# Between Empiricism and Rationalism: A Layer of Perception Modeling Fuzzy Sets as Intermediary in Philosophy of Science

Rudolf Seising

Medical University of Vienna, Core Unit for Medical Statistics and Informatics, Spitalgasse 23, A-1090 Vienna, Austria
Rudolf.seising@meduniwien.ac.at

**Abstract.** In philosophy of science we find two epistemological traditions: rationalism and empiricism. Rationalists believe that the criterion of knowledge is not sensory but intellectual and deductive whereas from the empiricist point of view the source of our knowledge is sense experience. Bridging this gap between these theories of knowledge has been a problem in philosophical approaches, both past and present. This philosophical paper focuses on using fuzzy sets and systems (FSS), computing with words (CW), and the computational theory of perceptions (CTP) as methodologies to help bridge the gap between systems and phenomena in the real world and scientific theories. It presents a proposal in which fuzzy methods are used to extend the so-called structuralist view of scientific theories in order to represent the relation of empiricism and theoretical structures in science.

**Keywords:** Philosophy of science, epistemology, rationalism, empiricism, fuzzy sets, computing with words, computational theory of perceptions, theory, reality, perceptions.

## 1 Introduction

In science we have a traditional division of work: on the one hand we have fundamental, logical and theoretical investigations and on the other hand we have experimental and application side examinations. The theoretical work in science is using logics and mathematics to formulate axioms and laws. It is linked with the philosophical view of rationalism whereas the other aspects of science using experiments to find or prove or refute natural laws have their roots in the philosophical empiricism.

In both directions – from experimental results to theoretical laws or from theoretical laws to experimental proves or refutations – scientists have to bridge the gap that separates theory and practice in science.

Beginning as early as the $17^{th}$ century, a primary quality factor in scientific work has been a maximal level of exactness. Galileo and Descartes started the process of giving modern science its exactness through the use of the tools of logic and mathematics.

The language of mathematics has served as a basis for the definition of theorems, axioms, definitions, and proofs. The works of Newton, Leibniz, Laplace and many others led to the ascendancy of modern science, fostering the impression that scientists were able to represent all the facts and processes that people observe in the

O. Castillo et al. (Eds.): Theor. Adv. and Appl. of Fuzzy Logic, ASC 42, pp. 101–108, 2007.
springerlink.com 

world, completely and exactly. But this optimism has gradually begun to seem somewhat naïve in view of the discrepancies between the exactness of theories and what scientists observe in the real world.

From the empiricist point of view the source of our knowledge is sense experience. John Locke used the analogy of the mind of a newborn as a "tabula rasa" that will be written by the sensual perceptions the baby has later. In Locke's opinion this perceptions provide information about the physical world. Locke's view is called "material empiricism" whereas the so called idealistic empiricism was hold by Berkeley and Hume: there exists no material world, only the perceptions are real.

This epistemological dispute is of great interest for historians of science but it is ongoing till this day and therefore it is of great interest for today's philosophers of science, too. Searching a bridge over the gap between rationalism and empiricism is a slow-burning stove in the history of philosophy of science. In this paper, Lotfi Zadeh's hierarchy stack of methodologies, fuzzy sets and systems (FSS), computing with words (CW) and the computational theory of perception (CTP), is recommended to build a bridge over this gap.

In my original research work on the history of the theory of fuzzy sets and systems (FSS) I could show that Lotfi A. Zadeh established this new mathematical theory in 1964/65 to bridge the gap that reflects the fundamental inadequacy of conventional mathematics to cope with the analysis of complex systems [1, 2, 3].

In the last decade of the 20th century Zadeh set up computing with words (CW) [4] and the computational theory of perceptions (CTP) [5, 6] and he erected the methodologies of CTP and CW on the basic methodology of FSS.

In this non-historical but philosophical paper this methodology stack for bridging the gap between real and theoretical systems will be examined from a philosophical point of view. To this end, the so-called structuralist approach of scientific theories in the philosophy of science will first be reviewed in section 2 and then this approach will be modified in section 3 – i.e. it will be "fuzzified" – by extending the structuralist framework with fuzzy sets and fuzzy relations to model perceptions of observers. This approach provides a new view of the "fuzzy" relationship between empiricism and theory.

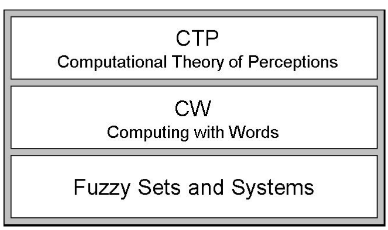


**Fig. 1.** Lotfi Zadeh's hierarchical stack of methodologies: FSS, CW, CTP

## 2 The Structuralist View of Theories

Two trends in obtaining systematic rational reconstructions of empirical theories can be found in the philosophy of science in the latter half of the 20th century: the *Carnap* *approach*[1] and the *Suppes approach*[2]. In both, the first step consists of an axiomatization that seeks to determine the mathematical structure of the theory in question. However, whereas in the Carnap approach the theory is axiomatized in a formal language, the Suppes approach uses informal set theory. Thus, in the Suppes approach, one is able to axiomatize real physical theories in a precise way without recourse to formal languages. This approach can be traced back to Patrick Suppes' proposal in the 1950s to include the axiomatization of empirical theories of science in the metamathematical programme of the French group "Bourbaki" [7].

Later, in the 1970s, Joseph D. Sneed[3] developed informal semantics meant to include not only mathematical aspects, but also application subjects of scientific theories in this framework, based on this method. In his book [8], Sneed presented the view that all empirical claims of physical theories have the form "*x* is an *S*", where "is an *S*" is a set-theoretical predicate (e.g., "*x* is a classical particle mechanics"). Every physical system that fulfils this predicate is called a model of the theory. For example, the class $M$ of a theory's models is characterized by empirical laws that consist of conditions governing the connection of the components of physical systems. Therefore, we have models of a scientific theory, and by removing their empirical laws, we get the class $M_p$ of so-called potential models of the theory. Potential models of an empirical theory consist of *theoretical terms*, i.e. observables with values that can be measured in accordance with the theory. This connection between theory and empiricism is the basis of the philosophical "problem of theoretical terms".

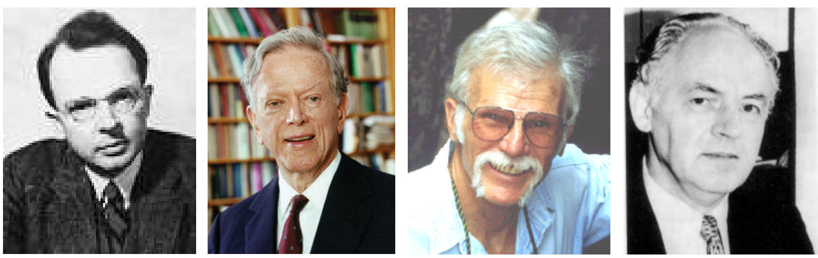

**Fig. 2.** From left to right: Rudolf Carnap, Patrick Suppes, Joseph Sneed, Wolfgang Stegmüller

If we remove the theoretical terms of a theory in its potential models, we get structures that are to be treated on a purely empirical layer; we call the class $M_{pp}$ of these structures of a scientific theory its "partial potential models". Finally, every physical theory has a class $I$ of intended systems (or applications) and, of course, different intended systems of a theory may partially overlap. This means that there is a class $C$ of

---

1 The German philosopher Rudolf Carnap (1891-1970) was a professor in Vienna (1926-1931) and a member of the Vienna Circle. He was a professor in Prague (1931-1935), Chicago (1936-1952), at the Institute for Advanced Study in Princeton (1952-1954), and at the University of California in Los Angeles (1954-1970).

2 The American mathematician and philosopher Patrick Suppes (born in 1922) was and is a professor at Stanford University in the USA.

3 The American physicist and philosopher Joseph D. Sneed is a professor at the Colorado School of Mines in the USA.

constraints that produces cross connections between the overlapping intended systems. In brief, this structuralist view of scientific theories regards the core $K$ of a theory as a quadruple $K = \langle M_p, M_{pp}, M, C \rangle$. This core can be supplemented by the class $I$ of intended applications of the theory $T = \langle K, I \rangle$.[4] To make it clear that this concept reflects both sides of scientific theories, these classes of $K$ and $I$ are shown in Fig. 2. Thus we notice that $M_{pp}$ and $I$ are entities of an empirical layer, whereas $M_p$ and $M_{pp}$ are structures in a theoretical layer of the schema.

Now this approach of the structuralist view of theories will be extended by using fuzzy sets and fuzzy relations to represent perceptions as important components in the interpretation of scientific theories. This will be very suitable in future invetstigations in the philosophy of science, because in new theories of the 20[th] century, such as relativity theory and quantum mechanics in physics, the observer and his/her perceptions play a central and important role [9].

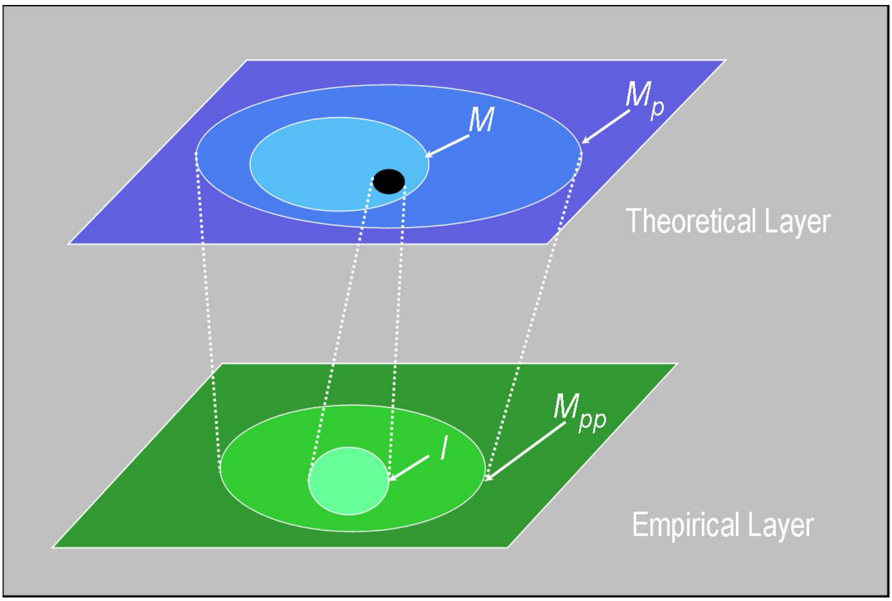


**Fig. 3.** Empirical and theoretical structural layers in the analysis of scientific theories

## 3 A Fuzzy Layer as Intermediary in Philosophy of Science

The proposed modification of the structuralist approach in philosophy of science pertains to the empirical layer in Fig. 3. A distinction can be made between real systems and phenomena, on the one hand, and perceptions of these entities, on the other. Thus a lower layer – the real layer – is introduced and the former empirical layer is renamed the "fuzzy layer", as the partial potential models and intended systems are not

[4] Sneed, Wolfgang Stegmüller, C. Ulises Moulines, and Wolfgang Balzer, developed this view into a framework intended to analyze networks of theories and the evolution of theories [10].

real systems because a minimal structure is imposed by the scientist's observations. These are perception-based systems and thus must be distinguished from real systems and phenomena that have no structure before someone imposes one upon them.

Now there is a layer of perceptions between the layer of real systems and phenomena and the layer of theoretical structures. In accordance with Lotfi A. Zadeh's computational theory of perceptions (CTP), perceptions in this intermediate layer can be represented as fuzzy sets. Whereas measurements are crisp, perceptions are fuzzy, and because of the resolutions achieved by our sense organs (e.g. aligning discrimination of the eye), perceptions are also granular – in 2001 Zadeh wrote in the *AI Magazine*: "perceptions, in general, are both fuzzy and granular or, for short *f*-granular [6]. Fig. 4 shows Zadeh's depiction of *crisp* (*C*) and *fuzzy* (*F*) granulation of a linguistic variable.

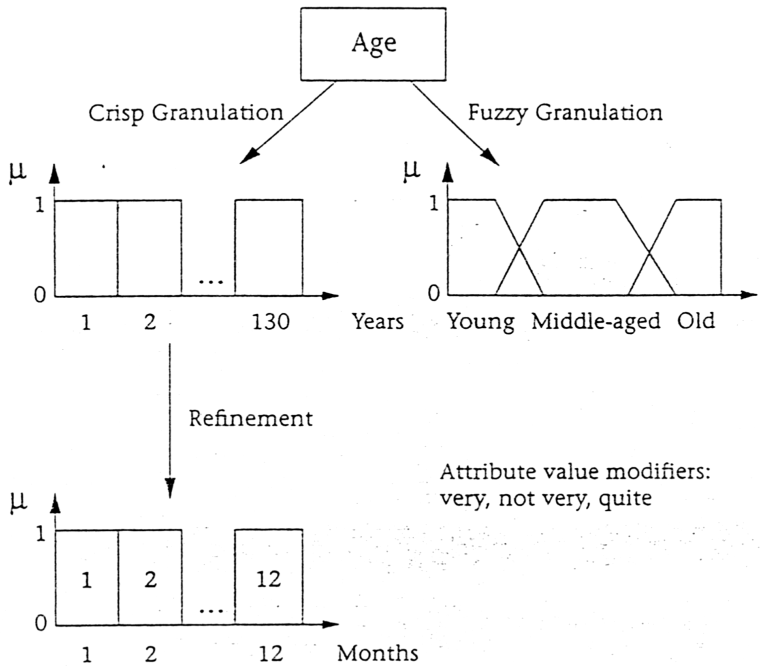


**Fig. 4.** Empirical and theoretical structural layers in the analysis of scientific theories [6]

When Zadeh established CTP on the basis of computing with words (CW), which in turn is based on his theory of fuzzy sets and systems [5], he earnestly believed that these methodologies would attain a certain importance in science: "In coming years, computing with words and perceptions is likely to emerge as an important direction in science and technology." [15]. Taking Zadeh at his word, his methodologies of fuzzy sets and computing with words and perceptions are here incorporated into the structuralist approach in the philosophy of science. As discussed above, a fuzzy layer of perceptions is inserted between the empirical layer of real systems and phenomena, and the theoretical layer, where there are structures of models and potential models. Thus the relationship of real systems and theoretical structures has two dimensions: fuzzification and defuzzification.

### 3.1 Fuzzification

Measurements are crisp and perceptions are fuzzy and granular. To represent perceptions we use fuzzy sets, e.g. $A^F$, $B^F$, $C^F$, .... It is also possible that a scientist observes not just a single phenomenon, but interlinked phenomena, e.g. two entities move similarly or inversely, or something is faster or slower than a second entity, or is brighter or darker, or has an analogous smell, etc. Such relationships can be characterized by *fuzzy-relations* $f^F$, $g^F$, $h^F$, ….

### 3.2 Defuzzification

"Measure what is measurable and make measurable what is not so" is a sentence attributed to Galileo. In modern scientific theories this is the way to get from perceptions to measurements or quantities to be measured. Here this transfer is interpreted as a defuzzification from perceptions represented by fuzzy sets $A^F$, $B^F$, $C^F$, … and relations between perceptions represented by fuzzy relations $f^F$, $g^F$, $h^F$, … to ordinary (crisp) sets $A^C$, $B^C$, $C^C$, … and relations $f^C$, $g^C$, $h^C$, … These sets and relations are basic entities for the construction of (potential) models of a scientific theory in the theoretical layer.

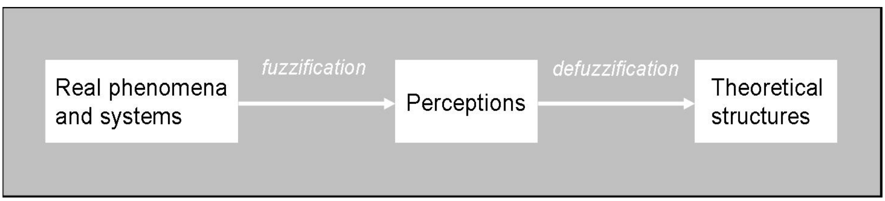


**Fig. 5.** Fuzzification and defuzzification in the fuzzy structuralist view

### 3.3 Theoretization Results from Fuzzification and Defuzzification

The serial operation of fuzzification and defuzzification (see Fig. 5) yields the operation of a relationship *T* that can be called "initial theoretization", because it transfers phenomena and systems in the real (or empirical) layer into structures in the theoretical layer (see Fig. 6).

In the structuralist view of theories, the general concept of theoretization is defined as an intertheoretic relation, i.e. a set theoretical relation between two theories *T* and *T'*. This theoretization relation exists if *T'* results from *T* when new theoretical terms are added and new laws connecting the former theoretical terms of theory *T* with these new theoretical terms of theory *T'* are introduced.

Successive addition of new theoretical terms establishes a hierarchy of theories and a comparative concept of theoriticity. In this manner the space-time theory arose from Euclidean geometry when the term "time" was added to the term "length", and classical kinematics developed from classical space-time theory when the term "velocity"

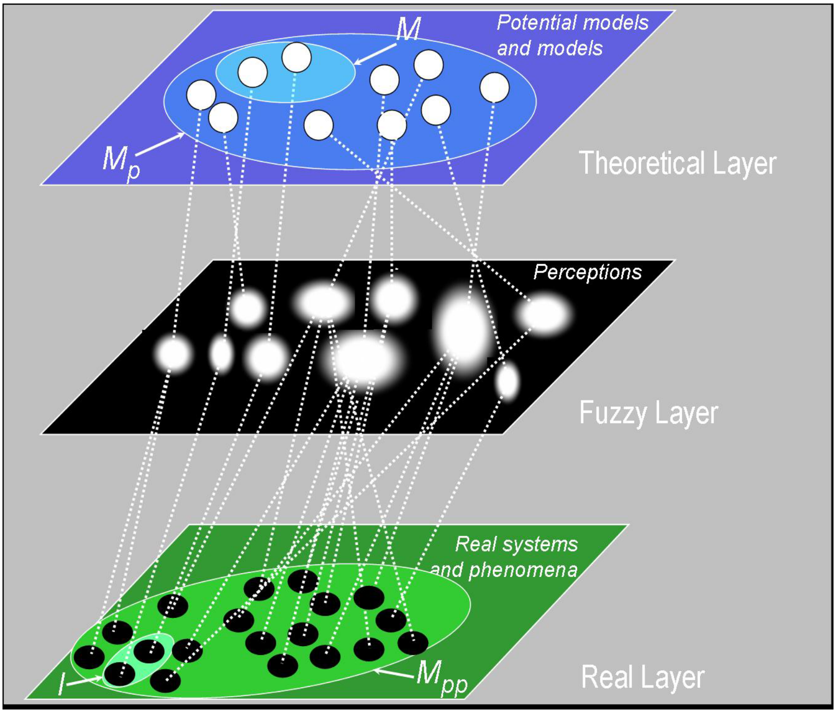


**Fig. 5.** Empirical, fuzzy, and theoretical layers of (fuzzy) structures in scientific research

was added. Classical kinematics turned into classical (Newtonian) mechanics when the terms “force“ and “mass“ were introduced.

The new theory *T’* adds a new theoretical layer to the old theory *T*. *T*-theoretical terms are not *T'*-theoretical but *T'*-non-theoretical terms, and reciprocally they may not be any of the T-non-theoretical terms. The old theory must not be changed in any way by the new theory. In this approach the higher terms are in the hierarchy, the more theoretical they are. The lower layers contain the non-theoretical base of the theory.

What is the situation in the lowest layer of this hierarchy? A theory *T* with theoretical terms and relations exists there, but it is not a theoretization of another theory. This theory *T* covers phenomena and intended systems with initial theoretical terms. This is an initial theoretization, because the *T*-theoretical terms are the only theoretical terms at this level. They have been derived directly as measurements of observed phenomena. This derivation was designated as “initial theoretization” above and it is a serial connection of fuzzification and defuzzification.

## 4 Conclusion

The computational theory of perceptions is an appropriate methodology to represent efforts of scientific research to bridge the gap between empirical observations and the abstract construction of theoretical structures.

In the classical, i.e. non-fuzzy, structuralist view of theories there is an empirical layer of real phenomena and systems that have some minimal structure and a theoretical layer of potential models and models that are fully structured entities. But there is no representation of the observer's role and his/her perceptions. The modified view of the structuralist approach presented in this paper as a proposal that will be worked out in detail in the near future comprises a layer of fuzzy sets and fuzzy relations as a means of dealing with the difference between real phenomena and systems on the one hand and the observer's perceptions of these real entities on the other. This extended structuralist view – which can be called the "fuzzy structuralist view" – of scientific theories may open up a new and fruitful way to understand scientific research.

**Acknowledgments.** I would like to thank Wolfgang Balzer (Munich), my former supervisor in the philosophy of science for his introduction to the structuralist view of theories and Dieter Straub (Munich) for important and enjoyable discussions. Many thanks go to Witold Pedrycz (Edmonton) for his encouragement to make a shift towards this interdisciplinary and science-oriented research project.

This work is partially supported by Grant # AB00158OFF from the Vienna Federal Data Center.

## References

1. Seising, R.: Die Fuzzifizierung der Systeme. Die Entstehung der Fuzzy Set Theorie und ihrer ersten Anwendungen – Ihre Entwicklung bis in die 70er Jahre des 20. Jahrhunderts. Stuttgart: Franz Steiner Verlag (Boethius: Texte und Abhandlungen zur Geschichte der Mathematik und der Naturwissenschaften Band 54) 2005. English edition: Seising R.: The Fuzzification of Systems. The Genesis of Fuzzy Set Theory and Its Initial Applications – Its Development to the 1970s. Berlin [u.a.]: Springer (2007, in print).
2. Seising, R.: Pioneers of Vagueness, Haziness, and Fuzziness in the 20th Century. In: Masoud Nikraves, Janusz Kacprzyk, Lotfi A. Zadeh (eds.): Forging the Frontiers I. Studies in Fuzziness and Soft Computing. New York [u.a.]: Springer (Series in Fuzziness and Soft Computing) (2007, in print).
3. Seising, R.: On the Absence of Strict Boundaries – Vagueness, Haziness, and Fuzziness in Philosophy, Medicine, and Science, Applied Soft Computing, Special Issue: Forging the Frontiers (2006, in print).
4. Zadeh, L. A.: Fuzzy Logic = Computing with Words, IEEE Transactions on Fuzzy Systems, 4, 2 (1996) 103-111.
5. Zadeh, L. A.: From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions, IEEE Trans. on Circuits and Systems-I: Fundamental Theory and Applications, 45, 1 (1999) 105-119.
6. Zadeh, L. A.: A New Direction in AI. Toward a Computational Theory of Perceptions. AI-Magazine, 22, 1 (2001) 73-84.
7. Bourbaki N. (pseudo.): Elements of Mathematics: Theory of Sets, Addison-Wesley, Reading, Mass. 1968.
8. Sneed, J. D.: The Logical Structure of Mathematical Physics, Dordrecht: Reidel (1971).
9. Seising, R.: Can Fuzzy Sets be Useful to (Re)Interpret Uncertainty in Quantum Mechanics? K. Demirli and A. Akgunduz (eds.): Proceedings of the 2006 Conference of the NAFIPS, June 3-6 (2006), Montréal, Canada (CD, ISBN: 0-7803-9188-8).
10. Balzer, W., Moulines, C. U., Sneed, J. D.: An Architectonic for Science. The Structuralist Program. Dordrecht: Reidel (1987).