Abductive Inference and Iterated Conditionals

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Summary. The first part of the paper aims to stressing the analogy between conditional inference and abductive inference, making evident that in both cases what is here called "reasonable" inference involves a choice between a finite set of incompatible conclusions, selecting the most information preserving-consequent in the case of standard conditionals and the most information-preserving antecedent in the case of abductive conditionals. The consequentialist view of conditionals which is endorsed in this perspective is then extended to cover the case of higher degree conditionals, introducing in the semantical analysis the notion of inferential agents reasoning about the activity of other inferential agents. It is then shown (i) that iterated conditionals are essential in the treatment of redundant causation (ii) that abductive conditionals are essential parts of iterated conditionals in the analysis of causal preemption (iii) that there is a widespread use of second-degree conditionals involving first degree abductive conditionals. The final section is devoted to remind that Peirce's original notion of abductive inference was actually defined in terms of second degree conditionals.

1 The Notion of ε-implication

There is no doubt that Stalnaker-Lewis conditional logics introduced an important change of paradigm in the study of conditional inference[1]. However, many features of the theorems involving Stalnaker-Lewis conditionals have been object of criticism inasmuch as the truth of such conditionals turns out to prescind from any kind of relevance or dependence nexus between the clauses. In front of such difficulty a natural move has been to go back to the tradition of the so-called "consequentialist" theory of conditionals originally proposed by Chisholm, Goodman and Reichenbach in the '40, when the tool of possible-worlds semantics was not yet developed. The present author has

[1] For a survey on conditional logics, see Nute [11], where Stalnaker-Lewis systems are termed "minimal change theories".

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defended the idea that the consequentialist view, in the form of what here will be called *reasonable* inference, grants a unified treatment of counterfactual, inductive and abductive conditionals. However, even in this trend of investigations, little attention has been given to two points which I will try to focus in the following sections: a) Higher degree conditionals, i.e. conditionals having other conditionals as antecedents or consequents, are essential to the reconstruction of scientific reasoning, even when they do not appear in the superficial structure of the statements. b) The consequentialist view is able to provide a clear and straightforward semantical interpretation of higher degree conditionals and grants a deeper understanding of the relation between counterfactual and abductive conditionals[2].

The treatment will be semiformal. However, it is useful to presuppose a formal language which may provide a joint formal representation of both probabilistic and modal notions. Fattorosi-Barnaba's and Amati's [5], for example, offers the instance of a system for additive graded modalities which may be chosen as a reference formal system. If $\Diamond^n A$ is intended as *A is probable at degree* $> n$, $\Diamond A$ is coincident with $\Diamond^0 A$ and $\Box A$ with $\Box^0 A$. $Pr(A) = n$ may be then put equivalent to $\neg\Diamond^n A \wedge \neg\Diamond^{1-n}\neg A$ and $Pr(B|A)$ may then be defined in terms of B and A in a standard way, i.e. as:

$$\frac{Pr(A \wedge B)}{Pr(A)} \quad (Pr(A) \neq 0)$$

In this linguistic framework other operators may be introduced by definition. For instance, if ε is a negligible value ≥ 0, we may define what we may call *ε-implication* in this way:

(1) $A \;_{\varepsilon}\!\rightarrow B =_{Df} Pr(B|A) = 1 - \varepsilon$

The notion of information content in terms of possible world semantics is that A ε-implies B iff "almost all the accessible A-worlds are B-worlds". The notion of information content (*Cont*) for sake of simplicity will be introduced here by definition as $Cont(B|A) =_{Df} 1 - Pr(B|A)$[3].

Some obvious properties of this notion are:

1. $Pr(A) = 1 - Cont(A)$
2. $Cont(A \wedge B) = Cont(A) + Cont(B) - Cont(A \vee B)$
3. $Cont(A \wedge B) \geq Cont(A)$

What about the information content of a physical law L? According to an extensionalist view of laws, a law L is an infinitary conjunction of statements. So in normal contexts it happens that if A is any finitary truth-functional and non contradictory statement, $Cont(L) > Cont(A)$. Let us recall that in

[2] For a first outline of this theory see Pizzi [14].
[3] For this definition see for instance Hintikka [2].

Carnap's inductive logic any physical law receives probability value 0 – so it has information content 1 – while in Hintikka's inductive logics laws receive a probability value which is different from 0 but anyway low[4]. These results give substance to the idea that every law will be normally more informative than every finite combination of single facts. Something should be said about the controversial question of second or higher degree probabilities, so about the content of probabilistic statements[5]. We are especially interested in giving a value to $Cont(A \;_{\varepsilon}\!\rightarrow B)$, i.e. to the content of $Pr(B|A) = 1 - \varepsilon$. There is no problem in introducing a **S4**-style axiom for Pr. i.e.

(2) $Pr(q|p) = 1 - \varepsilon \supset Pr(Pr(q|p) = 1 - \varepsilon) = 1 - \varepsilon$

From (2) of course, if ε is 0 and p is a tautology $\top$, we have:

(3) $\Box q \supset \Box\Box q$

This minimal principle is however not of help in evaluating the content of a conditional. As we will say in the next section, in evaluating an argument from A to B, we should take care of the laws which are essentially used in the argument itself. This suggests that the information value of a conditional should be proportional to the information content of the laws essentially involved in the derivation, and such a content, as already said, is very high. This criterion marks a difference between what we shall call rational and reasonable inference.

2 Rational and Reasonable Inference

A basic idea that we intend to develop here is that every rational inference rests on the choice of the best consequent in a set of consequents or in the choice of the best antecedent among a class of possible antecedents for a given consequent. But which is the *best* consequent or the *best* antecedent? We could leave this notion sufficiently vague or we can make it depend on some variable parameter of evaluation[6], but here we prefer a non-neutral policy: the best inferential conclusions will be here defined as the ones which are more information-preserving with respect to some given set of background knowledge **K**. Let us call **CR** (Corpus Rationale) the infinite set of all true statements, including the laws $L_1 \ldots L_n$. **CR** is closed under logical rules and may be thought as the infinite set of sentences describing the actual world w°. Let **K** be a finite subset of **CR** containing a finite set of true statements and a theory T consisting of a finite subsets of the laws in **CR**. Let use K to denote the conjunction of the members of **K**. $\mathbf{K}(A)$ will be a subset of **K**

[4] See Carnap [1] and Hintikka [3, 4].
[5] For a recent approach to higher degree probabilities see [6].
[6] See for instance Rescher [19].

revised in dependence of A. Of course $\mathbf{K}(\top) = \mathbf{K}$ More specifically, $\mathbf{K}(A)$ is a subset of $\mathbf{K}$ which is obtained by selecting the most informative statements of $\mathbf{K}$ compatible with A. Let us consider, to begin with, the case in which A and B are truth-functional statements. The symbol $>_C$ in wffs whose form is $A >_C C_i$ will be used for standard conditionals, namely for conditionals which are *factual*, *afactual* or *counterfactual*.

Given a certain finite set $\mathbf{K}$ with respect to which the conditional is evaluated as true or false, there are two possibilities to be considered:

(a1) $\Diamond(K \wedge A)$ (a2) $\neg\Diamond(K \wedge A)$

In the first case the conditional is afactual or factual, in the second case it is counterfactual[7]. We will say that $A >_C C_i$ is true or false with respect to K only if there are at least two statements C_i and C_j such that:

b.1 $\neg\Diamond(K \wedge A \wedge C_i \wedge C_j)$
b.2 There are at least two $\mathbf{K}_i, \mathbf{K}_j \subseteq \mathbf{K}$ such that $\Diamond(A \wedge K_i)$ and $\Diamond(A \wedge K_j)$
b.3 $(A \wedge K_i \;\; \varepsilon - implies \;\; C_i)$ and $(A \wedge K_j \;\; \varepsilon - implies \;\; C_j)$
b.4 $\underline{Cont(A \wedge K_i) \succ Cont(A \wedge K_j)}$

The underlined clause $b.4$ makes it clear that C_i is the preferred conclusion due to the fact that $\mathbf{K_i}$ is a subset of $\mathbf{K}$ which ε-implies C_i and has higher information content than the rival set $\mathbf{K_j}$. An instance of rational inference is offered by the choice between the following two counterfactuals:

(4) If Socrates were a donkey, Socrates would be four-legged: $A >_C C_1$
(5) If Socrates were a donkey, Socrates would be a two-legged donkey: $A >_C C_2$

Here $\mathbf{K_1}$ is {Every donkey is four-legged}, $\mathbf{K_2}$ is {Socrates is two legged}, $\mathbf{K}$ is {Socrates is not a donkey} $\cup$ $\mathbf{K}_1 \cup \mathbf{K}_2$. The theory T is the law in $\mathbf{K}_1$. The two conclusions C_1 and C_2 are incompatible, and a fortiori A, C_1, C_2 form an incompatible triad. The counterfactual (4) is "true" because its rival (5) relies on a set $\mathbf{K}_2$ which has a lower information content, due to the fact that this singular statement is less informative than the laws belonging to $\mathbf{K}_1$. The case of factual or afactual conditionals suggests that we have to extend our conditions by making the further assumption that $\mathbf{K}$ should be always contain inside T the metalaw known as principle of Uniformity of Nature (UN). As Goodman showed in the *grue-bleen* paradox, one could infer both C_i and C_j from the same premise A, but if we have in $\mathbf{K}$ also UN the conclusion, say, that emeralds will be blue after 3000 is incompatible with the consequence of UN stating that the properties of substances are spatio-temporal invariant[8].

[7] The factual conditionals, or since-conditionals, are conditionals such that A belongs to $\mathbf{K}$, while this is not required in afactual conditionals.

[8] In Rescher [19] a coherence theory of inductive reasoning is introduced. If 100 black ravens have been observed, this is compatible with the conclusion that

Remark 1. In case there is a tie between different subsets $\mathbf{K}_i$ and $\mathbf{K}_j$ leading to incompatible conclusions (as in the famous Bizet-Verdi case) we are not in conditions to choose between $Cont(A \wedge K_i)$ and $Cont(A \wedge K_j)$ since they are identical. So both conditionals are false. This is what has been called a *Gestalt Effect* in Pizzi [13].

The relativization to $\mathbf{K}$ of the truth of $A > C_i$ might be dropped by existential quantification over $\mathbf{K}$, in other words by saying that there is some $\mathbf{K}$ that has the mentioned properties. The minimal set of conditions b1-b4) is sufficient to define what we could call *rational* inference, but it is plausibile to require a further restriction. Such condition says that whenever any $K_i \wedge A$ ε-implies some C_i all the elements of $\mathbf{K}_i$ must have the property of being *essential* to such a derivation (otherwise the information content of $\mathbf{K}_i$ could turn out to be higher than the content of $\mathbf{K}_j$ simply for the occurrence in it of some irrelevant statement A). This means that, for every P belonging to some $\mathbf{K}_i$, we have to require also the following condition beyond b1–b4:

> c) For every P and every $\mathbf{K}_i$, if P belongs to $\mathbf{K}_i$ and $K_i \wedge A$ $\varepsilon -$ *implies* B, the conjunction of statements belonging to $\{\mathbf{K}_i - \{P\}\} \cup \{A\}$ does not ε-imply B.

Note that clause c) solves many cases of irrelevance due to the high probability of the conclusion. If for instance $Pr(C_i) = 1 - \varepsilon$, it follows that, for every consistent $A, Pr(C_i|A) = 1 - \varepsilon$, but this makes irrelevant every element of $\mathbf{K}_i$ since it is drawn simply via the laws of the background logic. In the same vein, notice that if $\mathbf{K}_i$ contains only B and A this does not legitimate $A >_C B$ even if $Pr(B|A \wedge K_i) = 1$ for every A and every B. This feature marks an important difference with Stalnaker-Lewis logics, since they accept the controversial law:

$$(A \wedge B) \supset (A >_C B)$$

Given the preceding conditions, we might distinguish between rational and reasonable inference, by asking that an inference is *reasonable* when and only when it satisfies beyond b1)-b4) the supplementary clause c). So every reasonable inference is also rational, but not vice versa[9].

the next raven will be black and also with the conclusion that the next raven will be of some other color. But since we have to choose, in Rescher's view, the "most plausible" subset, we are guided by a rule which says "When the initial evidence exhibits a marked logical pattern, then pattern- concordant statements are – ceteris paribus – to be evaluated as more plausibile than pattern-discordant ones" (p. 226). This criterion introduces a certain arbitrarity, while the Principle of the Uniformity of Nature appears to provide a firmer foundation to inductive reasoning.

[9] The distinction here drawn between rational and reasonable inference could be different if the comparison were made between epistemic utilities and not between information contents. On the notion of epistemic utility see Hintikka and Pietarinen [4].

Up to now we have characterized the properties of factual, afactual and counterfactual conditionals. What about abductive conditionals? Let us introduce the symbol $>_A$ to denote the abductive conditional. For instance $A >_A C_i$ may be "the match lit (A), so it has been scratched (C_i)" or also "if the match lit, this means that it has been scratched". The reliability of the conclusion lies in the fact that it is preferred in the set of other possible incompatible conclusions, as for instance the one embodied in the conditional "the match lit, so it has been put into fire". It is remarkable that abductive reasoning has important features in common with counterfactual reasoning, abstracting from the fact that the antecedent is normally belived to be true and not false. In both cases, in fact, we are faced with a rational choice between incompatible conclusions. Let A be "Smith has been killed". Let $\mathbf{K}$ be for example a set which includes

1. Smith has been killed in New York by only one person who had the keys of the room
2. No one had the keys except Brown and White
3. White was in Patagonia at the moment of the murder
4. White had a strong interest in killing Smith.
   A consequence of $\mathbf{K}$ is the disjunction:
   D : Brown is the murderer or Smith is the murderer ($C_1 \vee C_2$)
   Now there is a subset $\mathbf{K}_i$ of $\mathbf{K}$ i.e. $\{1, 2, 3\}$ such that jointly with A ε-implies
5. Brown is the murderer (C_1)
   while another subset $\mathbf{K}_j$ $\{1, 2, 4\}$ jointly with A ε -implies
6. White is the murderer (C_2)

Note that 5) and 6) are incompatible if conjoined with K, since the premise 1) in $\mathbf{K}$ states that only one person was responsible of the murder. The conditional "Smith has been killed, so from the given information [it is reasonable to conclude that] Brown is the murderer" ($A >_A C_1$) is a synthetic expression of what we call here an abductive conditional. What is the specific difference between the reasoning underlying a C-conditional and an A-conditional? Let us recall the schema of Hempel-Oppenheim's Statistical Inference: as is well known, such an inference requires a rule of high probability and also the essentiality of the items occurring in the antecedent. Any C-conditional $A >_C C$ implies that there is a potential *explanans* involving A conjoined with various true presuppositions K_i and an *explanandum* C. But in the case of abductive conditionals the inference, given a certain stock of true presuppositions, is not from the *explanans* to the *explanandum* but in the reverse direction. If A is an *explanandum*, given a certain amount of information represented by K_i, we have at least two *explanantia* $K_i \wedge C_i$ and $K_j \wedge C_j$. The schema of ε-implication is as before, with two important points of difference. In fact we have not as before b.4 $Cont(A \wedge K_i) \succ Cont(A \wedge K_j)$

but

b.4* $Cont(C_i \wedge K_i) \succ Cont(C_j \wedge K_j)$

Furthermore, the relation between C_i and A is always definable via ε-implication, but we have in place of b.3 what follows:

b.3* $C_i \wedge K_i$ ε -implies A and $C_j \wedge K_j$ ε -implies A.

The interesting point of agreement is that, in the case of the example, this choice is performed on the basis that the supposition C_i (that Brown is the murderer) implies the *explanandum* A thanks to a set of data which save more information than the alternative supposition C_j: so C_i is a component of the best explanation, in Hempel's sense, of A.

The two characterizations which we have given for $>_C$ and $>_A$ suggest that we could define an abstract notion of a reasonable inference. This step can be made in different ways. A possibility is to say that a conditional represents a reasonable inference if it is either a C-conditional $A >_C C$ satisfying the clauses b1)–c) of p. 371 or the converse abductive conditional $C >_A A$. We may define then a connective $>>$ as follows:

$$(Def >>) \quad A >> C =_{Df} A >_C C \vee C >_A A$$

A rough characterization of $>>$ is in saying that C is the best explanatory consequent of A or A is (part of the) best explanatory antecedent of C. Clearly $A >> C$ is independent from the converse $C >> A$, which is equivalent to $C >_C A \vee A >_A C$, so $>>$ is not a symmetric relation. We expect that the two following properties hold for $>>$:

TB1 $(A >> C) \supset \neg(A >> \neg C)$
TB2 $(A >> C) \supset \neg(\neg A >> C)$[10]

Needless to say, $>_A, >_C, >>$ are all non contrapositive. Furthermore, no one of them satisfies Modus Ponens. This is especially clear for the abductive conditionals. To say that A is the best available explanation of C does not mean that A is true given that C is true. In order to reach this conclusion we need a counterproof of A, or some independent evidence for it.

3 Iterated Conditionals and Causal Reasoning

To complete the theory of rational/reasonable inference an important detail needs to be added. The definition of rational and reasonable inference has been introduced in 2) with the restrictive clause that the antecedent clause A and the consequent C are truth-functional statements. But we have to face

[10] For this couple of formulas, often termed "Boethius' Theses", see Pizzi [15].

the possibility that a conditional contains another (negated or non-negated) conditional in the antecedent or in the consequent, giving rise to statements which have been named "embedded", "nested" or "iterated" conditionals. The first question to treat in this connection concerns the fact that it has been sometimes claimed that nesting of conditional antecedents lacks an independent sense. This skepticism is embodied in so called Generalized Stalnaker's Thesis.

$(GST)\ \ Pr(B > C/A) = Pr(C/A \wedge B) \quad (Pr(A \wedge B) \neq 0)$

But here we have to consider a famous counterexample suggested by R. Thomason:[11]

> (Th) If the glass would break if thrown against the wall, then it would break if dropped on the floor.

As Thomason remarked, the logical form of (Th) cannot be $A > B \models C > D$, but $(A > B) > (C > D)$. In fact (Th) exhibits the failure of weakening – which is typical of $>$, not of $\models$ – since the following conditionals is false:

> (Th^*) If the glass would break if thrown against the wall and the floor were covered with foam rubber, then it would break if dropped on the floor.

Stalnaker's Thesis indeed suggests that iterated antecedents might be paraphrased into a conjunction. If this were true we would have the permutation of antecedents as a theorem:

$(Perm)\ \ (A > (B > C)) \supset (B > (A > C))$

But it is clear that permutation does not work:

> (HAB) If you will have headache tomorrow, taking an aspirin you will feel better.

has a meaning which is different from

> (AHB) Taking an aspirin, if you will have headache tomorrow you will feel better.

The phrase "Taking an aspirin" in fact receives a different sense when it is in the scope of the supposition concerning an headache tomorrow and in a context in which such information does not exist. Other examples give evidence that in iterated conditionals a premise could be factual in one position, but not factual in a different position. For instance, the supposition "the lamp is alight" may be factual or afactual in the following conditional

> (LSD) If the lamp is alight, then if you switch off the light we will be in the dark.

but not in the permutated variant

[11] Quoted in van Fraassen [20].

(SLD) If you switch off the light then, if the lamp is alight we will be in the dark.

(SLD) in fact appears to be meaningless or false while (LSD) appears to be true.

In the present section our aim is to outline an analysis of iterated conditionals, both standard and abductive, in the framework of a consequentialist view of conditionals. A useful step is to introduce a more analytical formal language in which the symbol $>$ (which we now stipulate to stand ambiguously for $>_A$ and $>_C$) is indexed by some variables $a, b, c \ldots$ representing intuitively arbitrary rational inferential agents. We add the assumption that every agent $a, b, c \ldots$ is biunivocally associated to a certain set $\mathbf{K}_a, \mathbf{K}_b, \mathbf{K}_c \ldots$ of presupposed information. We will have then an infinite number of conditional operators $>_a, >_b, >_c \ldots$ The intuitive meaning of $A >_a C$ is that the agent a correctly infers C from A (with respect to a certain set $\mathbf{K}_a$ associated to a). Then $(A >_a C) >_b (R >_c Q)$ means then "b reasonably infers, from the fact that a reasonably infers C from A, that c reasonably infers Q from R". The involved sets of information are $\mathbf{K}_a, \mathbf{K}_b, \mathbf{K}_c$. The move from indexed $>$ to non-indexed $>$ is provided by the existential quantification on the variables for agents. In other words, $A > B$ can be made equivalent to $\exists x(A >_x B)$. The intuitive meaning of $A > B$ is then that there is some agent x who reasonably infers B from A and from the background knowledge at his disposal. According to this definition $A > (B > C)$ means then $\exists x(A >_x \exists y(B >_y C))$: For some x, x reasonably infers from A that (for some y, y infers reasonably C from A). A negated conditional $\neg(A > B)$ amounts then to $\neg\exists x(A >_x B)$, (i.e. must be understood as saying that no subject x infers reasonably B from A) and is obviously different from $A > \neg B$. Some remarks are in order.

i. The notion of degree of a conditional is the usual one adopted in conditional logic[12]. We stipulate that if $A >_d B$ is the conditional having the highest degree in a nested formula, the information set $\mathbf{K}_d$ is the *basic* information set: in other words every other information set considered in the formulas is coincident with $\mathbf{K}_d$ save for revisions introduced by the suppositions occurring in lower-degree conditionals (see point iv).
ii. An assumption should be introduced to calculate the information content of $\exists x(A >_x B)$. If $>_x$ stands for *reasonable* inference, it depends on the laws of nature essentially involved in the inference of B from A, so it is natural to think that the information content of $A > B$ is as least as high as the content of the physical laws which are essential to such inference. Of course $Cont\neg(A > B)$ equals $1 - Cont(A > B)$: if no rational subject can make an inference from A to B this makes $A > B$ something which is epistemically vacuous (so something having content near to 0).

[12] The conditional degree of a statement S may be simply calculated by 1) replacing $>$ with strict implication 2) eliminating the symbols for strict implications in favor of $\Box$ and truth functional operators and 3) calculating the modal degree of the resulting wff.

iii. In order to make an inference about other inferences we need the special laws which are the *meta-laws* governing the inferential behavior of rational subjects. Such laws are obviously part of the ordinary stock of background knowledge, i.e. of **CR**, but we will also assume that such laws belong to the T in $\mathbf{K}_x$. Some of such laws describe the already defined behavior of any rational subject in calculating information and drawing the "best" conclusion. Other important laws, however, rule the way in which any agent takes into account what other agents know or do.
iv. A metalaw which here we are willing to endorse – but could be ignored in different approaches – is that revision is cumulative: in other words any new supposition S made by some subject y should be added to an information set modified by the suppositions made by all $x_1 \ldots x_n$ in lower degree conditionals.

Let us for instance consider the following nested formula and suppose for sake of simplicity that $>$ is a C-conditional:

(6) $A >_a ((D >_b C) \wedge \neg(H >_c R))$

Here $>_a$ has degree two, while $>_b$ and $>_c$ have degree one. If $\mathbf{K}$ is the background information set, $\mathbf{K}_a$ is here the basic information set, in the sense that it contains the part of $\mathbf{K}$ known by the subject a. Then:

1. The inference of a is performed by adding A to $\mathbf{K}_a(A)$
2. The inference of b is performed by adding D to $\mathbf{K}_a(A)(D)$[13]
3. The inference of c is performed by adding H to $\mathbf{K}_a(A)(H)$

Since it is different to add B to $\mathbf{K}(A)$ and to add A to $\mathbf{K}(B)$ this makes clear why $A > (B > C)$ is different from $B > (A > C)$.

This cumulative character of the suppositions should be made explicit by suitable axioms. In the light of the preceding interpretation, for instance, it should be natural to have at our disposition at least two principles, the first of which is obvious:

Ax1 $(A > (A > B)) \equiv A > B$
Ax2 $(A > (B > C)) \supset (A > (A \wedge B > C))$

We may now go back to the question whether nesting is pleonastic or not in the reconstruction of scientific reasoning. Our claim is that nesting is essential to give a correct understanding of important features of scientific arguments. An important argument in favor of the essentiality of nesting concerns causal redundancy in the frame of a counterfactual theory of causation. If e_1 and e_2 are symbols for token events identified by their instant of occurrence and O is an operator forming propositions from token events, our claim is that there is an unlimited number of causal notions, which may differ at least in two features: 1) the degree of the counterfactual that expresses the relation between

[13] For the definition of $\mathbf{K}(A)$, to be obiviously extended to $\mathbf{K}$b(A), see page 367. It is understood that if $\mathbf{K}(A) = \mathbf{K}'$, $\mathbf{K}(A)(D) = \mathbf{K}'(D)$.

cause(s) and effect 2) the additional qualification expressing the explanatory strength of the causes with respect of the effect[14].

The statement $Oe_1 \wedge Oe_2 \wedge (\neg Oe_1 >_C \neg Oe_2)$ (to be read "e_1 is causally relevant for e_2") defines the minimal notion of causality, in the double sense that the conditional has degree one and no supplementary qualification is transmitted. But following the given theory we may define, among other notions, a two-place notion of *causal concurrency* which is as follows:

(CC) if e_1 had not occurred then, in absence of e_2, e_3 had not occurred.

So two concurring causes for e_3 are e_1 and e_2: $\neg Oe_1 >_C (\neg Oe_2 >_C \neg Oe_3)$ Standard examples of overdetermination are clear expressions of concurrency in the given sense: if the first killer had not fired a shot to Smith, the second would have killed Smith (which means: if also the second had not fired his shot then Smith would have not died). The paraphrase of this iterated conditional in terms of inferences performed by rational subjects is not difficult and will be omitted.

What to say about the kind of asymmetrical redundancy called preemption? We assume that preempting is a case of concurrency, but it is an asymmetrical concurrency. A standard definition of preemption says that a cause prevents the action of some other potential cause which would have reached the same effect. As argued in Pizzi [16], preemption should not be confused with causal anticipation or causal delay. The Sarajevo shots have been a triggering cause for the First World War, but the common opinion is that a macroevent classifiable as the First World War would have anyway taken place soon after, in absence of the shots, due to some other potential causes. So strictly speaking this is not a case of pre-emption because the effect-events involved are different. We remark anyway that a full description of causal anticipation might be realized by using an additional statement which has in any case the form of a second degree conditional:

(7) If the Sarajevo shots had not caused the First World War in t, some other event would have caused the First World War in some instant t' posterior to t.

In other cases the seeming preemption is not *anticipated* causation but *delayed causation*. The famous case of the thirsty traveler may be classified in this category, provided we make the reasonable assumption that poison is quicker than dehydration[15]. However, we can imagine a case in which the

[14] For this theory see Pizzi [16].

[15] The story says that a traveler has to cross desert with a can full of water, but two enemies try to kill him – the first by making a hole in the can, the second by poisoning the water. The traveler dies without touching the water. If the traveler had not died thirsty he would have died poisoned. But the poison is normally quicker than thirst , so we can say that he would had died poisoned before than the moment in which he really died. So in a sense the hole delayed his death.This

poison takes exactly the same time as thirst in killing the victim or a case in which we are unable to calculate the time of action, so in this case we conventionally stipulate that the two processes take the same time. In this case the first event preempts the other even they are not overdetermining. There is no doubt, to begin with, that the two causes are symmetrically concurring, since it is true both

(8) $\neg Oe_1 >_C (\neg Oe_2 >_C \neg Oe_3)$

and

(9) $\neg Oe_2 >_C (\neg Oe_1 >_C \neg Oe_3)$

But now we have to add the supplementary qualification that one of the two causes preempts the other. What does it mean to preempt? A naïve idea is that to preempt means to interrupt a causal chain. Now a causal chain is often seen as a transference of some quantity (speed, weight, force, energy, ...) from a three-dimensional object to another (as in so-called transference theory of causation). This idea has surely an appeal for physicists and for Aristotelian philosophers, but is insufficiently general. Negative events such as silence, darkness, fast, etc... may be causes or effects, and they are at the origin of the transference of nothing. In the example of the traveler, to say that he died by thirst (absence of water) is to give the example of a negative event causing something. If we switch off the light of the lamp this implies that the pressing of the button causes an interruption of electric current, and this is not clearly a transference of anything.

The idea that we want to propose here is that what preemption blocks is not a causal flux but a possible inference from the effect to one of the causes[16]. More clearly, the inference which is blocked in the case of preemption is the abductive inference from the effect to the preempted cause. When a preempting cause leaves a track in the effect, this means that there is something in the effect which makes an abduction possible for some or all inferential agents: but this inference becomes impossible from the effect to the preempted cause. From the fact that the can has been perforated some rational x infers that no y can infer from the fact that the victim died in the known conditions (empty can etc.) that the victim was poisoned. The correct formal rendering of this simple idea is not straightforward because we have at least two possibilities of formal rendering:

asymmetry suggested to R. Smullyan the idea that the poisoner is really more guilty than the perforator.

[16] This is not the proper place to make a comment about the philosophical controversy over preemption/overdetermination. According to the ideas of Lewis and Bunzl no genuine case of overdetermination exists. In fact either 1) the compared effects are really different or 2) a cause preempts another one, as when an electron on a wire prevents another electron to reach the bulb. The following example of of trumping preemption states a case in which we have premption without the interruption of any transmission of energy or of other entities.

a1) $Oe_1 >_C \neg(Oe_3 >_A Oe_2)$
a2) $(Oe_3 >_A Oe_2) >_C \neg Oe_1$

The two formulas are not equivalent since $>_C$ and $>_A$ are not in general contrapositive. Both a1) and a2) actually look suitable to the case of what Jonathan Schaffer in [18] called *Trumping Preemption*. Let us suppose that a major and a sergeant are in front of a corporal, both shout "Charge!" at the same time, and the corporal soon charges (Oe_3). From the rules of military code we understand that the major's order (Oe_1), not the sergeant's order (Oe_2), caused the corporal's decision to charge (*ubi maior minor cessat*). After examining Lewis' and Ramachandran's theories, Schaffer concludes that no one of these theories is able to treat this kind of pre-emption. Our proposal appears to be free from the mentioned difficulties since it does not postulate any interruption of any causal chain. The reason why we say that the major's order preempts the sergeant's order is that, given the mentioned circumstances and the order of the general Oe_1, such an event disallows a correct abduction from the effect Oe_3 to the order of the sergeant Oe_2. Let us simply recall that abduction is inference to the best explanation, and that the military law by which a soldier obeys the order of the higher-degree military is part of the background theory T.

The asimmetry of preemption is then granted by the falsity of $Oe_2 >_C \neg(Oe_3 >_A Oe_1)$ and of $(Oe_3 >_A Oe_1)$.

4 The Role of Iteration in Abductive Reasoning

So what we showed up to now is: i) that iterated conditionals are essential to reconstructing causal reasoning and ii) that abductive conditionals may occur as subclauses of iterated conditionals which are important ingredients of complex causal statements. However, a statement like (a2), in which the abductive conditional occurs in antecedent position, is an additional statement and strictly speaking is not part of the statements which identify the core of the causal statement. We recall here an important remark contained in Goodman, which states that every counterfactual is equivalent to a factual conditional[17]. Goodman suggests that every counterfactual, as for instance

(10) If the match had been scratched it would have lit $(A >_C B)$

is equivalent to

(11) Since that match was not lit it was not scratched $(\neg B >_A \neg A)$

If this remark were correct, every counterfactual would be equivalent to the contrapositive abductive conditional. Unfortunately the development of the semantics for conditional logics made us familiar with the already mentioned

[17] See Goodman [7].

idea that conditionals are not in general contrapositive. This means that not every case of iterated counterfactual may be generally turned into an iterated abductive conditional, even this transformation may be a legitimate one in a class of cases which can be exactly delimitated. It is however remarkable that abductive conditionals may occur in iterated conditionals independently from the equivalence with some iterated counterfactual. Some simple examples are in order.

Suppose we accept that C is the best explanation of B ($B >_A C$). Suppose that we know that B' is similar to B in important features and C' is similar to C in important features, given the same informations which make $B >_A C$ a true conditional. So some inferential agent x could infer from $B >_A C$ that $B' >_A C'$: so it turns out that the second degree conditional $(B >_A C) >_C (B' >_A C')$ is a true conditional. The inferential laws which are involved are analogical laws of inference. Note that $B >_A C$ may be a counterfactual or afactual antecedent of $>_C$, and that the schema of the argument to apply here is similar to Thomason' example.

Examples of the previous schema are not difficult to construct when B and C stand for singular propositions expressing real possibilities. For instance, if the best explanation of my cold in the circumstances of yesterday is that I have been under the rain without umbrella, the best explanation of your cold in the same circumstances may be a token-event of the same kind.

But the same schema holds for any $B > C$ which is a counterfactual or counterlegal conditional. Counterlegal or counterpossible suppositions are legitimate suppositions in scientific reasoning[18]. An instance is the following. We know that every planet has an elliptical rotation round the Sun, so to suppose that some planet, say Venus, has a circular rotation round the Sun is to suppose something which is impossible. This means that we should remove from our background knowledge one of Kepler's laws, i.e. the law that every planet has an elliptical orbit, and replace it with a different law. However, we have no reason to reject the more general law.

(12) All planets have orbits of the same form

which is a generic variant of the rejected Keplero's law. It follows then the following conditional is true: If planets had a circular rotation (A) some agent x would infer from this that some rational agent y supposing that Alpha Centauri were a planet (B) would infer that Alpha Centauri would have a circular rotation (C):

(13) $A >_C (B >_C C)$

But note that also the abductive conditional would be appropriate in this example:

(14) $(B >_C C) >_A A$

[18] See Pizzi [17].

In fact, if someone is able to infer correctly from the supposition that Alpha Centauri is a planet (B) that Alpha Centauri has a circular orbit (C), the best explanation of this strange argument would be the belief in the false law that planets have a circular orbit. This use of abduction in iterated conditionals will of course seem far-fetched. However, an use of nested abduction may occur more naturally in other contexts: for instance when we want to test the normality of background conditions. A rational agent in normal conditions infers from the existence of smoke the existence of past or present fire. But smoke may be caused in non- normal conditions by other kinds of phenomena (for instance by frozen carbon dioxide, i.e. dry ice). So we could say

(15) If some x were able to exclude from other informations that fire was present, some y would conclude from this and the presence of smoke that dry ice was present.

The form of (15) is $(A >_C \neg B) >_C (C >_A D)$. With a further step, it is not difficult to find examples of abductive statements construed over lower-degree abductive statements. For instance, if someone can make an inference from smoke to dry ice, we might conclude abductively that in those circumstances an abduction from smoke to fire is impossible: $(C >_A D) >_A \neg(C >_A B)$. And this is a meta-abduction about abductions.

5 Abductive Conditionals and Standard Second Degree Conditionals

In the preceding sections we have treated $>_C$ and $>_A$ as operators belonging to the same family, i.e. as subspecies of the same species. But a famous quotation in which Peirce introduces his notion of abduction makes us reflect deeper on the properties of abductive conditionals. In fact the most quoted definition of abduction introduced by Peirce is the following

(16) "The surprising fact, F, is observed; But if H were true, F would be a matter of course. Hence, There is reason to suspect that H is true" [12, 5.189].

Neglecting the condition that C should describe a "surprising" (i.e. improbable) fact, a *prima facie* formal paraphrase of Peirce's analysis is as follows:

(17) $A >_A C$ is true if and only if $C \wedge (A >_C C) \models A$.

But this rendering could be criticized along the same lines used by Thomason against a similar paraphrase of embedded conditionals. In fact, the consequence relation used here is not monotonic, since the addition in the antecedent of some supplementary information D might destroy the validity of the inference. Thus we are justified in supposing that the correct formal rendering is offered by the equivalence

(18) $(A >_A C)$ if and only if $(C \wedge (A >_C C)) >_C A$.

This rendering, however, could also be questioned since $>_C$ appears not to be a proper formalization of "There is reason to suspect". On the other hand, "there is reason to suspect" cannot be an abductive conditional since this would make the definition a circular one. A possible way out is that a proper rendering of the intended meaning would be to put in place of the second occurrence of A the formula $Pr(A) > \delta$, where δ is some threshold probability value. Since we are willing to treat probability as a particular kind of graded modality (see section 1), this is simply a way to say that A is true in a reasonably great class of (epistemically) possible worlds. We have also to remark that the only way to grasp the idea that A is the best explanatory factor of C (an idea which is not explicit in Peirce's words) is to state that A is more information-preserving than every other rival hypotheses $A'_1 \dots A'_n$, in the terms which have been formulated in section 3. But this or some other qualification does not modify the second order characterization of abduction which is clearly implicit in Peirce's proposal. As a matter of fact, Peirce's definition opens the road to an inquiry about the relations between $>_C$-conditionals and $>_A$-conditionals which may usefully amplify the limits of the theory which has been outlined in the present paper.

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