A Complete Method to Incomplete Information Systems

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Abstract. In the paper, we present a novel method for handling incomplete information systems. By the proposed method we can transform an incomplete information system into a complete set-value information system without loss any information, and we discuss the relationship between the reducts of incomplete information system and the reducts of it's complements. For incomplete decision tables, we introduce two complete methods according to different criterions of certain factor of decision rules, i.e., maximal sum complement and maximal conjunction complement of certain factor of decision rules.

Keywords: Rough sets, incomplete information systems, knowledge reduction.

1 Introduction

Rough sets theory, introduced by Pawlak [1], has been conceived as a tool to conceptualize and analyze various types of data, in particular, has important applications to artificial intelligence and cognitive sciences, as a tool for dealing with vagueness and uncertainty of facts, and in classification [2,3,4,5].

A concept related to rough set is information system (attribute-value system). According to whether or not there are missing data (null values), information system can be classified into two categories: complete and incomplete. A incomplete information system contains null value for at last one attribute, a null value may be some value in the domain of the corresponding attribute, however, it is unknown. Here we consider the case in which a null value means an applicable value. Some important results have recently been obtained for incomplete information system by knowledge acquisition methodologies [3,5,6,7,8,9].

There are several ways in which null value may be handled in [10,11,12]. The simplest method to hand null value is to remove objects with unknown values or replace null values with most common values [10] in the original system. More complex approaches which provide strategies to deal with null values in terms of statistics are studied [13], in which it is suggested to predict the null values on the basis of values of other attributes of an object and relevant class information.

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The problem of rules extraction from incomplete information system was discussed in the context of the Rough Sets [3,6,7,10,14]. Modeling uncertainty caused by the appearance of unknown values by means of fuzzy sets was described in [3]; A methodology of rules generation from original incomplete systems was discussed in [6,7]; A learning algorithm is proposed [14], which can simultaneously derive rules from incomplete system and estimate the missing values in the learning process.

In the paper, we describe a new method for handling incomplete information system. By the proposed method we can transform an incomplete information system into a complete set-value information system. For decision tables, we show two complete methods according to different criterions of certain factor of decision rules, i.e., maximal sum complement of certain factor and maximal conjunction complement of certain factor. The relationship between the reducts of incomplete information system and the reducts of it's complements are also discussed in details.

2 Incomplete Information Systems

An information system (IS) is an ordered triplet $\mathcal{I} = (U, AT, f)$, where U is a finite nonempty set of objects and AT is a finite nonempty set of attributes, $f_a : U \to V_a$ for any $a \in AT$, where V_a is the domain of attribute a.

It may happen that some of attribute values for objects are missing. To indicate such a situation a distinguished value, so-called *null value*, is usually assigned to those attributes. We denote special symbol $*$ to indicate that the value of an attribute is unknown. Here, we assume that an object $x \in U$ possesses only one value for an attribute a ($a \in AT$). Thus, if the value of an attribute a is missing, then the real value must be one of value of V_a. An IS in which values of all attributes for all objects from U are known is called complete, it is called incomplete otherwise.

Example 1. An incomplete IS $\mathcal{I} = (U, AT, f)$ is presented in Table 1.

Table 1. An incomplete IS

U	a_1	a_2	a_3
x_1	1	1	1
x_2	1	*	1
x_3	2	1	1
x_4	1	2	*
x_5	1	*	1
x_6	2	2	2
x_7	1	1	1

From Table 1, we have a set-value IS $\mathcal{I}_F = (U, AT, F)$, see Table 2, where $F = \{F_a : U \to \mathcal{P}(V_a) |\ a \in AT\}$,

$$F_a(x) = \begin{cases} \{f_a(x)\} & f_a(x) \neq *, \\ V_a & f_a(x) = *. \end{cases}$$

Table 2. The set-value IS $\mathcal{I}_F$

U	a_1	a_2	a_3
x_1	{1}	{1}	{1}
x_2	{1}	{1,2}	{1}
x_3	{2}	{1}	{1}
x_4	{1}	{2}	{1,2}
x_5	{1}	{1,2}	{1}
x_6	{2}	{2}	{2}
x_7	{1}	{1}	{1}

In the following, we only consider the set-value IS that derived from incomplete IS.

Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS and $A \subseteq AT$. Let

$$R^*_A = \{(x, y) \in U \times U | \ \forall \ a \in A, F_a(x) \cap F_a(y) \neq \emptyset\},$$

we denote $[x]^*_A = \{y \in U | (x, y) \in R^*_A\}$. In general, U/R^*_A do not constitute a partition of U, they may overlap.

Let $A \subseteq AT$, we say A is a reduct of $\mathcal{I}_F$, if $R^*_A = R^*_{AT}$ and $R^*_{A-\{a\}} \neq R^*_{AT}$ $(\forall \ a \in A)$.

Definition 1. *Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS, $X \subseteq U$, $A \subseteq AT$, a pair of lower and upper approximations, $\underline{A}(X)$ and $\overline{A}(X)$, is defined by*

$$\underline{A}(X) = \{x \in U | \ [x]^*_A \subseteq X\}, \quad \overline{A}(X) = \{x \in U | \ [x]^*_A \cap X \neq \emptyset\}.$$

Theorem 1. *Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS, $X, \ Y \subseteq U$, $A \subseteq B \subseteq AT$, then*

(1) $\underline{A}(\emptyset) = \overline{A}(\emptyset) = \emptyset$, $\overline{A}(U) = \underline{A}(U) = U$;
(2) $\underline{A}(X \cap Y) \subseteq \underline{A}(X) \cap \underline{A}(Y)$, $\overline{A}(X \cup Y) \supseteq \overline{A}(X) \cup \overline{A}(Y)$;
(3) $X \subseteq Y \Rightarrow \underline{A}(X) \subseteq \underline{A}(Y)$, $X \subseteq Y \Rightarrow \overline{A}(X) \subseteq \overline{A}(Y)$;
(4) $\underline{A}(X \cup Y) \supseteq \underline{A}(X) \cup \underline{A}(Y)$, $\overline{A}(X \cap Y) \subseteq \overline{A}(X) \cap \overline{A}(Y)$;
(5) $\underline{A}X \subseteq X \subseteq \overline{A}X$;
(6) $\underline{A}X \subseteq \underline{B}X$, $\overline{A}X \supseteq \overline{B}X$.

Proof. It immediately follows from Definition 1.

Definition 2. *Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS. We denote*

$$D(x, y) = \{a \in AT | \ F_a(x) \cap F_a(y) = \emptyset \ (x, y \in U)\},$$

then $D(x, y)$ is called discernibility attribute set of $\mathcal{I}_F$, and $\mathcal{D} = (D(x, y) : x, y \in U)$ is called discernibility matrix of $\mathcal{I}_F$.

Theorem 2. *Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS and $A \subseteq AT$, then $R^*_A = R^*_{AT}$ iff $A \cap D(x, y) \neq \emptyset$ $(\forall D(x, y) \neq \emptyset, x, y \in U)$.*

Proof. Suppose that $A \cap D(x,y) \neq \emptyset$ ($\forall D(x,y) \neq \emptyset, x,y \in U$), then $\exists\ a \in A$, such that $a \in D(x,y)$, which implies $F_a(x) \cap F_a(y) = \emptyset$, i.e. $(x,y) \notin R^*_A$. Thus, $R^*_A \subseteq R^*_{AT}$. On the other hand, it is evident that $R^*_{AT} \subseteq R^*_A$. Therefore, $R^*_A = R^*_{AT}$.

Conversely, assume that $R^*_A = R^*_{AT}$, then $[x]^*_A = [x]^*_{AT}$ ($\forall\, x \in U$). If $y \notin [x]^*_{AT}$, then $y \notin [x]^*_A$. Thus $\exists\ a \in A$, such that $F_a(x) \cap F_a(y) = \emptyset$, which implies $a \in D(x,y)$. Therefore, $A \cap D(x,y) \neq \emptyset$ ($\forall D(x,y) \neq \emptyset$).

We denote

$$\triangle = \bigwedge_{(x,y)\in\ U\times U} \bigvee D(x,y),$$

then $\triangle$'s prime implications determine reducts uniquely for set-value IS (see [15]).

Example 2. Table 3 is the discernibility matrix of Table 2, where, values of $D(x_i, x_j)$ for any pair (x_i, x_j) of objets from U are placed.

Table 3. The discernibility matrix of $\mathcal{I}_F$

x/y	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1			a_1	a_2		$a_1a_2a_3$	
x_2			a_1			a_1a_3	
x_3	a_1	a_1		a_1a_2	a_1	a_2a_3	a_1
x_4	a_1		a_1a_2			a_1	a_2
x_5			a_1			a_1a_3	
x_6	$a_1a_2a_3$	a_1a_3	a_2a_3	a_1	a_1a_3		$a_1a_2a_3$
x_7			a_1	a_2		$a_1a_2a_3$	

From the Table 3, we have

$$\triangle = a_1 \wedge a_2 \wedge (a_1 \vee a_2 \vee a_3) \wedge (a_1 \vee a_3) \wedge (a_1 \vee a_2) \wedge (a_2 \vee a_3) = a_1 \wedge a_2.$$

Thus, $\{a_1, a_2\}$ is the unique reduct of set-value IS.

Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS. We denote

$$f^{'} = \{f^{'}_a : U \rightarrow V_a,\ f^{'}_a(x) \in F_a(x),\ (a \in AT, x \in U)\},$$

then $f^{'}$ is called a selection of F.

It is easy to see that $\mathcal{I}_{f'} = (U, AT, f^{'})$ is a complement of the original incomplete IS. Let F^* denotes the set of all selections of F. Then, $S_F = \{(U, AT, f^{'}) : f^{'} \in F^*\}$ is the set of all the complements of the original incomplete IS.

Example 3. Table 4 is a selection of set-value IS $\mathcal{I}_F = (U, AT, F)$ presented in Table 2.

Table 4. A selection $\mathcal{I}_{f'} = (U, AT, f')$

U	a_1	a_2	a_3
x_1	1	1	1
x_2	1	2	1
x_3	2	1	1
x_4	1	2	2
x_5	1	1	1
x_6	2	2	2
x_7	1	1	1

Let $\mathcal{I} = (U, AT, f)$ be an incomplete IS. We denote

$$Y^* = \{(x, a) |\ x \in U,\ a \in AT,\ f_a(x) = *\},\ B^* = \{a \in AT |\ \exists\ x \in U, f_a(x) = *\}.$$

The number of all the complement of $\mathcal{I}$ is $\prod\limits_{(x,a)\in Y^*} |V_a|$, i.e., $|S_F| = \prod\limits_{(x,a)\in Y^*} |V_a|$, where $|\cdot|$ denotes the cardinal of a set. We can select a complement from S_F according to different criterions.

Example 4. In *Example 1*, since $Y^* = \{(x_2, a_2),\ (x_4, a_3),\ (x_5, a_2)\}$, then $\prod\limits_{(x,a)\in Y^*} |V_a| = 2 \times 2 \times 2 = 8$.

Theorem 3. *Let $\mathcal{I}_{f'} \in S_F,\ \ A \subseteq AT$. We denote $R_A^{f'} = \{(x, y) \in U \times U |\ \forall\ a \in A, f_a'(x) = f_a'(y)\}$, then $R_A^* = \bigcup\limits_{f' \in\ F^*} R_A^{f'}$.*

Proof. For any $f' \in F^*$, we have $R_A^{f'} \subseteq R_A^*$. Thus $\bigcup\limits_{f' \in F^*} R_A^{f'} \subseteq R_A^*$. On the other hand, for any $(x, y) \in R_A^*$, we can easily conclude that $F_a(x) \cap F_a(y) \neq \emptyset$ $(\forall\ a \in A)$. Hence $\exists\ f' \in F^*$, such that $f_a'(x) = f_a'(y)$ $(\forall\ a \in AT)$, which implies $(x, y) \in R_A^{f'}$. Therefore, $R_A^* \subseteq \bigcup\limits_{f' \in\ F^*} R_A^{f'}$.

We denote $[x]_A^{f'} = \{y \in U |\ (x, y) \in R_A^{f'}\}$, by Theorem 3 we have $[x]_A^* = \bigcup\limits_{f' \in\ F^*} [x]_A^{f'}$.

Theorem 4. *Let $\mathcal{I}_{f'} \in S_F$ and $A \subseteq AT$, if $\forall\ f' \in\ F^*$, $R_A^{f'} = R_{AT}^{f'}$, then $R_A^* = R_{AT}^*$.*

Proof. It is immediately from Theorem 3.

Theorem 5. *Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS and $B^* \subseteq A \subseteq AT$, then $R_A^* = R_{AT}^*$ iff $R_A^{f'} = R_{AT}^{f'}$, $\forall\ f' \in\ F^*$.*

Proof. It is evident that $R^{f'}_{AT} \subseteq R^{f'}_{A}$ ($\forall\ f' \in\ F^*$), and we only need to prove that $R^{f'}_{A} \subseteq R^{f'}_{AT}$. For any $(x,y) \in R^{f'}_{A}$, by Theorem 3 we have $(x,y) \in R^*_A$. Since $R^*_A = R^*_{AT}$, then $\exists\ R^{f^1}_{AT} \in R^*_{AT}$ such that $(x,y) \in R^{f^1}_{AT}$, i.e. $f^1_a(x) = f^1_a(y)$ ($\forall\ a \in AT$). It is easy to see that $\forall\ f^1, f^2 \in F^*$ there always are

$$f^1_a(x) = f^2_a(x),\ (\forall\ a \in AT - B^*,\ \forall\ x \in U).$$

Therefore,

$$f'_a(x) = f'_a(y)\ (\forall\ a \in AT - B^*). \tag{1}$$

On the other hand, since $(x,y) \in R^{f'}_{A}$, then

$$f'_a(x) = f'_a(y)\ (\forall\ a \in A). \tag{2}$$

By Eq. (1) (2), we have $f'_a(x) = f'_a(y)$ ($\forall\ a \in AT$), i.e., $(x,y) \in R^{f'}_{AT}$. Therefore,

$$R^{f'}_{A} \subseteq R^{f'}_{AT}.$$

Conversely, it follows immediately from Theorem 4.

Theorem 6. *Let $\mathcal{I}_F = (U, AT, F)$ be a set-value IS, A is a reduct of $\mathcal{I}_F$. We denote $C = A \cup B^*$, then $R^{f'}_{C} = R^{f'}_{AT}$, $\forall\ f' \in F^*$.*

Proof. Since $R^*_A = R^*_{AT}$ and $A \subseteq C$, then $R^*_C = R^*_{AT}$; on the other hand, since $B^* \subseteq C$, by Theorem 5 we have that $R^{f'}_{C} = R^{f'}_{AT}$, ($\forall\ f' \in F^*$).

Theorem 7. *Let $\mathcal{I}_{f'} = (U, AT, f') \in S_F$, A is a reduct of $\mathcal{I}_{f'}$ and $A \cap B^* = \emptyset$. We denote $C = A \cup B^*$, then $R^{f'}_{C} = R^{f'}_{AT}$, $\forall\ f' \in F^*$.*

Proof. It is similar to the proof of Theorem 5.

3 Incomplete Decision Tables

A decision table (DT) is an IS $\mathcal{I} = (U, AT \cup \{d\}, f)$, where d ($d \notin AT$ and $* \notin V_d$) is a distinguished attribute called the decision, and the element of AT are called conditions. A DT is called complete, if it is a complete IS; it is incomplete otherwise.

Example 5. An incomplete DT is presented in Table 5, similar to incomplete IS, from Table 5, we have a set-value DT $\mathcal{I}_F = (U, AT \cup \{d\}, F)$, see Table 6.

In the following, we only consider the set-value DT derived from incomplete DT.

Let $\mathcal{I}_F = (U, AT \cup \{d\}, F)$ be a set-value DT, we denote

$$f' = \{f'_a : U \to V_a,\ f'_a(x) \in F_a(x) \text{ and } f'_d(x) = F_d(x),\ (a \in AT, x \in U)\},$$

then f' is called a selection of F.

Table 5. An incomplete DT $\mathcal{I} = (U, AT \cup \{d\}, f)$

U	a_1	a_2	a_3	d
x_1	1	1	1	1
x_2	2	*	2	2
x_3	1	2	1	2
x_4	1	1	*	1
x_5	2	2	2	1
x_6	1	1	1	2
x_7	1	*	1	1
x_8	2	1	2	1

Table 6. A set-value DT $\mathcal{I}_F = (U, AT \cup \{d\}, F)$

U	a_1	a_2	a_3	d
x_1	{1}	{1}	{1}	1
x_2	{2}	{1,2}	{2}	2
x_3	{1}	{2}	{1}	2
x_4	{1}	{1}	{1,2}	1
x_5	{2}	{2}	{2}	1
x_6	{1}	{1}	{1}	2
x_7	{1}	{1,2}	{1}	1
x_8	{2}	{1}	{2}	1

Similar to incomplete IS, we can compute the set of all the complements of the original incomplete DT.

In set-value DT, R^*_{AT} is defined as in set-value IS, we denote

$$R_d = \{(x, y) \in U \times U |\ F_d(x) = F_d(y)\},$$

then $\mathcal{I}_F$ is called consistent, if $R^*_{AT} \subseteq R_d$; it is inconsistent otherwise.

Let $\mathcal{I}_F$ be a consistent DT and $A \subseteq AT$, A is called a reduct of DT, if $R^*_A \subseteq R_d$ and $R^*_B \nsubseteq R_d$ $(\forall\ B \subset A)$.

Let $\mathcal{I}_F = (U, AT \cup \{d\}, F)$ be a consistent set-value DT. We denote

$$D_d(x, y) = \begin{cases} \{a \in AT : F_a(x) \cap F_a(y) = \emptyset\}, & F_d(x) \neq F_d(y), \\ \emptyset, & F_d(x) = F_d(y). \end{cases}$$

then $D_d(x, y)$ is called discernibility attribute set of DT, and $\mathcal{D}_d = (D_d(x, y) : x, y \in U)$ is called discernibility matrix of DT.

We denote

$$\triangle = \bigwedge_{(x,y) \in\ U \times U} \bigvee D_d(x, y),$$

then $\triangle$ determine reducts uniquely for consistent set-value DT.

Knowledge hidden in data contained in decision tables may be discovered and expressed in the form of decision rule $t \to s$, where

$$t = \wedge(c, v), c \in AT, v \in V_c \backslash \{*\} \text{ and } s = \vee(d, w), w \in V_d.$$

We denote

$$||t|| = \{x \in U|\ f_a(x) = v, (a, v) \in t\}, \quad ||s|| = \{x \in U|\ f_d(x) = w, (d, w) \in s\},$$

where $||t||$ be the set of objects of property $\wedge(c, v)$ $(c \in AT, v \in V_c)$, and $||s||$ be the set of objects of property $\vee(d, w)$ $(w \in V_d)$, then

$$||t \vee s|| = ||t|| \vee ||s||, \quad ||t \wedge s|| = ||t|| \wedge ||s||.$$

Let $r:\ \wedge(c, v) \rightarrow \vee(d, w)$ be a decision rule, we denote

$$cer_{\mathcal{I}_F}(s \rightarrow t) = \frac{card(||s \wedge t||)}{card(||s||)},$$

then $cer_{\mathcal{I}_F}(s \rightarrow t)$ is called the *certainty factor* of rule r.

Let $\mathcal{I}_{f'} = (U, AT \cup \{d\}, f^{'}) \in S_F$ and $U/R_d = \{D_1, D_2, \cdots, D_n\}$. A membership distribution function $\mu_A^{f^{'}} : U \rightarrow [0, 1]^n$ is defined as follows [16]:

$$\mu_A^{f^{'}}(x) = (D(D_1/[x]_A^{f^{'}}, \ldots, D(D_n/[x]_A^{f^{'}})), \quad x \in U$$

where

$$D(D_i/[x]_A^{f^{'}}) = \frac{|D_i \bigcap [x]_A^{f^{'}}|}{[x]_A^{f^{'}}}.$$

It is evident that $D(D_i/[x]_A^{f^{'}})$ is the *certainty factor* of the rule

$$\bigwedge_{a \in A} (a, f_a^{'}(x)) \rightarrow (d, f_d^{'}(D_i)).$$

Let $x \in U$, we denote

$$m_A^{f^{'}}(x) = \max\{D(D_i/[x]_A^{f^{'}}) : i \leq n\} = D(D_j/[x]_A);$$

$$\eta_A^{f^{'}}(x) = \{D_j : m_A^{f^{'}}(x) = D(D_j/[x]_A^{f^{'}})\}.$$

Let $D_j \in \eta_A^{f^{'}}(x)$, then $D(D_i/[x]_A^{f^{'}}) \leq D(D_j/[x]_A^{f^{'}})$ $(\forall\ D_i \in\ U/R_d)$, i.e., the *certainty factor* of rule $\bigwedge_{a \in A} (a, f_a^{'}(x)) \rightarrow (d, f_d^{'}(D_j))$ is maximal in all the rules supported by object x. Rule $\bigwedge_{a \in A} (a, f_a^{'}(x)) \rightarrow (d, f_d^{'}(D_j))$ is called maximal confidence rule supported by object x.

We denote

$$M_A^{f^{'}} = \sum_{[x]_A \in\ U/R_A} m_A^{f^{'}}(x), \quad m_A^{f^{'}} = \bigwedge_{[x]_A \in\ U/R_A} m_A^{f^{'}}(x).$$

Let

$$M_A^{f^1} = \max\{M_A^f : f \in F^*\}, \quad m_A^{f^2} = \max\{m_A^f : f \in F^*\},$$

then f^1 is called maximal sum selection of *certainty factor*, and f^2 is called maximal conjunction selection of *certainty factor*.

Let $\mathcal{I}_F = (U, AT \cup \{d\}, F)$ be a set-value DT, we select $f^1, f^2 \in F^*$ such that

$$M_A^{f^1} = \max\{M_A^{f'} : f' \in F^*\}, \quad m_A^{f^2} = \max\{m_A^{f'} : f' \in F^*\},$$

then we have the two complete DT

$$(U, AT \cup \{d\}, f^1), \quad (U, AT \cup \{d\}, f^2).$$

In all the selection of F, the sum of *certainty factor* of rules hidden in $(U, AT \cup \{d\}, f_1)$ is maximal, the conjunction of *certainty factor* of rules hidden in $(U, AT \cup \{d\}, f_2)$ is maximal.

Example 6. In *Example 5*, from Table 6, we select $\mathcal{I}_{f^1} \in S_F$ (see Table 7).

Table 7. DT $\mathcal{I}_{f^1} = (U, AT \cup \{d\}, f^1)$

U	a_1	a_2	a_3	d
x_1	1	1	1	1
x_2	2	1	2	2
x_3	1	2	1	2
x_4	1	1	2	1
x_5	2	2	2	1
x_6	1	1	1	2
x_7	1	1	1	1
x_8	2	1	2	1

It can be easily checked that $M_{AT}^{f'} \leq M_{AT}^{f^1}$, $m_{AT}^{f'} \leq m_{AT}^{f^1}$ $(\forall\, f' \in F^*)$. Therefore, f^1 not only is a maximal sum selection of *certainty factor*, but also a maximal conjunction selection of *certainty factor*.

4 Conclusions

In the paper, a new method is proposed to handling incomplete information systems. By the proposed method we transform an incomplete information system into a complete set-value information system, in which we discussed the problems of set approximation and attribute reduction. For incomplete decision tables, we introduce two complete methods according to different criterions of certain factor of decision rules, i.e., maximal sum complement and maximal conjunction complement of certain factor of decision rules. The relationship between the reducts of incomplete information system and the reducts of it's complements are also discussed in details. This paper may provide a new, different understanding and representations to incomplete information systems.

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