 \cap R_3 = \{(x_2, x_3), (x_2, x_6), (x_8, x_3), (x_8, x_6)\}.$

Let $V = \{y_1, y_2, y_3, y_4, y_5, y_6\}$. Define a mapping as follows:

x_1, x_4, x_{11}	x_2, x_8	x_3, x_6	x_5	x_7, x_9, x_{10}	$x_{12}, x_{13}, x_{14}, x_{15}$
y_1	y_2	y_3	y_4	y_5	y_6

Then $f(\mathbf{R}) = \{f(R_1), f(R_2), f(R_3)\}$, where
$f(R_1) = \{(y_2, y_3), (y_4, y_2), (y_4, y_3), (y_5, y_6)\}$,
$f(R_2) = \{(y_1, y_6), (y_2, y_3), (y_4, y_2)\}$,
$f(R_3) = \{(y_1, y_5), (y_2, y_3), (y_4, y_3), (y_6, y_4)\}$.

And $(V, f(\mathbf{R}))$ is the $f-$ induced relation information system of $(U, \mathbf{R})$. It is very easy to verify that f is a homomorphism from $(U, \mathbf{R})$ to $(V, f(\mathbf{R}))$.

We can see that $f(R_1)$ is superfluous in $f(\mathbf{R}) \Leftrightarrow R_1$ is superfluous in $\mathbf{R}$ and that $\{f(R_2), f(R_3)\}$ is a reduct of $f(\mathbf{R}) \Leftrightarrow \{R_2, R_3\}$ is a reduct of $\mathbf{R}$. Therefore, we can reduce the original system by reducing the image system and reduce the image system by reducing the original system. That is, the reductions of the original system and image system are equivalent to each other.

5 Conclusions

In this paper, we point out that a mapping between two universes can induce a binary relation on one universe according to the given relation on the other universe. For a relation information system, we can consider it as a composition of some generalized approximation spaces on the same universe. The mapping between generalized approximation spaces can be explained as a mapping between the given relation information systems. A homomorphism is a special mapping between two relation information systems. Under the condition of homomorphism, we discuss the characters of relation information systems, and find out that the reductions of the original system and image system are equivalent to each other. These results may have potential applications in knowledge reduction, decision making and reasoning about data, especially for the case of two relation information systems. Our results also illustrate that some characters of a system are guaranteed in explanation system, i.e., a system gain acknowledgement from another system.

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