Modern statistical methods for accessing the hardening process of concrete

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1. Introduction

In order to investigate the probability of cracking of hardening concrete in terms of the maximal allowable crack width, a full-scale concrete element has been evaluated. For this, evaluation, In order to investigate this issue, a concrete element has been simulated which measures a thickness of 1m and a height of 3m. The length deformations of the wall are considered to be fully restraint by 100%. The ambient conditions are taken into account, as well as the concrete mix parameters (see Table 1). TEMPSPAN, which is an acronym for TEMPeratuur (temperature) and SPANningen (stresses), is used to calculate the hardening stresses which develop during hardening.

2. Cracking criterions

Cracking criterions are used to define the moment of cracking of a hardening concrete element. A very commonly used criterion is the strength criterion. This criterion describes the ratio between the actual tensile stress and the actual tensile strength at the moment of cracking:

$$\xi = \frac{\sigma}{f_{ctm}} \tag{1}$$

For the tensile splitting strength of a normal strength concrete, Lokhorst [1] developed an experimentally-based refinement for this criterion For this he discovered that the uniaxial tensile strength is 0.9 * tensile splitting strength:

$$f_{ctm} = 0.9 * f_{ctm,sp} \tag{2}$$

Where: f_{ctm} = uniaxial tensile strength
$f_{ctm,sp}$ = mean concrete tensile splitting strength

With accounting for the rate of loading, this criterion holds:

$$\text{Cracking criterion} = 0.9 \cdot f_{ctm,sp,long} = 0.9 \cdot 0.85 \cdot f_{ctm,sp,short} \cong 0.75 \cdot f_{ctm,sp} \quad (3)$$

Experiments (on plain concrete) have shown a stress/strength ratio at failure ranging between 0.75 for slow loading and 0.88 for fast loading. For high strength reinforced concrete, Sule [2] developed a cracking criterion, based on the 5% failure criterion. For this she also accounted for the rate effects and assumed a standard deviation of 0.09 times the standard deviation:

$$\text{Cracking criterion} = 0.6 \cdot f_{ctm,sp} \quad (4)$$

In practise an often used criterion is:

$$\text{Cracking criterion} = 0.5 \cdot f_{ctm} = 0.45 \cdot f_{ctm,sp} \quad (5)$$

3. Level II: First Order Second Moment Method (FOSM)

In order to investigate the influence of different cracking criterions on the required amount of reinforcement a Level II First Order Second Moment Method (FOSM) and a Monte Carlo approach has been conducted. The average results of the hardening stress and strength development calculated with TEMPSPAN, representing an element in the middle of the wall, are provided in Fig. 1.

Due to its simplicity, the so called first order second-moment method is a very frequently used method for calculating the probability of cracking. This method uses the first two moments, i.e. mean value and standard deviation, which represent the stresses and strengths of the hardening process. Assuming the resistance R

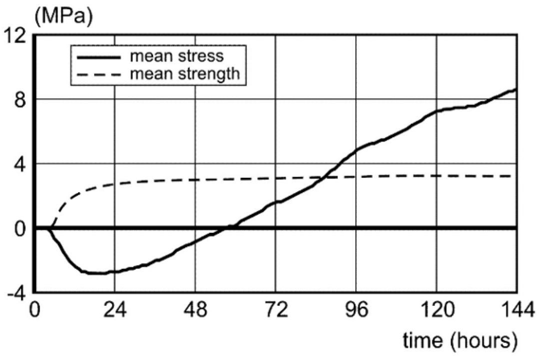

Fig. 1. Mean stress and strength

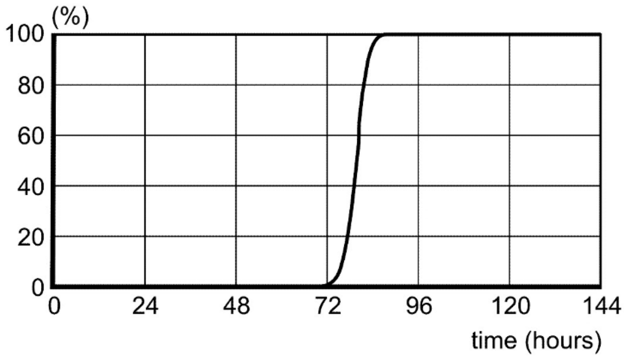

Fig. 2. Probability of failure.

and the load S to be the second-moment random variables which exhibit a normal distribution with coefficients of variation of 10% and 8%, respectively, and that cracking occurs, at an average stress level of 75% of the actual tensile splitting strength, the probability of failure of the first crack occurrence is:

$$P_f = P_f\left\{u < \frac{0-\mu_Z}{\sigma_Z}\right\} \tag{6}$$

Where:

$$\sigma = \mu \cdot V; \qquad \sigma_R = (0.75 \cdot f_{ct}) \cdot V_R; \qquad \sigma_S = \sigma_{ct} \cdot V_S$$

$$\mu_Z = \mu_R - \mu_S; \qquad \mu_Z = 0.75 \cdot f_{ct} - \sigma_{ct}; \qquad \sigma_Z = \sqrt{\sigma_R^2 + \sigma_S^2} \tag{7}$$

For this particular case, without any variation, the element in the center of the wall will crack 83 hrs after casting (see Fig. 2). With variations for the stress and strength assumed to be 10% and 8%, the probability of cracking, starts to increase from 0% after 76 hrs to 100% after 90hrs (figure 2).

4. Crack width calculations

When assuming a not completely developed cracking pattern, the crack width can be calculated from the strain difference between the concrete and the reinforcement at the moment cracking of the concrete. For the maximum crack width, it holds that:

$$w_{max} = \frac{\phi}{4 \cdot \omega_s \cdot E_s} \cdot \left[\sigma_{scr} - \frac{f_{ctm}}{2}\left(\frac{1}{\omega_s} + n\right)\right] \tag{8}$$

in which:

ϕ: – diameter of the reinforcement bars [mm]
ω_s: – reinforcement ratio [%]
E_S: – modulus of elasticity of the reinforcement [MPa]
σ_{scr}: – steel stress in the crack [MPa]
f_{ctm}: – tensile strength of the concrete [MPa]
n: – ratio between elastic modulus and reinforcement [–]

The steel stress in the crack can be calculated by:

$$\sigma_{scr} = f_{ctm} \cdot \left(\frac{1}{\omega_s} + n \right) \tag{9}$$

For example, for an actual tensile strength (2.5 MPa) and actual modulus of elasticity at the moment of cracking (36,7 GPa), the reinforcement ratio, needed to control the maximum crack width at 0.20mm, is calculated at a value of 0.966% for reinforcement bars of ϕ12mm. With an adopted cracking criterion was acc. (3).

For a representative range of cracking criterions, ranging from 0.5 to 0.8, the minimum required reinforcement has been calculated. The results are presented in figure 3. For lower values of the cracking criterion (= 0.75 * $f_{ctm,sp}/\sigma$), lower values oft the reinforcement ratio, needed to obey the ultimate crack width of 0.20mm[1]. This can be attributed to the lower tensile strength and modulus of elasticity at younger ages of the concrete (see Fig. 3). So, in case it is uncertain which cracking criterion should be allowed, it is recommended use a higher cracking criterion, which implicitly results in a higher reinforcement ratio.

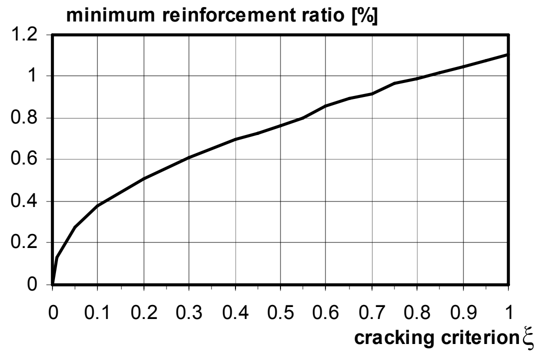

Fig. 3. Minimum required reinforcement ratio's for different cracking criterions.

[1] For this calculations an arbitrary value for the ultimate crack width has been adopted.

5. Level III: Monte Carlo approach (MC)

The Level III probability of failure is determined by means of a Monte Carlo approach. With this calculation procedure, all probability density functions of all strength and load variables are considered. It links the cracking reliability of an element directly to the probability density functions of the stochastic input parameters. Results of previous tests (and other data) can be used to establish the probability density functions of the input parameters of the early age cracking problem. This probability distribution knowledge can than be used to generate samples of the numerical data. A flow chart of a Crude Monte Carlo approach as used in [3,4] is shown in Fig. 4. The flowchart also indicates the level of the partial safety factor (PSF) and the level of the first order second moment (FOSM).

The simulation results are used to estimate a probability of failure of a particular sample. Since all input parameters are considered as random variables, the estimated probability itself can be treated as a random variable as well [5]. The uncertainty in the estimation of the probability decreases as the total number of simulations increases. For a required reliability of 95% (Vp=0.05) and a maximum relative error of 0.1, the required number of simulations should exceed [5, 6]:

$$n > 400 \cdot \left(\frac{1}{P_f} - 1 \right) \tag{10}$$

For an arbitrary estimated average probability of failure of 0.5, the number of simulations required for this calculation should be at least n > 400. With 28 random distributed variables (See Table 1), this number becomes 400*28=11200 uniformly distributed random numbers.

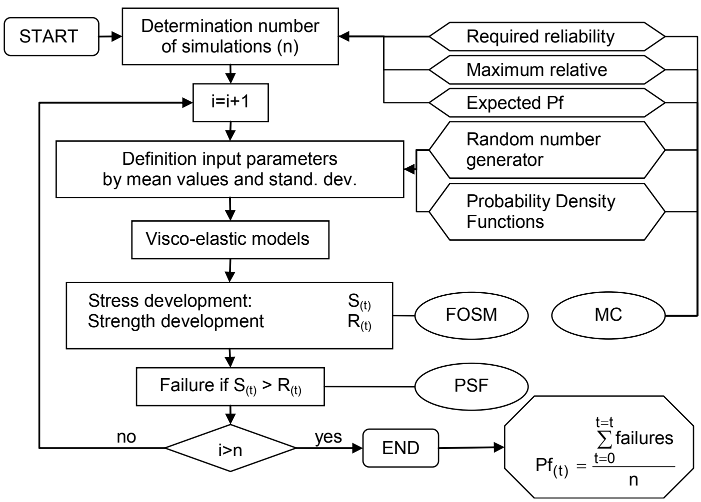

Fig. 4.: Flowchart of the Monte Carlo approach.

Table 1. Input data for the Monte Carlo Approach

Variable	Mean value	Stand. Dev.	Variable	Mean value	Stand. Dev.
Material parameters			**External parameters**		
dens_gravel [kg/m^3]	2650	26.5	windspeed [m/s]	2	0.2
dens_sand [kg/m^3]	2650	26.5	mean_surr. temp [K]	293	2
dens_cement [kg/m^3]	3150	31.5	ampl_surr. temp [K]	10	2
Ea [KJ/mol]	45.7	4.113	initial_concrete_temp [K]	293	2
alpha_c [K^{-1}]	1.2*10^{-5}	4.8*10^{-7}	construction_width [m]	1.00	0.005
Q_max [kJ/kg cement]	440	9.4	construction_height [m]	3.00	0.005
Density_concrete [kg/m^3]	2500	25	Restraint [-]	1.00	0.1
lambda_formwork [W/mK]	0.17	0.017	d_formwork [m]	0.02	0.001
R [J/mol.K]	8.315	0.8315	**Calculation parameters**		
Mix parameters			d_age [-]	0.35	0.035
Air [%]	1	0.1	n_tension [-]	0.30	0.03
Gravel [kg]	695	6.95	n_compression [-]	0.30	0.03
Sand [kg]	1236	12.36	Emod_aggregate [MPa]	55000	5500
Cement [kg]	350	3.5	Emod_particle[MPa]	55000	5500
Water [kg]	150	1.5	E_fictitious [MPa]	31000	3100

The simulations performed by the level III calculation results in sets of data, which consist of a cracking time, tensile strength of the concrete at the cracking time, modulus of elasticity of the concrete at the cracking time and other data. This data can be used to the calculated crack width for each run (acc. to eq. 8). With these crack widths, a probability function can be constructed. For this function, most frequently used cracking criteria (ξ) (from 0.5 to 0.75) are adopted, representing probabilities of failure of 1^{-3} and 0.5, respectively (see Fig. 5) [1]. The results of these probabilities of crack exceedance are provided in Fig. 6. From the results it can be observed that when assuming an arbitrary allowable maximum crack width of 0.2 mm and when applying the crack-width as provided by equation (8), the probability of exceedance curves show that there still is a substantial risk that larger cracks will be found in a structure. Especially for the lower stress/strength ratios, i.e. 0.5 (cracking criterion ξ), the probability on larger cracks increases substantially. This shows that in case of critical structures, viz. water tight structures, the cracking criterion in relation to the desired maximum crack width should be treated carefully.

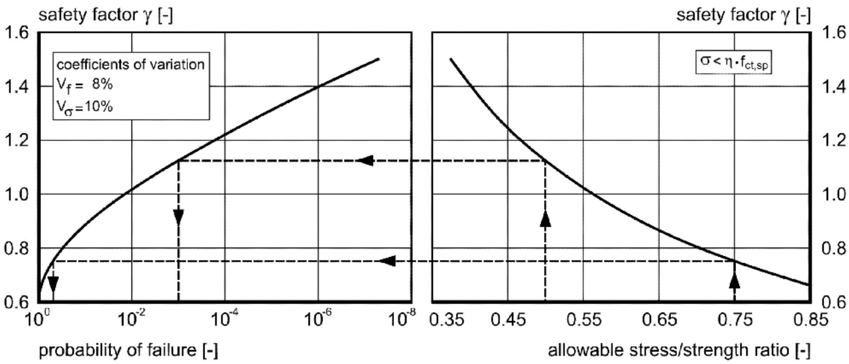

Fig. 5. Probability of failure versus maximum allowable stress/strength ratio

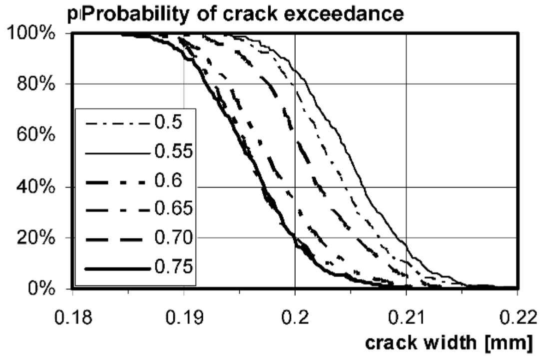

Fig. 6. Probability of crack exceedance for different levels of crack criterions.

6. Conclusions

For lower cracking criterion, less reinforcement is required for obeying the allowable cracking width. Whenever high reinforcement ratios are available in a structure, a relatively high allowable level of the cracking criterion can be adopted as well.

A Monte Carlo approach can be applied to calculate the crack width distributions for hardening concrete elements. The probability of exceeding provides the opportunity to calculate the ultimate crack width that can likely develop in a structural element.

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