A Novel Fractal Image Coding Based on Quadtree Partition of the Adaptive Threshold Value

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Abstract. Fractal image coding is a novel technique for still image compression. Compared with the distance between the range block and the matching domain block, setting of the initial threshold value is one of the most difficult problems in Fisher Quadtree-based fractal image coding. In this paper, a novel fractal image coding based on Quadtree partition of the adaptive threshold value is proposed. Considering the input image feature fully, we put forward the computation derivation process of the adaptive threshold value progressively and declare that the adaptive threshold value has the direct proportion with the variance of the current range block. Experimental results show that compared with Fisher Quadtree-based fractal coding for the same image, the proposed coding scheme obtains better performance including the improved quality of the decoded image, shorter compression time and higher compression ratio.

Keywords: fractal image coding, Quadtree partition, adaptive threshold.

1 Introduction

Fractal image compression, which is based on the IFS (Iterated Function System) proposed by Barnsley[1], is a novel approach to image coding. Its performance relies on the presence of self-similarity between the regions of an image. Since most images process a high degree of self-similarity, fractal compression contributes an excellent tool for compressing then[2-3]. Recently, there are several methods[4] subsequently proposed to improve the performance of fractal image compression. In the range and domain block mapping, several other functions have been proposed in the literatures. Besides, various approaches are also proposed to reduce the searching within the domain pool. Among all fractal block coding schemes, the technique of variable-size blocking is included to compromise the compression ratio and the level of quality. Range block segmentation is important to code image for saving bit rate. Quadtree segmentation is a common method to partition image, since its flexibility and less overhead. In Fisher Quadtree-based fractal image coding[5-6], the threshold value compared with the distance between the range block and the searching domain block is setup by manual experimental experience fixedly. Hence experimental results will be greatly influenced via the threshold minor variety. How to get the adaptive threshold value corresponding to the current range block is one of the most difficult problems in the fractal image coding.

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In this paper, a novel fractal image compression based on Quadtree partition of the adaptive threshold value is proposed. Considering the input image feature fully, we put forward the computation derivation process of the adaptive threshold value progressively and declare that the adaptive threshold value has the direct proportion with the variance of the current range block. Experimental results show that we improve the performance of Quadtree segmentation by adapting the threshold value among each level of the Quadtree.

The balance of the paper is organized as follows: theoretical foundations including basic fractal image coding and Fisher Quadtree-based coding are stated in Section 2. Proposed methodology is described in Section 3. Experimental results and discussion is reported in Section 4. Conclusion is included in Section 5.

2 Theoretical Foundations

2.1 Basic Fractal Image Coding

Fractal image coding makes good uses of image self-similarity in space by ablating image geometric redundant. Fractal coding process is quite complicated but decoding process is very simple, which makes use of potentials in high compression ratio. The main theory of fractal image coding is based on Local Iterated Function System, attractor theorem, and collage theorem. Regard original compressible image as attractor, how to get LIFS parameters is main problem of fractal coding.

We explain the basic procedure for the fractal image coding [7].

1. A given image I is divided into non-overlapping M range blocks of size $B \times B$ and into arbitrarily located N domain blocks of size $2B \times 2B$. The range blocks are numbered from 1 to M, and represented by $R_i(1 \le i \le M)$. Similarly, the domain blocks are from 1 to N, and represented by $D_j(1 \le j \le N)$.
2. For each range block R_i, the best matched domain $D_k(1 \le K \le N)$ and an appropriate contractive affine transformation τ_{ik} which satisfy the following equation are found through

$$d(R_i, \tau_{ik}(D_k)) = \min d(R_i, \tau_{ij}(D_j)) \tag{1}$$

Where τ_{ij} is an contractive affine transformation from the domain block D_j to the range block R_i; the distortion measure $d(R_i, \tau_{ij}(D_j))$ is the Mean Square Error (MSE) between the range block R_i and the contractive domain block $\tau_{ij}(D_j)$. The contractive affine transformation τ_{ij} is composed of two mappings ϕ_j and θ_{ij} as follows:

$$\tau_{ij} = \theta_{ij} \circ \phi_j \tag{2}$$

The first mapping ϕ_j is the transformation of domain-block size to the same size as range blocks. This transformation can be described as follows: The domain block

D_j is divided into non-overlapping unit blocks of size 2×2; and each pixel value of the transformed block $\phi_j(D_j)$ is an average value of four pixels in each unit block in D_j. The second mapping θ_{ij} consists of two steps: The first step transforms the block $\phi_j(D_j)$ by one of the following eight transformations: rotation around the center of the block $\phi_j(D_j)$, through 0^0,$+90^0$,$+180^0$, and $+270^0$, and each rotation after orthogonal reflection about mid-vertical axis of the block $\phi_j(D_j)$.Those eight transformations are called isometries. The second step is the transformation p_{ij} of pixel values of a block obtained by the first step. This transformation p_{ij} is defined as

$$p_{ij}(v) = s_{ij}v + h_{ij} \tag{3}$$

where v is a pixel value of the block obtained by the first step, and the parameters s_{ij} and h_{ij} are computed by the least square analysis of pixel values of the range block R_i and the block obtained by the first step. We call the parameters s_{ij} and h_{ij} a scaling coefficient and an offset, respectively.

The LIFS parameters listed below are encoded:

(1) Parameters to indicate a location of the best matched domain block;
(2) A parameter to indicate an isometric on the best matched domain block;
(3) A scaling coefficient and an offset.

The proposed method quantizes these LIFS parameters [7].

2.2 Fisher Quadtree-Based Fractal Image Coding

In the basis automatic fractal image coding, they cannot represent the image features well because an image is divided into the blocks of a fixed size. In fact, if a larger block is used, it is difficult to find a good matching for areas, which have fine details and complex regions such as Lena's eye. It will lead to loss in quality in the decoding image. On the other hand, if a smaller block is used, although a good matching can be found, as many smooth areas such as Lena's background are divided to many blocks while they are well fitted with a larger block, the number of blocks will increase. Hence, it causes less compression ratio as well as larger computation time.

Jacquin and Fisher gained a good deal of enlightenment from the Quadtree method of a grey image and solved the above problem by means of the Quadtree method to separate such an image[5-7]. Quadtree segmentation expresses an image with a tree structure[8]. Fig.1 shows this spatial structure. On the top it is a father node which has 4 son-nodes corresponding to 4 blocks of image. And each son-node has 4 son-nodes of its own which maps to the 4 quadrants of next subblock image. The root of Quadtree is original image. Fig.2 shows quadrants of a block and their labels. Fig.3 shows the divided image and its corresponding Quadtree.

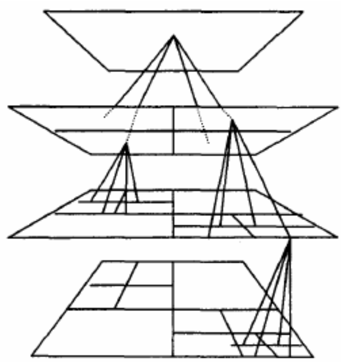


Fig. 1. Image quadtree segmentation

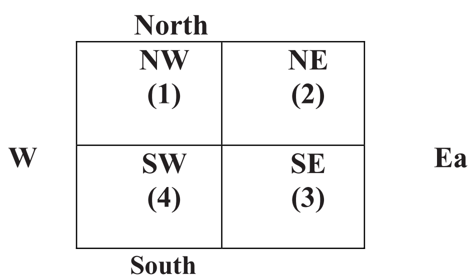
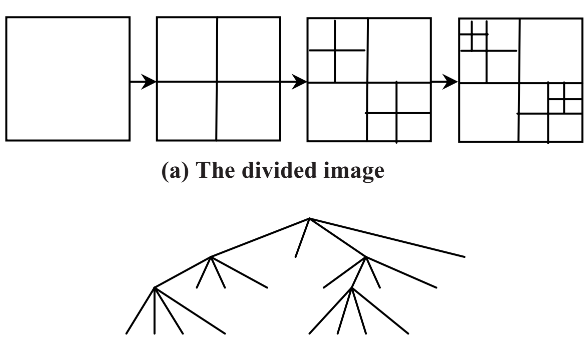


Fig. 2. Quadrants of a block and their labels

Fisher Quadtree-based fractal image coding is described as followed:

Before division, we first set maximal and minimal depth of the Quadtree and a maximal allowable fixed threshold to decline the number of range blocks. Then we continuously partition a range into four square ranges of the same size by the Quadtree



(a) The divided image

(b)Corresponding Quadtree

Fig. 3. The divided image and its Quadtree

method until minimal depth is met. An optimum matching block will be can be marked as D_j and the range corresponding with it can be marked as R_i and the partition is not done again. Otherwise they are further partitioned into four ranges. This process continues until the minimal depth is met.

3 Proposed Methodology

In Fisher Quadtree-based fractal image coding, the fixed threshold value decides the number of the range block partition and affects coding efficiency directly. The larger value the fixed threshold is, the more numbers range blocks with large size has, and compression ratio improves while PSNR of the decoded image decreases. The smaller value the fixed threshold is, the less numbers range blocks with large size has, and PSNR of the decoded image improves while compression ratio decreases. The threshold value is the key to Fisher Quadtree-based fractal image compression[9].

Considering the input image feature fully, we put forward the computation derivation process of the adaptive threshold value progressively and declare that the adaptive threshold value has the proper proportion with the variance of the current range block. Some basic definitions is described as following:

Average grey value of the range block R_i is defined as $\bar{a}$:

$$\bar{a} = \frac{1}{N}\sum_{i=0}^{N-1} a_i \tag{4}$$

Average grey value of the domain block D_i is defined as $\bar{b}$:

$$\bar{b} = \frac{1}{N}\sum_{i=0}^{N} b_i \tag{5}$$

Variance of the range block R_i is defined as σ_r^2 :

$$\sigma_r^2 = \frac{1}{N}\sum_{i=0}^{N-1} (a_i - \bar{a})^2 \tag{6}$$

Variance of the range block D_i is defined as σ_d^2 :

$$\sigma_d^2 = \frac{1}{N}\sum_{i=0}^{N-1} (b_i - \bar{b})^2 \tag{7}$$

Covariance between the range block and the domain block is defined as $\mathrm{cov}(a,b)$:

$$\mathrm{cov}(a,b) = \frac{1}{N}\sum_{i=0}^{N-1} (a_i - \bar{a})(b_i - \bar{b}) \tag{8}$$

The distance of Mean Square Error (MSE) between the range block and the searching domain block is defined as $dis(a,b)$:

$$dis(a,b)=\frac{1}{N}\sum_{i=0}^{N-1}[a_i-(sb_i+h)]^2 \tag{9}$$

Relative covariance between the range block and the domain block is defined as $\mathrm{cov}_R^2(a,b)$:

$$\mathrm{cov}_R^2(a,b)=\frac{\mathrm{cov}^2(a,b)}{\sigma_d^2} \tag{10}$$

From the given definition, conclusion is obtained that according to a given range block, function $dis(a,b)$ the distance between the range block and the searching domain block is relative with $\mathrm{cov}_R^2(a,b)$ and also satisfied with form,

$$dis(a,b)=\sigma_r^2-\mathrm{cov}_R^2(a,b) \tag{11}$$

Here, for each of the range block, σ_r^2 is a constant and function $dis(a,b)$ has only be relative with $\mathrm{cov}_R^2(a,b)$. Moreover, function $dis(a,b)$ has relation with the function $f(x)=m\cdot e^{-k^2x^2}\ (a>0,k>0)$ in close coordination. When condition is included as follows: $m=\sigma_r^2, mk^2=1$, $dis(a,b)$ is main component for the Maclaurin expansion of function $f(x)$.

$$f(x)=m-mk^2x^2+R_n(x) \tag{12}$$

Here, $R_n(x)$ is the rest component for the Maclaurin expansion of function $f(x)$. Given $m=\sigma_r^2, mk^2=1, x=\mathrm{cov}_R(r,d)$, we can get form

$$f(cov_R(a,b))=\sigma_r^2-cov_R^2(a,b)+R_n(cov_R(a,b))=dis(a,b)+R_n(cov_R(a,b)) \tag{13}$$

Form (13) shows the threshold of $dis(a,b)$. Fig.4 shows function $f(x)$ has relation with the function $dis(a,b)$.

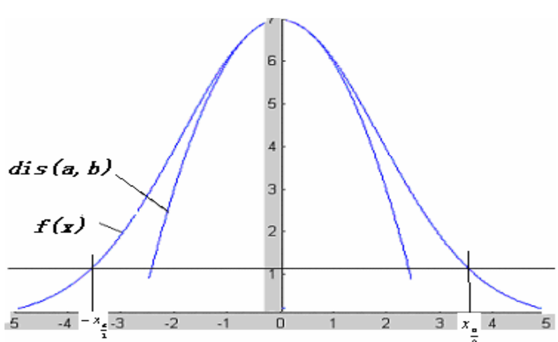


Fig. 4. Function f(x) relation to dis(a,b)

During the interval range $(-\infty,+\infty)$, definite integral of *f(x)* is $\int_{-\infty}^{=\infty} f(x)dx = \frac{m}{k}\sqrt{\pi}$. In probability theory , believe metric of $(1-\alpha)$ of the function *f(x)* is $\int_{-x_{\frac{a}{2}}}^{x_{\frac{a}{2}}} f(x)dx = (1-a)\int_{-\infty}^{+\infty} f(x)dx$. So $f(x_{\frac{a}{2}}) = \sigma_r^2 e^{-\frac{x_{\frac{a}{2}}^2}{2}}$. Finally we regard $f(x_{\frac{a}{2}}) = \sigma_r^2 e^{-\frac{x_{\frac{a}{2}}^2}{2}}$ as the adaptive threshold of $dis(a,b)$.

4 Experimental Results

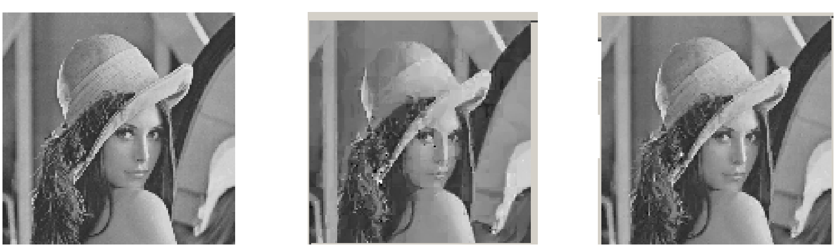
All the experiments are carried out on a computer with Intel 2.5Ghz and 512M RAM in the Win2000 professional operating system and VC6.0 language is used. Original image is classical 128×128 grey-level Lena face image coded with 8 bits per pixel.

An optimal bit allocation strategy is as follows: 14 bits for the location of the matched domain block (horizontal and vertical coordinate), 3 bits for isomorphic types, 5 bits for contrast scaling and 7 bits for the offset, 3 bit for the depth of the Quadtree. For each of the range block, fractal code includes 32 bits allocation via writing into a text file as a fractal coding file. During the iteration process of the image decoding, those grey value either exceeding integer 255 or less than integer 0 is replaced by the average of its four neighbors to avoid block diverging.

In Fisher Quantree-based fractal image coding, our setting is that the maximal range size is 64×64, and the minimal range size is 2×2.The fix threshold value is setup up by real number 0.5 and 0.2 respectively. Table.1 shows the detail experimental data and Fig.5 shows the decoded image of 10 iteration.

We can see from the experimental data. The larger value fixed threshold is, the more numbers range blocks with large size has, then compression ratio improves while PSNR of the decoded image decreases. The smaller value fixed threshold is, the less numbers range blocks with large size has, then PSNR of the decoded image improves while compression ratio decreases. For obtaining the matching domain of Lena face image, range blocks of eye and fair regions are partitioned into small size and range blocks of shoulder and background are partitioned into large size.

In our coding scheme (fractal image compression based on Quadtree partition of the adaptive threshold value), the maximal and the minimal range block size is as above. The believe metric is setup up by real number 0.92 and 0.96 respectively.



(1)original image, (2)decode image with 0.5 theshold, (3)decoded image with 0.2 threshold

Fig. 5. Fisher Quadtree-based fractal image coding

Table 1. Fisher Quadtree-based fractal image coding

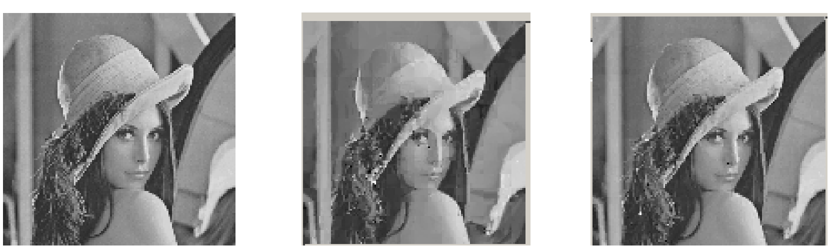
Fixed threshold	0.5	0.2
Different size Range numbers (maximal range size 64*64, minimal range size 2*2)	Total Range number: 460 4*4 range number: 292 8*8 range number: 163 16*16 range number: 5	Total range number: 1021 2*2 range number: 340 4*4 range number: 595 8*8 range number: 86
Coding time	55 S	135 S
Compression ratio	9.82:1	4.42:1
PSNR	25.8	32.2

Table.2 shows the detail experimental data of our coding scheme and Fig.6 shows the decoded image of 10 iteration of our coding scheme.

We can see from the experimental data. For each range block, the adaptive threshold has direct proportion with its variance. The larger variance of the range block is, the larger adaptive threshold is, and the range block may be a midrange or an edge block. The smaller variance of the range block is, the smaller adaptive threshold is, and the range block may be a smooth or shade block. Hence, our coding scheme can be better adapted with human vision characteristic and obtains less distortion of the decoded image. Compared with Fisher coding for the same image, the proposed scheme obtains better performance including the improved quality of the decoded image, shorter compression time and higher compression ratio.

Table 2. Our coding scheme

Believe metric	0.92	0.96
Different size Range numbers (maximal range size 64*64, minimal range size 2*2)	Total range number:420 4*4 range number: 272 8*8 rang number: 144 16*16 range number: 4	Total range number: 986 2*2 range number: 321 4*4 range number: 603 8*8 range number: 62
Coding time	49 S	128 S
Compression ratio	9.91:1	4.62:1
PSNR	26.9	33.6



(1) original image, (2)decoded image with 0.92 believe metric, (3)decoded image with 0.96 believe metric

Fig. 6. Our coding scheme

5 Conclusion

In this paper, a novel fractal image compression based on Quadtree partition of the adaptive threshold value is proposed. We put forward the computation derivation process of the adaptive threshold value progressively and declare that the adaptive threshold value has the direct proportion with the variance of the current range block, so that threshold rely on manual experimental experience is solved. Experimental results show that we improve the performance of Quadtree segmentation by adapting the threshold value among each level of the Quadtree. How to reduce computing variance time of each range block and make effective classification searching among the region, how to make use of the fractal characteristic to encode and decode image, such as fractal dimension[10] and other related topics[11-14], are our future work.

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