Near Sets. Toward Approximation Space-Based Object Recognition

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Abstract. The problem considered in this paper is how to recognize objects that are qualitatively but not necessarily spatially near each other. The term *qualitatively near* is used here to mean closeness of descriptions or distinctive characteristics of objects. The solution to this problem is inspired by the work of Zdzisław Pawlak during the early 1980s on the classification of objects by means of their attributes. In working toward a solution of the problem of the approximation of sets that are qualitatively near each other, this article considers an extension of the basic model for approximation spaces. The basic approach to object recognition is to consider the degree of overlap between families of perceptual neighbourhoods and a set of objects representing a standard. The proposed approach to object recognition includes a refinement of the generalized model for approximation spaces. This is a natural extension of recent work on nearness of objects. A byproduct of the proposed object recognition method is what we call a near set. The contribution of this article is an approximation space-based approach to object recognition formulated in the context of near sets.

Keywords: Approximation space, feature, near set, object recognition, perceptual neighborhood.

An approximation space ... serves as a formal counterpart of perception ability or observation.
– Ewa Orlowska, March, 1982.

1 Introduction

The problem considered in this paper is how to recognize objects that are qualitatively but not necessarily spatially near each other. The term *qualitatively near* is used here to mean closeness of descriptions or distinctive characteristics of objects. The term *object* denotes something perceptible. If we choose *shading* as a feature and let $B_{shading}(x) = \{y \mid shading(x) = shading(y)\}$, then the objects in Fig. 1 can be partitioned, where the objects in $B_{shading}(x)$, *i.e.*, equivalence class containing objects that are *descriptively* indiscernible from x,

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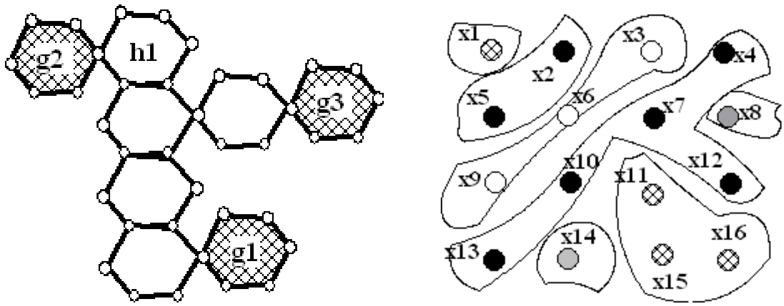
are not adjacent to each other (see, *e.g.*, $B_{shading}(x1) = \{x1, x11, x15, x16\}$ in Fig. 1.2 or $B_{shading}(g1) = \{g1, g2, g3\}$ in Fig. 1.1).



1.1: Hexagonal Objects

1.2: Pixel Neighborhoods

Fig. 1. Non-Adjacent Objects with Matching Descriptions

The solution to this problem is inspired by the work of Zdzisław Pawlak during the early 1980s on the classification of objects [22] and elaborated in [25,26,27]. In working toward a solution of the problem of the approximation of sets that are qualitatively near each other, this article considers an extension of the basic model for approximation spaces. The basic approach is to consider families of perceptual neighborhoods containing objects with matching descriptions that are possibly space-independent. A *perceptual neighborhood* is an equivalence class containing observed sample objects with matching descriptions. The proposed approach to object recognition is a straightforward extension of the rough set approach, where approximation can be considered as formal counterpart of perception [18] in the context of families of perception granules (neighborhoods). The term *perception granule* comes from [48]. A byproduct of the proposed approximation method is what we call a near set.

The approach to classifying objects such as those in Fig. 1.2 contrasts sharply with the approach to defining neighborhoods with an Adjacency relation in [7]. For example, the hexagons with mesh interiors (*g1, g2, g3*) in Fig. 1.1 are descriptively near each other but spatially non-adjacent. The refinement of approximation spaces in [28] is close to what is known as a nearness space [11,48] with the exception of the distinction between attributes and features as well as covering $\mathcal{F}$ (family of neighborhoods) that underly the approach to approximation spaces in this article. In addition, the proposed approach of nearness of objects [28] is not restricted to the neighborhood of a point x and $x \in Cl(A)$ (closure of A) as in [48], since we consider the nearness of objects that are not points. The contribution of this article is an approach to approximating a set based on the union of families of sets of objects with matching descriptions, which provides a foundation for near sets.

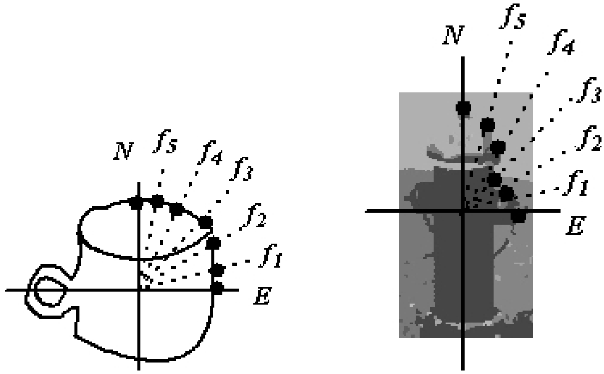
This article is organized as follows. The distinction between features and attributes is explored in Sect. 2. An approach to pattern recognition is briefly presented in Sect. 3. A refinement of the generalized approximation space model is given in Sect. 4. Sample near sets extracted from ethogram tables are presented in Sect. 5.

2 Features and Measurements

Underlying the study of near sets is an interest in classifying sample objects by means of probe functions associated with object features. The term *feature* was originally identified with the cast of a face [14]. More recently, the term *feature* is defined as the make, form, fashion or shape (of an object) [19]. This term comes from the Latin term *factura*, i.e., *facture*, which means the action or process of making an object or the result of an action or process (e.g., a work of art, image made with a digital camera). In effect, the term *feature* characterizes some aspect of the makeup of an object. From a philosophical perspective that can be traced back to Kant [15], features highlight an interest in the *appearances* of objects rather than calling attention to the properties or qualities that are somehow inherent in objects. The term *feature* is commonly used in pattern recognition theory [21], statistical learning theory [45], reinforcement learning [34], neural computing [4], science (e.g., ethology [16,33,35]), image processing [13,5], biotechnology, industrial inspection, the internet, radar, sonar, and speech recognition [9]. More recently, the term *feature* has been used in rough set theory [5,33,34,35,28,30].

Historically, semantically, and philosophically, there is a distinction between the terms *feature* and *attribute*. An *attribute* is a quality regarded as characteristic or inherent in an object [19]. In philosophy, an attribute is a property of an object (e.g., spatial extension of a piece of wax). The term *attribute* is commonly used in database theory [44], data mining [47], and philosophy [12]. In rough set theory [25], an attribute is treated as a partial function, which is a relation that associates each element of a set of objects (domain) with at most one element of a value set (codomain) [49].

It was Zdzisław Pawlak who proposed classifying objects by means of their attributes considered in the context of an approximation space [22]. The proposed approach to classifying objects can also be explained in terms of features. Implicit in the original work of Pawlak is a distinction between features (*makeup, appearance*) of objects and knowledge about objects. The knowledge about an object is represented by a measurement associated with each feature of an object. It can observed that a feature is an invariant characteristic of objects belonging to a class [46]. The distinction between features and corresponding measurements associated with features is usually made in the study of pattern recognition (see, *e.g.*, [17,21]). Let A denote a set of features for objects in a set X. For each $a \in A$, we associate a function f_a that maps X to some set V_{f_a} (range of f_a). The value of $f_a(x)$ is a measurement associated with feature a of an object $x \in X$. The function f_a is called a *probe* [21]. By $Inf_B(x)$, where $B \subseteq A$ and $x \in U$ we denote the *signature* of x, i.e., the set $\{(a, f_a(x)) : a \in B\}$. If the set $B = \{a_1, \ldots, a_m\}$, then Inf_B is identified with a vector $(f_{a_1}(x), \ldots, f_{a_m}(x))$ of probe function values for features in B.



2.1: Prototype Image 2.2: Sample Image

Fig. 2. Image Patterns

3 Approach to Pattern Recognition

The problem considered here is to determine whether there is a correspondence between an object in a prototype image $\mathcal{I}$ (*e.g.*, cup in Fig. 2.1) and an object in a sample image I_1 (*e.g.*, fire hydrant in Fig. 2.2). By way of illustration, consider *contour* as a helpful feature in considering the form of various objects. Let $I_1, \mathcal{I}, f$ denote sample image, prototype image, probe function associated with *contour*, respectively. Then, following the approach suggested in [21], pattern recognition is defined for real-valued probe functions in

$$\mathcal{I} \approx (I_1)T \Leftrightarrow \forall f.|f(\mathcal{I}) - f(I_1)| < \varepsilon, \varepsilon \in [0, 1],$$

where $\mathcal{I}$ is approximately the same as I_1 after some transformation T *iff* the differences between pairs of probe function values is less than some threshold.

4 Approximation Spaces and Object Recognition

This section introduces a view of approximation spaces defined in a slightly modified manner in comparison with the original definition in [38]. Any generalized approximation space (GAS) is a tuple

$$GAS = (U, A, N_r, \nu_B),$$

where U is a universe of objects, A, a set of probe functions, N_r, a neighbourhood family function and ν_B is an overlap function defined by

$$\nu_B : \mathcal{P}(U) \times \mathcal{P}(U) \longrightarrow [0, 1],$$

where $\mathcal{P}(U)$ is the powerset of U. ν_B maps a pair of sets to a number in $[0,1]$ representing the degree of overlap between the sets of objects with features defined by B, and $\mathcal{P}(U)$ is the powerset of U [39]. For each subset $B \subseteq A$ of probe functions, define the binary relation $\sim_B = \{(x,x') \in U \times U : \forall f \in B, f(x) = f(x')\}$. Since each $\sim_B$ is, in fact, the usual Ind_B (indiscernibility) relation, for $B \subset F$ and $x \in U$, let $[x]_B$ denote the equivalence class containing x, *i.e.*,

$$[x]_B = \{x' \in U : \forall f \in B, f(x') = f(x)\} \subseteq U.$$

If $(x,x') \in \sim_B$ (also written $x \sim_B x'$), then x and x' are said to be *indiscernible* with respect to all feature probe functions in B, or simply, B-*indiscernible*. Then define a family of neighborhoods $N_r(A)$, where

$$N_r(A) = \bigcup_{B \subseteq P_r(A)} [x]_B,$$

where $P_r(A) = \{B \subseteq A \mid |B| = r\}$ for any r such that $1 \leq r \leq |A|$. That is, r denotes the number of features used to construct families of neighborhoods. For the sake of clarity, we sometimes write $[x]_{B_r}$ to specify that the equivalence class represents a neighborhood formed using r features from B. Families of neighborhoods are constructed for each combination of probe functions in B using $\binom{|B|}{r}$, *i.e.*, $|B|$ probe functions taken r at a time. Information about a sample $X \subseteq U$ can be approximated from information contained in B by constructing a $N_r(B)$-lower approximation

$$N_r(B)_* X = \bigcup_{x : [x]_{B_r} \subseteq X} [x]_{B_r},$$

and a $N_r(B)$-upper approximation

$$N_r(B)^* X = \bigcup_{x : [x]_{B_r} \cap X \neq \emptyset} [x]_{B_r}.$$

Then $N_r(B)_* X \subseteq N_r(B)^* X$ and the boundary region $BND_{N_r(B)}(X)$ between upper and lower approximations of a set X is defined to be the complement of $N_r(B)_* X$, *i.e.*

$$BND_{N_r(B)}(X) = N_r(B)^* X \backslash N_r(B)_* X = \{x \in N_r(B)^* X \mid x \notin N_r(B)_* X\}.$$

Remark 1. **What is a Near Set?** A set X is termed a "near set" relative to a chosen family of neighborhoods $N_r(B)$ *iff* $|BND_{N_r(B)}(X)| \geq 0$. This means every rough set is a near set but not every near set is a rough set. Object recognition and the problem of the nearness of objects have motivated the introduction of near sets (see, *e.g.*, [28,29]).

4.1 Object Recognition

It is now possible to formulate a basis for object recognition, which parallels the traditional formulation of pattern recognition. Assume $N_r(B)_*X$ defines a standard for classifying perceived objects. The notation $B_j(x)$ denotes a member of the family of neighborhoods in $N_r(B)$, where $j \in B$. Put

$$\nu_j(B_j(x), N_r(B)_*X) = \frac{|B_j(x) \cap N_r(B)_*X|}{|N_r(B)_*X|},$$

(called *lower rough coverage*) where ν_j is defined to be 1, if $N_r(B)_*X = \emptyset$. Let $\mathcal{O}, \mathcal{O}_{id}$ denote sample object and standard object, respectively. Then recognition of sample objects that are approximately the same as $\mathcal{O}_{id}$ is defined by comparing overlap function values in

$$\mathcal{O} \approx (\mathcal{O}_{id})T \Leftrightarrow |\nu_j(\mathcal{O}, N_r(B)_*X) \ - \nu_B(\mathcal{O}_{id}, N_r(B)_*X| < \varepsilon,$$

where $\varepsilon \in [0, 1]$. The sample object $\mathcal{O}$ is approximately the same as $\mathcal{O}_{id}$ after some transformation T *iff* the difference in coverage values is less than some threshold. An image-based model for object recognition is given in [10].

4.2 Percepts and Perception

The set $N_r(B)$ contains a set of percepts. A *percept* is a byproduct of perception, *i.e.*, something that has been observed [19]. For example, a member of $N_r(B)$ represents *what has been perceived about objects belonging to a neighborhood, i.e.*, observed objects with matching probe function values. Collectively, $N_r(B)$ represents a *perception*, a product of perceiving. Perception is defined as the extraction and use of information about one's environment [1]. This is basic idea is represented in the *sample objects*, *perceptual neighborhoods* and *judgemental percepts* columns in Fig. 3. In this article, we are focusing on the perception of acceptable objects.

4.3 Sensing, Classifying, and Peceptual Judgement

Sensing provides a basis for probe function measurements commonly associated with features such as colour, contour, shape, arrangement, entropy, and so on. A probe function can be thought of as a model for a sensor. Classification combines evaluation of a disposition of sensor measurements with judgement (apprehending the significance of a vector of probe measurements for an observed object). The result is a higher level percept, which has been traditionally called a decision. In the context of percepts, the term *judgement* means a conclusion about an object's measurements rather than an abstract idea. This form of judgement is considered *perceptual.* Perceptual judgements provide a basis for the formulation of abstract ideas (models of perception, rules) about a class (type) of object. Let D denote a feature called *decision* with a probe $d_B : X \times B \longrightarrow \{0, 1\}$, where X denotes a set of sample objects; B, a set of probe functions; 0, "reject perceived object" and 1, "accept perceived object". A set of objects d with matching perceptual judgements (*e.g.*, $d_B(x) = 1, x \in X$ for an acceptable object) is a mathematical model representing the abstract notion *acceptable*.

5 Near Sets From Ethograms

This section briefly considers particular near sets derived from an ethogram. An *ethogram* is a set of descriptions of behaviour patterns of a species [2], which is fundamental in ethology [43]. In this work, an ethogram is represented by a decision system that provides a record of observations of episodic behaviour of a swarm. The form of ethogram in Table 1 was introduced in [33,35] and elaborated in [31,34]. An *episode* is a sequence of states that terminates. During a swarm episode, an ethogram table is constructed, which provides the basis for an approximation space such as the one represented in Fig. 3.

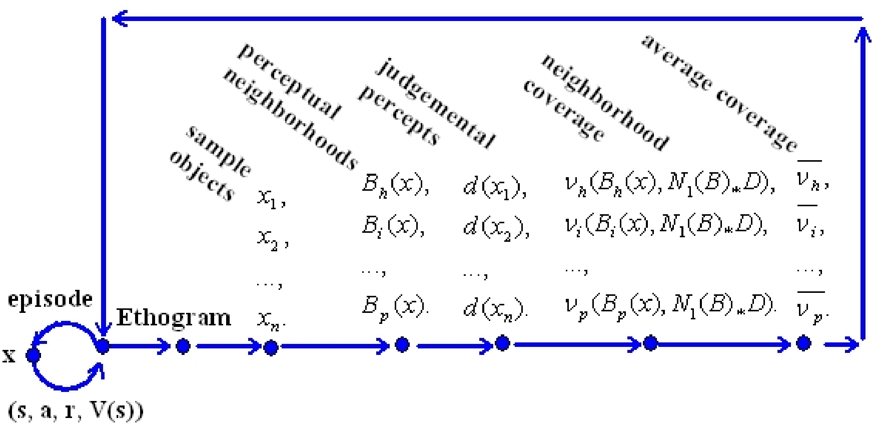


Fig. 3. Approximate Adaptive Learning Cycle

Let $s, a, p(s,a), r$ denote the state, action, action preference and reward associated with a previous action by an observed organism. Define a *behaviour* to be a collection $(s, a, p(s,a), r)$ at any one time t, and let d denote a decision (1 = choose action, 0 = reject action) for acceptance of a behaviour. Let $U_{beh} = \{x_0, x_1, x_2, \ldots\}$ denote a set of behaviours. Decisions to accept or reject an action are made by the actor during the learning process; let d denote a decision (0=reject, 1=accept). Often ethograms also exclude $p(s,a)$ or include a column for "proximate cause" (see [43]). Let $S = \{k, \ell\}$ be the collection of two states, and let $A = \{i, j, k\}$ be the set of possible actions, with $A(k) = \{h, i\}$, $A(\ell) = \{i, j\}$.

The calculations are performed on the feature values shown in the first four columns of Table 1. Put $B = \{s, a, p(s,a), r\}$. Let $U_{beh} = \{x_0, x_1, \ldots, x_9\}$ and let $D = \{x \in U : d(x) = 1\} = \{x_0, x_3, x_4, x_6, x_8\}$ be the decision class. Then

Table 1. Sample Ethogram

x_i	s	a	$p(s,a)$	r	d
x_0	k	h	0.0	0.75	1
x_1	k	i	0.0	0.75	0
x_2	ℓ	i	0.0	0.1	0
x_3	ℓ	j	0.0	0.1	1
x_4	k	h	0.0	0.75	1
x_5	k	i	0.0	0.75	0
x_6	ℓ	i	0.010	0.9	1
x_7	ℓ	j	0.025	0.9	0
x_8	k	h	0.01	0.75	1
x_9	k	i	0.056	0.75	0

Case 1. N_1**, 1-Feature Neighborhoods**

Let $D = \{x \in \ Object \mid d(x) = 1\} = \{x_0, x_3, x_4, x_6, x_8\}$, $B = \{s, a, p(s,a), r\}$, and observe

$B_{s_k}(x_0) = \{x_0, x_1, x_4, x_5, x_8, x_9\}$, $B_{s_\ell}(x_2) = \{x_2, x_3, x_6, x_7\}$,
$B_{a_h}(x_0) = \{x_0, x_4, x_8\}$, $B_{a_i}(x_1) = \{x_1, x_2, x_5, x_6, x_9\}$, $B_{a_j}(x_3) = \{x_3, x_7\}$,
$B_{p_{0.0}}(x_0) = \{x_0, x_1, x_2, x_3, x_4, x_5\}$,
$B_{p_{0.01}}(x_6) = \{x_6, x_8\}, B_{p_{0.025}}(x_7) = \{x_7\}, B_{p_{0.056}}(x_9) = \{x_9\}$
$B_{r_{0.1}}(x_2) = \{x_2, x_3\}$, $B_{r_{0.75}}(x_0) = \{x_0, x_1, x_4, x_5, x_8, x_9\}$, $B_{r_{0.9}}(x_6) = \{x_6, x_7\}$,

$(N_1(B))_* D = B_{a_h}(x_0) \cup B_{p_{0.01}}(x_6) = \{x_0, x_4, x_6, x_8\}$,
$(N_1(B))^* D = B_{s_k}(x_0) \cup B_{s_\ell}(x_2) \cup B_{a_h}(x_0) \cup B_{a_i}(x_1) \cup B_{a_j}(x_3) \cup B_{p_{0.0}}(x_0) \cup B_{p_{0.01}}(x_6) \cup B_{p_{0.75}}(x_0) \cup B_{r_{0.1}}(x_2) \cup B_{r_{0.9}}(x_6) = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$,
$BND_{N_1(B)} D = \{x_1, x_2, x_5, x_7, x_9\}$.

Using $(N_1(B))_* D$ together with each block $B_{f_v}(x), f \in B, f(x) = v$, we obtain

$\nu_{s_k}(B_{s_k}(x_0), (N_1(B))_* D) = \frac{3}{4}$, $\nu_{s_\ell}(B_s(x_2), (N_1(B))_* D) = \frac{1}{4}$,
$\nu_{a_h}(B_{a_h}(x_0), (N_1(B))_* D) = \frac{3}{4}$, $\nu_{a_i}(B_{a_i}(x_1), (N_1(B))_* D) = \frac{1}{4}$,
$\nu_{a_j}(B_{a_j}(x_3), (N_1(B))_* D) = 0$,
$\nu_{p_{0.0}}(B_{p_{0.0}}(x_0), (N_1(B))_* D) = \frac{1}{2}$, $\nu_{p_{0.01}}(B_{p_{0.01}}(x_6), (N_1(B))_* D) = \frac{1}{2}$,
$\nu_{p_{0.025}}(B_{p_{0.025}}(x_7), (N_1(B))_* D) = 0$, $\nu_{p_{0.056}}(B_{p_{0.056}}(x_9), (N_1(B))_* D) = 0$,
$\nu_{r_{0.1}}(B_{r_{0.1}}(x_2), (N_1(B))_* D) = 0$, $\nu_{r_{0.75}}(B_{r_{0.75}}(x_0), (N_1(B))_* D) = \frac{3}{4}$,
$\nu_{r_{0.9}}(B_{r_{0.9}}(x_6), (N_1(B))_* D) = \frac{1}{4}$.

Recently, we have found that lower coverage obtained in this manner has proved to be useful in solving image pattern recognition problems (see, *e.g.*, [10]). Next, we obtain the average lower coverage for each feature, which indicates that features a and r are more important than s and $p(s,a)$ (it happens that this matches our tuition about the content of an ethogram, where actions and rewards have greater weight).

$\overline{\nu_s} = \frac{\frac{3}{4}+\frac{1}{4}}{2} = 1$, $\overline{\nu_a} = \frac{\frac{3}{4}+\frac{1}{4}+0}{3} = 0.3$, $\overline{\nu_p} = \frac{\frac{1}{2}+\frac{1}{2}+0+0}{4} = 0.25$ $\overline{\nu_r} = \frac{0+\frac{3}{4}+\frac{1}{4}}{3} = 0.3$.

In sum, notice that all of the features are used to construct families of neighborhoods, but not in the usual way, since features are considered separately to construct feature-based neighborhoods. The lower and upper approximations are obtained by taking into account feature-based families of neighborhoods. The set D is both a near set as well as a rough set. This does not always happen. Average lower coverage has proved to be useful in reinforcement learning (see, *e.g.*, [34,31]). In the remaining cases, only the approximation sets and boundary set are given.

Remark 2. **Significance of the Lower Approximation**
This goes back to Archimedes, who suggested approximating the unknown area of a bounded region in the plane by summing the areas of all of the small rectangles entirely contained inside the bounded region. Each rectangle inside the bounded region is well-understood, since we know that it is inside the bounded region, *i.e.*, there is no part of an inner rectangle that is outside the bounded region (we know an inner rectangle belongs entirely inside the bounded region). Also notice that the bounded region provides a basis for evaluating all rectangles, those inside, overlapping or entirely outside the bounded region. Analogously, each perceptual neighborhood $B_j(x)$ contained in the lower approximation of a set D is well-understood because the objects in $B_j(x)$ are entirely contained inside the set of perceptual judgements D, assuming that $D = \{x \mid d(x) = 1,\ \textit{i.e., accept behaviour associated with } x\}$. That is, based on knowledge represented by $B_j(x)$, the sample objects in $B_j(x) \subseteq D$ are known to have acceptable behaviours. For this reason, $(N_1(B))_*D$ can be used as a norm or standard in evaluating all of the perceptual neighbourhoods gathered together during an episode. That is, we can measure the extent that the objects in each perceptual neighbourhood overlap with the acceptable objects in $(N_1(B))_*D$.

Case 2. N_2**, 2-Feature Neighborhoods**
$(N_2(B))_*D =$
$B_{sa}(x_0) \cup B_{sp}(x_6) \cup B_{sp}(x_8) \cup B_{ap}(x_0) \cup B_{ap}(x_3) \cup B_{ap}(x_6) \cup B_{ap}(x_8) \cup B_{pr}(x_6) \cup B_{pr}(x_8) =$
$\{x_0, x_3, x_4, x_6, x_8\}$,
$(N_2(B))^*D =$
$B_{sa}(x_2) \cup B_{sa}(x_3) \cup B_{sp}(x_0) \cup B_{sp}(x_2) \cup B_{pr}(x_0) \cup B_{pr}(x_2) \cup B_{sr}(x_0) \cup B_{sr}(x_2) \cup B_{sr}(x_6) \cup$
$B_{sr}(x_8) = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$,
$BND_{N_2(B)}D = \{x_1, x_2, x_5, x_7, x_9\}$.

Case 3. N_3**, 3-Feature Neighborhoods**
$(N_3(B))_*D =$
$B_{sap}(x_0) \cup B_{sap}(x_3) \cup B_{sap}(x_6) \cup B_{sap}(x_8) \cup B_{apr}(x_0) \cup B_{apr}(x_3) \cup B_{apr}(x_6) \cup B_{apr}(x_8) \cup$
$B_{spr}(x_0) \cup B_{spr}(x_3) \cup B_{spr}(x_6) \cup B_{spr}(x_8) \cup B_{sar}(x_0) \cup B_{sar}(x_3) \cup B_{sar}(x_6) =$
$\{x_0, x_3, x_4, x_6, x_8\}$,
$(N_3(B))^*D = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_8\}$,
$BND_{N_3(B)}D = \{x_1, x_2, x_5\}$.

Case 4. N_4**, 4-Feature Neighborhoods**
$(N_4(B))_*D = (N_4(B))^*D =$
$B(x_0) \cup B(x_3) \cup B(x_6) \cup B(x_8) = \{x_0, x_3, x_4, x_6, x_8\}$,
$BND_{N_4(B)}D = \emptyset$.

This case is interesting because D is a near set but not a rough set.

In sum, D is a near set as well as a rough set in cases 1, 2 and 3. D is a near set but not a rough set in case 4 and 4 (quadruple feature families of neighborhoods). The lower approximation in several cases equals D, which means the objects in D are known with certainty for certain but not all feature combinations.

6 Conclusion

It is Zdzisław Pawlak's original 1981 paper on classification of objects by means of attributes that has led to the introduction of near sets and the proposed approach to object recognition. In this approach, the focus is on the comparison between families of perceptual neighborhoods containing observed sample objects with matching descriptions and perception granules representing a standard. The standard we have in mind is the lower approximation of a set of sample objects representing perceptual judgements, *i.e.*, objects judged to be acceptable. This has led to a refinement of the generalized approximation space model to include families of neighborhoods. Object recognition is defined in terms of a measure of the degree of overlap between perceptual neighborhoods and a set of objects constituting a standard. The *feature identification* and *feature extraction* are currently the subject of intense research in connection with solving object recognition, ethology and reinforcement learning problems. It is conjectured that near sets will be useful in solving a number of object recognition problems.

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