Evolutionary Combinatorial Programming for Discrete Road Network Design with Reliability Requirements

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Abstract. This paper examines the formulation and solution of the discrete version of the stochastic Network Design Problem (NDP) with incorporated network travel time reliability requirements. The NDP is considered as a two-stage Stackelberg game with complete information and is formulated as a combinatorial stochastic bi-level programming problem. The current approach introduces the element of risk in the metrics of the design process through representing the stochastic nature of various system components related to users' attributes and network characteristics. The estimation procedure combines the use of mathematical simulation for the risk assessment with evolutionary optimization techniques (Genetic Algorithms), as they can suitably address complex non-convex problems, such as the present one. The implementation over a test network signifies the potential benefits of the proposed methodology, in terms of intrinsically incorporating stochasticity and reliability requirements to enhance the design process of urban road networks.

Keywords: Network Reliability, Stochastic Discrete Network Design, Game Theory, Mathematical Simulation, Genetic Algorithms.

1 Introduction

The role of the design of transportation networks has been recognized as a crucial element to foster the mobility of people and goods in growing metropolitan areas. The conflicting goals of saving scarce resources, such as land and public funds, allocated to road infrastructure and accommodating the increased demand for passenger and freight transportation prompt the need for deploying compromise and efficient design solutions. In addition to the supply of adequate capacity, the volatile traffic conditions caused by recurrent congestion phenomena and incidents amplify the complexity of the design problem, since they raise risk concerns with regard to the reliability of the provided transportation services. Further risk concerns also emerge from the need for making urban transportation networks capable to provide sufficient lifelines during unexpected, emergency situations, due to man-made or physical disasters.

The question of determining the optimum network design, typically referred to as the Network Design Problem (NDP), can be traditionally addressed through two

M. Giacobini et al. (Eds.): EvoWorkshops 2007, LNCS 4448, pp. 678–687, 2007.

different mathematical forms (for general reviews, see [1], [2]). The first form refers to the Continuous-NDP (C-NDP), where the capacity of the system is treated as a continuous variable and can be expressed in terms of vehicles, passengers and unit loads. The second form refers to the Discrete-NDP (D-NDP), which is formulated in terms of discrete (integer or binary) variables, such as the number of new links, in the case of network expansion, or the number of lane additions, in the case of network enhancement etc. Despite that current research on the NDP is particularly active, the majority of existing studies deals with the continuous form, which can be regarded as a relaxation of the discrete one. Moreover, although several reliability considerations have been incorporated into the structure of the C-NDP (see [3], [4], [5]), no such attempt has been made for the case of the D-NDP.

The present study addresses the reliable D-NDP, i.e. the D-NDP with reliability requirements. The solution of such a problem is infrastructure related, since it corresponds to the number of added lanes and new links, which may have a greater bearing to designing the required civil works, in comparison to the solution of the corresponding C-NDP. The total travel time reliability is considered here as a network quality performance indicator, since it reflects the ability of the network to respond to different states of the system. The study provides a formulation and a solution algorithm for the reliable D-NDP, whose application is illustrated for a simplified network with typical urban road settings. Section 2 presents the formulation of the reliable D-NDP. Section 3 analyzes the components of the network reliability and describes a simulation method for performing the risk assessment. Section 4 presents an evolutionary algorithm for the efficient solution of the complex D-NDP. Section 5 includes the results obtained from the application of the method into the test network and Section 6 concludes.

2 Formulation of the Reliable D-NDP

Similar to many other transportation planning problems, the network design process is essentially affected by decisions made on multiple hierarchical levels, concerning both the demand and supply properties of the system [6]. The design process of transportation networks (system) is regarded as a game among two players namely the system designer and the system users, whose decisions made individually affects both their performance. The particular structure of the above game has the form of a two-stage leader-follower Stackelberg game with perfect information, with the system designer to be the leader imposing modifications on the network attempting the optimization of the system performance while the users reacting as followers to alternative design plans. The formulation of such games is usually has the form of bi-level programming problems, where optimum strategies are sought by taking into account a number of constraints, including those of physical feasibility and budget availability, while considering the demand and supply attributes of the system as known but not necessarily fixed. This study extends the standard game-theoretic, bi-level programming formulation of the D-NDP so that include reliability requirements.

Consider a network composed of L links, R origins and S destinations. The travel demand q_{rs} gives rise to equilibrium flows f_k^{rs} along path $k \in K_{rs}$ connecting

$r-s$ pair and to equilibrium flows $x_a(y)$ along link a, with δ_{ak}^{rs} be the path-link incidence variable and $c_{rs}^k \in C^{rs}$ be the cost of traveling along the k th path between $r-s$ pair. The travel cost, at equilibrium state, of some link a with capacity y_a is denoted as $c_a(x_a(y))$, with y be the total network link capacity, and w_a is a binary decision variable of link a, as follows: *1* if link a is added to the network or an extra lane is added to link a, and *0* otherwise. Also, $V_a(w_a)$ denotes the monetary expenditures for adding a link a or a lane on link a, B is the total available construction budget for network capacity improvement, and θ is a factor converting monetary values to travel times. Then, the *Upper-Level Problem*, which comprises the objectives of the optimum NDP, and the *Lower-Level Problem*, which provides the path and link equilibrium flows, can be given as follows:

Upper-Level Problem:

$$\min_y F(x,y) = \sum_{a\in A} \left(E[c_a(x_a(y), w_a) x_a(y)] + \theta V_a(w_a) \right) \tag{1}$$

$$\text{Subject to} \qquad w_a \in \{0,1\}, \qquad \forall\, a \in A \tag{2}$$

$$\sum_{a\in A} (V_a(w_a)) \le B, \qquad \forall\, a \in A \tag{3}$$

$$P(\sum_{a\in A} (c_a(x_a(y), w_a)\; x_a(y)) \le T) \le Z, \quad \forall\, a \in A \tag{4}$$

Lower-Level Problem:

$$\min_x G(x) = -\sum_{rs} q_{rs}\, E\left[\min_{k\in K_{rs}} \{c_{rs}^k\} | C^{rs}(x) \right] + \sum_a x_a c_a(x_a) - \sum_a \int_0^{x_a} c_a(\omega) d\omega \tag{5}$$

$$\text{Subject to} \qquad f_k^{rs} = P_k^{rs}\, q_{rs}, \qquad \forall \;\; k, r, s \tag{6}$$

$$x_a = \sum_{rs} \sum_k f_k^{rs}\, \delta_{ak}^{rs}, \quad \forall\, a \in A \tag{7}$$

$$f_k^{rs}, x_a \ge 0, \qquad \forall\, a \in A \tag{8}$$

In the Upper-Level Problem, $F(x,y)$ represents the objective function of the NDP, wherein the first component refers to the travel cost expressed in terms of the expectation E of network *Total Travel Time* (*TTT*), and the second component corresponds to the total expenditures (in time units) for capacity improvements. The choice set defined in relationship (2) represents the binary selection of link (or lane) addition, while inequality (3) imposes budgetary restrictions. The reliability requirements are introduced in constraint (4), through restricting the probability of the *TTT* to be lower than or equal to a pre-specified upper limit T, with Z defining the acceptable confidence interval ($0 \leq Z \leq 1$) for this hypothesis. Such a condition essentially depicts the stability of the system [7].

The Lower-Level Problem, which consists of functions (5) to (8), performs the trip demand assignment process, based on the expected (perceived) value E of the path travel cost c_{rs}^{k}. Specifically, it estimates the response of users to the capacity improvements made at the Upper-Level Problem, through determining the probability P_k^{rs} that a traveler chooses to use path k between $r-s$ pair. The Stochastic User Equilibrium (SUE) model [8] is used here for the assignment of demand onto the network and the solution Method of Successive Averages (MSA) is employed to calculate equilibrium flows.

3 Modeling Reliability Assessment in the D-NDP

The operational performance of transportation systems typically relies on variables of uncertain nature, as their values are influenced by random events and human decision-making processes. The risk involved in the operation of transportation networks can be mainly attributed to the uncertainty pertaining to four different components: the demand, the supply (capacity), the level of service (link travel time) and the users' characteristics (route choice behavior). By and large, travel demand patterns in urban transportation networks can be considered as recurrent, at typical operating conditions. Nonetheless, several disturbances can be observed, as expressed by seasonal or random spatial and temporal variations of demand between origins and destinations, due to special (planned or unexpected) events in different network localities, causing significant fluctuations in link travel times. These disturbances are additionally influenced by several characteristics of travelers, which are mostly related to factors involved in the route choice decision-making process, including perception of travel cost, value of travel time and driving behavior. Capacity fluctuations is also a typical phenomenon in transportation networks, which can emerge from several factors, such as changes in the composition of traffic, congestion effects and other, random phenomena, like accidents, road workzones and adverse weather conditions. The problem of network reliability degradation considering fluctuations in link capacities has been extensively investigated in the literature (see [9], [10], [11]).

The current study provides a mathematical simulation framework for representing the stochastic properties and, in turn, the fluctuations of the values of variables, i.e., demand, supply and travel time, which affect the system performance. In particular, the demand is considered here as a random variable following the normal distribution $N(\mu_{rs}, \sigma_{rs}^2)$, with μ_{rs} denoting its mean for each $r-s$ pair and σ_{rs}^2 denoting its variance. Despite the fact that travel time can be also described by the normal distribution [7], an alternative, more explanatory assumption is made here, assuming that link speeds follow a multinomial normal distribution correlated with the speeds of the neighboring links. A similar assumption is adopted for the distribution of link capacities.

The current framework connects link travel time to link speed and capacity fluctuations and it enables to express the interaction between the costs of each link. In this way, link travel time variability (which is the result) is intrinsically modeled, in the structure of the D-NDP, with regard to its causal phenomenon (which is the link capacity and speed variability). More specifically, the Lower-Level Problem enables the estimation of the statistical properties of the *TTT*, i.e. its mean value and variance, which are subsequently fed to the Upper-Level problem, through iterating the solution of the assignment procedure, comprising the set of link and path equilibrium flows, the values of origin-destination demand, link capacity, and the link free flow travel time, in accordance with the stochastic characteristics assigned to these variables, as described previously.

The estimation of the statistical properties of *TTT* and, hence, the reliability assessment, are performed through the simulation method of the *Latin Hypercube* sampling. In comparison to other simulation methods, such as that of Monte Carlo simulation, Latin Hypercube is based on a stratified random procedure which provides an efficient way to capture the properties of the stochastic variables from their distributions, namely, it produces results of higher accuracy without the need for increasing the sampling size, and it allows to model correlations among different variables. In particular, the procedure of Iman and Conover [12] is followed here in order to produce correlated random numbers from the normal distribution, based on the Cholesky decomposition of the correlation matrix. The assumptions concerning the usage of the simulation method in the network design process are:

i) The duration of changes in link speeds and capacities allows users to re-estimate route choices, and
ii) Link speed reduction is due to random events (like accident, physical disaster or other) which affect a locality of the network and, hence, link capacities and speeds are correlated with those of neighboring ones.

4 Evolutionary Algorithm for Solving the Reliable D-NDP

The D-NDP, as well as the C-NDP, can be generally characterized as problems of increased computational complexity. This complexity arises from the fact that bi-level programming problems, even for simple linear cases, are Non-deterministic Polynomial-time (*NP*)-hard problems [13]. In particular, the D-NDP is a *NP*-hard, non-convex combinatorial problem [14], since its set of constraints involves non-linear formulations, such as those of the SUE assignment of the Lower-Level Problem. Several algorithms, appropriate for addressing complex combinatorial (integer or mixed-integer programming) problems, have been hitherto proposed and implemented to solve the D-NDP. Such algorithms include the branch-and-bound method [15], Lagrange relaxation and dual ascent procedures [1] and a method based on the concept of support function [16].

The present study uses evolutionary strategies, in particular, a Genetic Algorithm (GA) [17] to address the difficulties of obtaining optimal solutions within the proposed framework of the D-NDP. GAs have been extensively used in solving complex, *NP*-hard, combinatorial problems. Furthermore, they have been widely used in various bi-level programming problems (see [18]) and, especially, in addressing the C-NDP [19]. This wide applicability of GAs can be attributed to their convenience in handling variables of stochastic nature and multiple constraints in a seamless way, without requiring information about the nature of the problem but only about the performance of a ‘fitness’ function for various candidate states.

GAs are population-based stochastic, global search methods. In the context of the D-NDP, an individual of the population corresponds to alternative binary codings of the link and lane additions. For every individual of the population, a Latin Hypercube simulation is performed altering the travel demand, link travel time and capacities, based on the framework described in Section 3, in order to estimate *TTT* reliability. The steps of the solution procedure are given to the pseudo-code shown below:

Step 1. (Initialization)

Produce an initial random population of candidate feasible solutions (link capacity improvements or link additions) and select the properties of the genetic operators

DO UNTIL CONVERGENCE:

Step 2. (Path Enumeration)

Perform path enumeration for every candidate solution

Step 3. (Simulation)

Estimate the *TTT* reliability for every candidate solution by Latin Hypercube simulation.

Step 4. (Genetic Evolution Process)

4.1 Check for the consistency of constraints and estimate the ‘fitness function’ (1) of each candidate solution.

4.2 Perform a stochastic selection of the ‘fittest’ solution set and the crossover operation among the selected ‘individuals’

4.3 Perform the mutation of individuals

4.4 Produce a new population of genetically improved candidate solutions

Though the extended use of meta-heuristic techniques such as GAs for solving complex problems, the solutions provided by them have met some skepticism. This is mainly because they are heavily dependent on initial conditions and random search processes incorporated into them. In order to confront this problem, multiple runs of the GA are performed in this study to confirm that the solution provided is optimal (or adequately near-optimal).

5 Test Application and Results of the Algorithm

The proposed methodology for solving the reliable D-NDP is implemented into a test network. The specific network layout has been used in [16] and is composed of a single origin-destination pair (from node #1 to node #12), 12 nodes, 17 existing links and 6 candidate new links. Fig. 1 shows the complete configuration of the test network, including a total of 23 links and 25 paths, after making all possible improvements, i.e. adding the links figured #18-23. The network is considered as fully degradable, in the sense that all links exhibit fluctuations in speed and capacity. The complexity of this specific combinatorial problem, although the small scale of the given network, can be considered as high, since it contains L=23 links (variables), yielding $2^L=2^{23}=8388608$ possible combinations (alternative construction plans) of network capacity improvement.

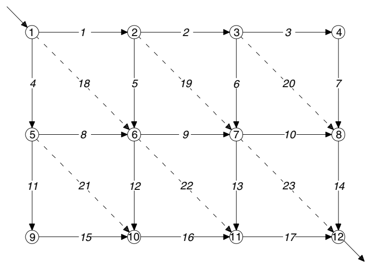

Fig. 1. The test network layout: existing links (*solid lines*) available for lane additions and potential new links (*dashed lines*)

In the current study, the link travel time t_a at some link a is expressed as a function of the random free-flow travel time t_a^f, traffic flow x_a and random capacity y_a at this link and is carried out by using the standard formulation of the Bureau of Public Roads (BPR), as follows:

$$t_\alpha = t_a^f\left(1+\beta\left(\frac{x_\alpha}{y_a}\right)^m\right), \tag{9}$$

where β and m are scale parameters depending on the operational characteristics of the network. Although the estimation of link travel time is based here on the BPR formula, which typically applies to uncongested road networks, other formulations could also be adopted for taking into account congestion effects, like queues or bottlenecks in the links of the network.

The capacity of each of the existing links is set equal to 20 vehicles per hour (veh/hr), while, after a lane addition, the capacity increases to 30 veh/hr. The capacity of each of the new links is set equal to 20 veh/hr. The demand between the origin-destination pair is set equal to 80 veh/hr. The cost of lane addition in each of the existing links is set equal to 30 monetary values, while the construction cost of each of the new links is set equal to 50 monetary values. This study adopts a conversion factor $\theta=1$. The free-flow travel time, which is proportional to the link length, is set equal to $t_a^f=1$ min, for the existing links, and $t_a^f=1.4$ min, for the new links. The complete reconstruction of the given network, which requires a lane addition to each of the 17 existing links and construction of 6 new links, amounts to a total of 810 monetary values, corresponding to 810 vehicle-minutes (veh-min), in time units, since

$\theta = 1$. Nonetheless, such a scenario may be considered as too expensive and, hence, impractical in real-world situations. For this reason, the half of this amount, i.e. 400 veh-min, is set here as the total available construction budget B, which can be regarded as sufficient for enhancing the capacity of the existing network. The estimation of the stochastic user-equilibrium link flows employs a total of 200 MSA iterations, which were found to be adequate for providing a stable solution to the Lower-Level Problem for the given test network.

An initial solution is first obtained by solving the D-NDP without reliability requirements. The assignment of the travel demand onto the initial network (with no link improvements or additions) results in a *TTT* equal to 886 veh-min. A total number of 50 runs of the solution algorithm are performed. The current GA employs a population of 50 individuals and its convergence criterion requires, on average, a number of 50 generations, which ensures that no further significant improvement of the objective function value can be achieved. For each individual, 200 iterations of the Latin Hypercube simulation are performed to obtain the stochastic properties of the system variables (see Section 3). Both the variances of the multinomial normal distributions of the free flow speed and link capacity are set here equal to the 20% of their corresponding theoretical (nominal) mean values. Similarly, the variance of the normal distribution of travel demand is set equal to the 20% of its mean (set) value.

The initial solution provides a construction plan encompassing the addition of the new links #18 and #23 and lane addition to the existing links #1, #9 and #17. The resulting construction cost amounts to 190 veh-min, which is lower than the total available budget (400 veh-min). The *TTT* reduces from 886 veh-min to 449 veh-min. Thus, the total cost (*TTT* + construction cost) comes to 449+190=639 veh-min. The existence of a considerable remaining (not allocated) portion of the total available budget, i.e., 400-190=210 veh-min, can be attributed to the fact that there is a threshold beyond which network capacity improvements become very expensive, in comparison to their contribution to the reduction of the *TTT*.

A new solution of the D-NDP is then obtained by imposing, as reliability requirement, the probability of the occurrence of the *TTT* to be higher than T =500 veh-min not to exceed 10%. This upper limit value expresses the 10% increment of the *TTT* value (449 veh-min) resulted from the initial problem solution. The new solution leads to the construction of two more links, i.e. links #19 and #22, in addition to the improvements resulted from the initial solution of the problem. The *TTT* is further reduced from 449 veh-min to 423 veh-min, while the construction cost is raised to 290 veh-min, which is still less than the total available budget. The raised construction cost is attributed to the need for increasing the amount of the sparse total network link capacity (system redundancy) in order to ensure the desired level of network reliability. The new solution results in the increase of the total cost from 639 veh-min to 423+290=713 veh-min. The resulting probability of the *TTT* to be higher than 500 veh-min was estimated to 9.94%, which is lower than the acceptable upper bound of 10%. As it is shown in the two histograms of Fig. 2, the dispersion of the *TTT* obtained from the initial solution without imposing reliability requirements, having $P(TTT \geq 500\,veh - \min) \approx 0.25$ (see left diagram) is considerably wider than the dispersion of the *TTT* obtained from solving the reliable D-NDP, having $P(TTT \geq 500\,veh - \min) < 0.1$ (see right diagram).

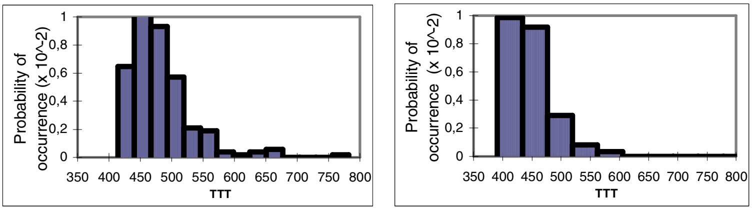

Fig. 2. Distribution of the *TTT* for the case without reliability requirements (*left*) and with reliability requirements (*right*)

6 Conclusions and Future Research

The current study provided a formulation and a solution algorithm for the Discrete-Network Design Problem (D-NDP). The formulation seeks the optimal network capacity improvements, subject to budgetary and physical restrictions, and, additionally, reliability requirements, in terms of the probability of the Total Travel Time (*TTT*) to be less than a pre-specified value. The model enables the inclusion of four different sources of uncertainty, i.e., demand, capacity, link travel time and route choice, into the reliability assessment, through applying the Latin Hypercube sampling simulation method. The estimation procedure uses a Genetic Algorithm, which is suitable for solving such types of stochastic combinatorial optimization problems. The test network application of the method demonstrated the beneficial impact of including reliability requirements into the standard bi-level programming formulation of the D-NDP. The benefits correspond to the reduction of the *TTT*, while satisfying the desired level of network reliability and the budget constraints.

Future developments of the method refer to the incorporation of other types of uncertainty, which affect network reliability, such as those concerning the information acquisition and the day-to-day (or period-to-period) adjustment of the decision-making process of users. Moreover, the current estimation framework could be extended into the more general case of multi-modal networks with multiple-class users, in order to address issues related to the sustainable network development. Finally, a comparison of the current evolutionary approach with other derivative-free algorithms suitable to handle such combinatorial problems could provide useful insight into the properties of the solution of the NDP.

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