A Macroscopic Model for Self-organized Aggregation in Swarm Robotic Systems

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Abstract. We study the self-organized aggregation of a swarm of robots in a closed arena. We assume that the perceptual range of the robots are smaller than the size of the arena and the robots do not have information on the size of the swarm or the arena. Using a probabilistic aggregation behavior model inspired from studies of social insects, we propose a macroscopic model for predicting the final distribution of aggregates in terms of the parameters of the aggregation behavior, the arena size and the sensing characteristics of the robots. Specifically, we use the partition concept, developed in number theory, and its related results to build a discrete-time, non-spatial model of aggregation in swarm robotic systems under a number of simplifying assumptions. We provide simplistic simulations of self-organized aggregation using the aggregation behavior with different parameters and arena sizes. The results show that, despite the fact that the simulations did not explicitly enforce to satisfy the assumptions put forward by the macroscopic model, the final aggregate distributions predicted by the macroscopic model and obtained from simulations match.

1 Introduction

Aggregation, defined as "the collecting of units or parts into a mass or whole" [1], can be considered as one of the fundamental behaviors of swarms. In nature, aggregation behaviors, observed in organisms ranging from bacteria to social insects and mammals[2], increase the survival chance of the swarm in hostile environments. Although some of these aggregations can be traced back to environmental cues, others are self-organized.

We believe that self-organized aggregation, that do not require a cue from the environment or centralized control, is an essential competence for swarm robotic systems[3,4]. In these systems, aggregation behaviors can act as precursors for more complex behaviors such as flocking, pattern formation or self-assembly[5]. However, like other behaviors that produce self-organization, engineering aggregation behaviors is a major challenge. Although the general structure of aggregation behaviors can be inspired from studies of social insects, the relationship

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of behavioral parameters, and environmental factors to the performance of self-organized aggregation remains an open problem.

In this paper, we propose a probabilistic aggregation behavior for swarm robotic systems and develop a macroscopic model to predict the performance of the aggregation behavior under different parameters of the swarm system.

2 Related Studies

2.1 Aggregation

Studies of aggregation can be grouped in three different but related fields; namely, social insect studies, control theory, and swarm robotics. In social insect studies, aggregation, a rather common phenomenon in ants, cockroaches, etc., is a rather well-studied phenomenon. In [6], Deneubourg et al. studied the aggregation behavior of cockroaches, that aggregate in hiding sites. They studied the modulation of the resting time, defined as the time a cockroach spends in an aggregation. They observed that the resting time of cockroaches is proportional to the number of individuals in the aggregate. Hence, individuals tend to spend more time in large aggregations, and that this provides a positive feedback for growth of aggregations. In another study[7], Jeanson et al. presented a model of aggregation in cockroach larvae in homogeneous conditions. It was observed that the behavior of individuals depend on the number of larvae in their close vicinity. The authors computed the parameters of their model through systematic experiments on the larvae, and showed that similar aggregations can be obtained in simulations using these parameters.

In control theory, aggregation is often referred as the gathering, the agreement or the rendezvous problem. In most of these studies[8,9], however, the robots are modeled as points without orientation neglecting even the physical embodiment of the robots. Also, it is usually assumed that all robots can perceive the location of all the other robots in the swarm. In these studies, the major axis of research focus on the convergence characteristics of aggregation methods. For instance, in [10], it was shown that explicit bounds on the swarm size and bounds on the time of convergence can be obtained for aggregation.

When perception range is limited, however, aggregation of robots into a single aggregate becomes a more difficult problem. Deterministic algorithms work only when there are no isolated robots in the swarm [11]. Even without isolated robots, it was noted that, convergence may take an infinitely long time [8] in some scenarios. In the same study, Flocchini et. al proposed an aggregation algorithm which can provide guaranteed aggregation in finite time that required limited visibility with distinguishable robots and a common orientation decided by the robots. In a similar approach proposed in [12], Lin et al. utilized the geometric constraints on the behavior of robots to develop an aggregation behavior.

In swarm robotic systems, the problem of engineering and evaluation of aggregation behaviors have been tackled by a number of studies. We would like to first note that, aggregation refers to the forming of aggregates by the robots themselves, and is fundamentally different from the aggregation of passive items

(like pucks) by a swarm of robots. We consider the latter problem as the clustering problem, and distinguish it from the aggregation problem.

In [13], one of the early studies on the problem, robots were required to form aggregates of pre-determined size around infrared beacons. Inspired by birds and frogs, the proposed method used a chorus consisting of individuals who can approximate the size of the aggregates using variations in sound. This method was also tested on systems without infrared beacons that trigger aggregation. The results indicated that, self-organized aggregation can be obtained with this method only in virtually noiseless environments.

In [14], Trianni et. al used genetic algorithms to study the evolution of neural networks to generate aggregation behaviors for a swarm of robots. Aggregation behaviors that were evolved in simulation, were partially tested on physical robots. It was shown that evolution was able to generate two different aggregation strategies: **(1)** static aggregation behaviors where robots remain still in aggregates, and **(2)** dynamic aggregation behaviors where robots continue moving in aggregates. It was shown that the evolved behaviors demonstrated a certain degree of scalability to generating aggregates in larger swarm sizes and larger arenas than the ones that the behaviors were evolved in.

Bahçeci et. al [15] investigated the use of evolutionary methods for developing aggregation behaviors. They systematically investigated the performance of behaviors evolved with different evolution parameters for the aggregation task. Based on the results of the systematic experiments, they proposed a number of rule of thumbs that can be used for evolving behaviors for swarm robot systems.

In a former study[16], we used a probabilistic aggregation behavior for studying aggregation in simulated swarm robotic systems. We investigated the effect of probabilistic parameters on aggregation performance through systematic experiments on a physics-based robot simulator, and identified different control parameters that lead to dynamic and static aggregation strategies.

2.2 Modeling

The engineering and evaluation of behaviors that generate self-organization in swarm robotic systems, such as self-organized aggregation, is a challenging problem. Although it is easy to propose generic behaviors for self-organization, it is hard to set their parameters and predict their performances for different swarm sizes and environments. Conducting systematic experiments is a difficult task, even with simulated swarm robotic systems, requiring a large amount of computation time. Despite this, however, the results obtained from such experiments provide little insight to the relationships between the performance of the behaviors and the parameters of the swarm system.

Constructing macroscopic models to describe the behavior of swarms can provide a viable approach to guide the design of swarm behaviors [17]. Macroscopic models, once developed, can provide explicit relationships between the parameters of the swarm system and its performance. Through such relationships, one can derive or evaluate the performances of a behavior under different parameters settings, and choose the optimum parameter values for a desired task. Despite

these advantages, however, building macroscopic models for swarm robotic systems is a challenging task and there are few studies in the literature.

Most of the existing macroscopic models[17,18,19] used rate equations and Markovian processes to model the behavior of swarm robotic systems. These models generally represent the environment and behavioral states with probabilistic variables and define the change of these variables. In [20], Martinoli et. al proposed a probabilistic model for puck clustering task. In this problem, a number of pucks that are initially dispersed within a bounded arena, are clustered by a swarm of robots. The robots grip pucks in the arena and transporting them closer to other puck clusters. The model developed in this study was validated against the simulated and physical robots through experiments. In [19], Agassounon et. al. extend the object aggregation model described in [20] to a macroscopic level.

In [21], Kazadi defined the global goals as mathematical constraints an synthesize behaviors as to satisfy them. The behavior of the system can be investigated using the goal constraints. Lee et. al applied this concept to robot aggregation in their recent work [22] and showed that a controller for aggregation can be constructed using results form the clustering algorithm.

3 Aggregation Problem

We define aggregation as the gathering of a swarm of robots, that are initially dispersed into a closed arena, into preferably a single aggregate. We assume that; **(1)** The arena is bounded. **(2)** The perceptual range of robots is smaller than the size of the arena and that the initial positioning of the individuals may not necessarily form a connected graph. **(3)** The individuals in the swarm do not have any knowledge regarding the size of the arena or the swarm.

The first assumption removes the possibility that some robots may get lost during the aggregation process and is very common in the aggregation studies with social insects[7] and swarm robotic systems[13,14]. The second assumption makes the aggregation problem more realistic and difficult. It also rules out any centralized coordination mechanisms that may be proposed for the problem. The third assumption rules out any solutions to the aggregation problem that may be specific for a particular swarm and/or arena size.

4 Aggregation Behavior

In this study, we use an aggregation behavior that is implemented as a finite state machine as shown in Figure 1(a). The behavior consists of three basic behaviors, namely *random walk*, *wait* and *approach*. The *random walk* behavior, once activated, controls the robot for a certain pre-specified duration, moving the robot in the arena randomly while avoiding other robots and the walls of the arena. After the expiration of the *random walk* behavior, if the robot has an aggregate in its perceptual view, the robot switches to the *approach* behavior, else, the robot switches into the *wait* behavior, creating a one-robot aggregate.

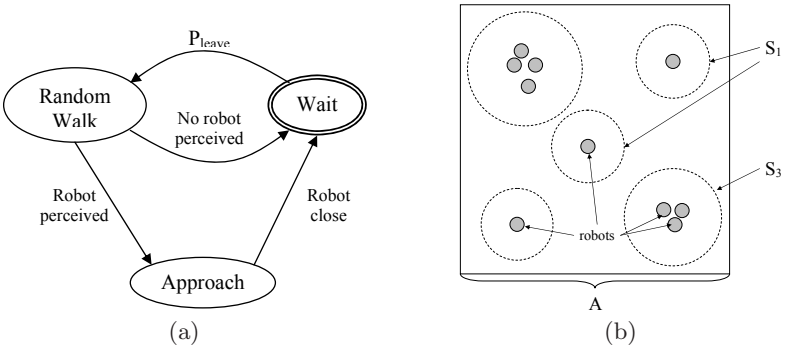

Fig. 1. (a) Aggregation behavior. Ovals display the simple behaviors and arrows represent the behavior transitions. **(b)** A sketch of the environment. The square frame represents the arena. The gray circles represent the robots and dashed circles represent the part of environment where the robot aggregate can be perceived by another robot, i.e S_m's. See text for more details.

In the *wait* behavior, the robot remains still with a certain probability to switch to the *random walk* behavior. The probability of leaving the *wait* behavior is denoted as P_{leave}. In the *approach* behavior, the robot moves toward the closest aggregate perceived. When the robot gets into the close proximity of the aggregate, the robot switches into the *wait* behavior.

We believe that the proposed aggregation behavior is consistent with the ones observed in natural swarms and is generic enough to represent different aggregation strategies through changes in transition probabilities. However, the performance comparison of different aggregation strategies that can be generated from this generic behavior, remains a challenge, and will be our motivation for constructing a macroscopic model.

5 A Macroscopic Model for Aggregation

We assume that there are n robots randomly placed in a closed arena of size A. We define an *aggregate* of robots as the group of robots who are in local proximity of each other, i.e. a connected group of robots who can sense each other through their proximity sensors. An aggregate which consists of m robots is called as an *m-aggregate*. The area within which an *m-aggregate* can be perceived by another robot is called as S_m, representing the area of the *attraction region* for the aggregate. Although, S_m would depend on the grouping of the *m-aggregate*, we will assume that all *m-aggregate*s have a rather compact grouping and that the area of their attraction regions can be approximated with a single S_m value. Figure 1(b) shows an exemplary sketch of the environment.

In the macroscopic model, the state of the swarm aggregation is denoted as a configuration $C_{a_1,a_2,..,a_k}$, where each a_k represent the existence of an a_k*-aggregate* in the arena and that $\sum_{i=1..k} a_k = n$. The configuration of a sample aggregation

state shown in Figure 1(b) is denoted as $C_{4,3,1,1,1}$ indicating that there exist five aggregates in the arena with sizes 4, 3, 1, 1 and 1. Such a representation corresponds to the notion of *partition* in mathematics which is formally defined as an unordered set of positive integers whose sum is n [23]. Finally, we would like to note that, in this representation the spatial positioning of the aggregates are ignored, and two distributions are considered equivalent if they have equal number of aggregates, all with the same size.

In order to analyze the evolution of the configuration of the robots, we need to consider possible changes in the size of aggregates. These changes occur when robots leave their current aggregates to form a new one-robot aggregate or to join an existing aggregate. Such changes can be modeled as transitions in the configuration of the swarm. Our ultimate goal is to construct a macroscopic model that can model these transitions to make predictions about the time evolution and performance of a certain aggregation strategy. However, within this paper, we will construct a constrained macroscopic model which will make two simplifying constraints; **(1)** Only single robot transitions happen among the aggregates. **(2)** The probability of robot transitions between two aggregates is independent of the distance in between.

The first constraint assumes that the probability of multi-robot transitions among aggregates is small and can be neglected to simplify the temporal analysis of the aggregation process. In swarm robotic systems, such an assumption can be approximately made to hold by choosing a small P_{leave} value for the aggregation behavior. The second constraint assumes that the probability for a robot, which left its current aggregate, to join another aggregate is independent of the distance between the two aggregates. This assumption is made to simplify the spatial analysis and can be considered to approximately hold when the duration of the *random walk* behavior is chosen long enough with respect to the size of the arena.

The changes in configurations can be modeled as probabilistic transitions and that a graph can be constructed to visualize them, as shown in Figure 2. In this graph, each node corresponds to a configuration and directed weighted edges represent the probabilistic transitions from one configuration to another. The central view of our macroscopic model is that, if one can compute the probability of transitions between these configurations from the parameters of the swarm system, it is possible to deduce the evolution of the aggregation in time. Here, the parameters of a swarm system consists of the parameters of the aggregation behavior, P_{leave}, the sensing characteristics of the robots through S_m values, the size of the swarm n, and the area of the arena A.

Note that the limitation of transitions to single-robot transitions simplifies the connectivity of the graph greatly reducing the complexity of the model. However, despite this, the number of possible configurations and the probabilistic transitions among them grows exponentially making it difficult, if not impossible, to compute the transition probabilities of the model.

Here, we would like to point out that, the ultimate goal of aggregation is to form a single aggregate that contains all the individuals in the arena. Hence, we propose to use the size of the largest aggregate in a configuration, as its performance

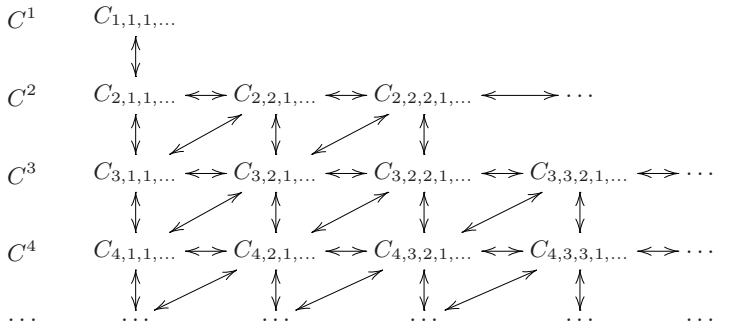

Fig. 2. Configuration graph re-ordered according to equivalence classes

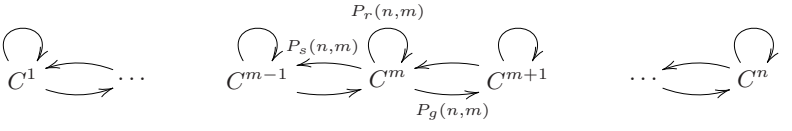

Fig. 3. Equivalence classes and the transitions between them

metric. With this metric, we can group all the different configurations with the same metric value, into equivalence classes. All the configurations that contain one or more *m-aggregate*s as their largest aggregate(s) can be lumped into a single equivalence class denoted by C^m. Hence, the transition graph of different configurations can be arranged according to their equivalence classes.

As a result of this grouping , in a swarm of size n, there can be only n equivalence classes. Also, using the single-robot transition assumption, we can limit the probabilistic transitions among these classes to transitions among consecutive equivalence classes in Figure 3.

Figure 4 shows all possible configurations of $n = 7$ robots as grouped into 7 equivalence classes. Note that, the spatial location of the aggregates is left out by our representation, and that the locations of the aggregates drawn on the figure is only exemplary.

With this representation, the system can only stay in one equivalence class or change into a neighboring equivalence class in one transition. The transitions between the equivalence classes will be denoted with three probabilities:

$\boldsymbol{P}_s(n,m)$: the probability that the largest aggregate shrinks.
$\boldsymbol{P}_g(n,m)$: the probability that the largest aggregate grows.
$\boldsymbol{P}_r(n,m)$: the probability that the largest aggregate remains the same.

5.1 Probabilities for Shrinking

Shrinking is defined as a change in the configuration such that the largest aggregate is reduced by one as the result of a single-robot transition. Effectively,

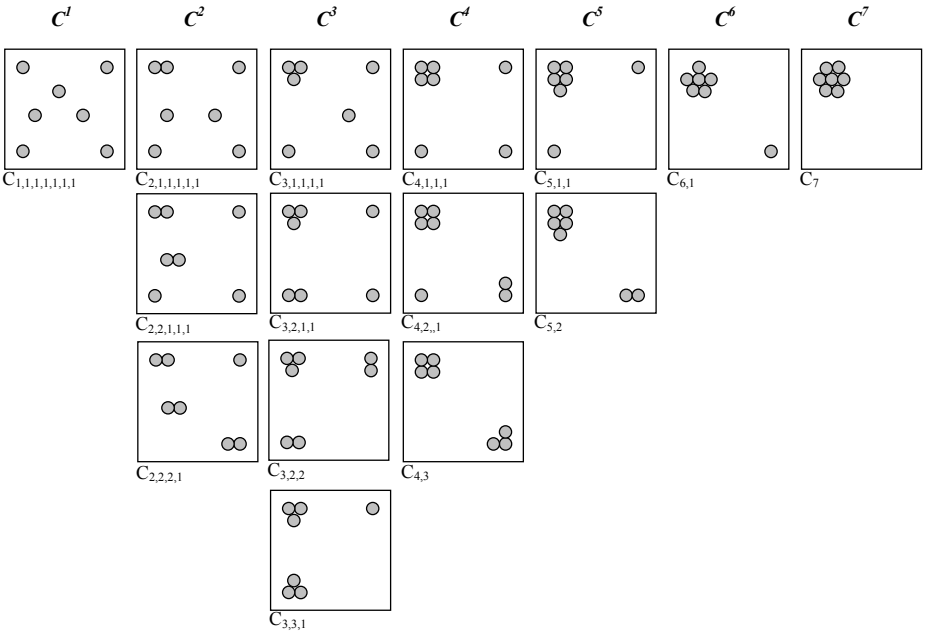

Fig. 4. All configurations of 7 robots grouped into equivalence classes

it means a transition from a configuration in equivalence class C^m to another configuration in equivalence class C^{m-1}.

First, note that the number of *m-aggregate*s must be 1 such that shrinking can occur. Let's call the number of *m-1-aggregate*s as k. If $k = 0$, meaning there is no aggregate with size $m - 1$ then there is a single aggregate the robot should not join for shrinking to occur, that is the aggregate it left. If $k > 0$ then the robot should not join any of these k aggregates and the aggregate it left. Therefore, the probability of shrinking when there are k *m-1-aggregate*s is

$$\frac{A - (k+1) \cdot S_{m-1}}{A},$$

which denotes the probability that the robot would not end up in the attraction regions of the *m-1-aggregate*s in the arena.

The transition probabilities between configurations in consecutive classes needs to be integrated over all the configurations in the initial class. Hence, the probability of being in a configuration that included only one *m-aggregate* and a certain number of *m-1-aggregates* over all possible configurations need to be calculated. In order to calculate these probabilities, we will depend heavily on number theory concepts. *A partition function* $q(n, m)$, gives the number of ways to write n in terms of positive integers where the largest one is less than or equal to m [24] . If we fix the first aggregate to size m, the number of ways to write the rest with the largest being m gives us the number of configurations in C^m. This is equivalent to $q(n - m, m)$.

In C^m, the number of configurations that contain k *m-1-aggregates* and a single *m-aggregate* can be calculated using the partition function

$$q(n - m - k(m-1), m-2).$$

This formulation guarantees that the rest of the aggregates are neither of size m nor of size $m-1$ by limiting the required largest aggregate size to $m-2$.

Combining these, we can derive the shrinking probability for C^m as:

$$\boldsymbol{P}_s(n,m) = \frac{m \cdot P_{leave}}{n} \cdot \sum_{k=0}^{\lfloor \frac{n-m}{m-1} \rfloor} \cdot \frac{q(n-m-k\cdot(m-1), m-2)}{q(n-m,m)} \frac{A-(k+1)\cdot S_{m-1}}{A},$$

where k ranges from 0 to the maximum number of aggregates of size $m-1$ in C^m, which can be computed as $\left\lfloor \frac{n-m}{m-1} \right\rfloor$.

Note that for $n = m$, corresponding to the case that there is a single cluster that contains all robots, $q(0,m) = 1$ as zero is considered to have a single partition which is the empty partition. Therefore, $P_s(n,n)$ is reduced to

$$P_{leave} \cdot \frac{A - S_{n-1}}{A},$$

which reflects the case that the single large cluster will shrink if any of the robots leaves the cluster and does not come back.

5.2 Probabilities for Growth

Growth is defined as a change in the configuration such that the largest aggregate is increased by one as the result of a single-robot transition. Effectively, it means a transition from a configuration in equivalence class C^m to another configuration in equivalence class C^{m+1}.

First, note that the growth probabilities depend on two factors: the size of the aggregate that the robot is and the number of *m-aggregate*s, denoted by t and k respectively. For given t and k, the probability of growth can be computed as:

$$\frac{\alpha}{q(n-m,m)} \cdot \frac{t \cdot P_{leave}}{n} \cdot \frac{k \cdot S_m}{A},$$

where α is the number of *t-aggregate*s in all configurations of C^m that also contain k *m-aggregate*s.

Here, the value of α can be computed using $\hat{p}(n,t)$ and $\hat{q}(n,m,t)$ functions. $\hat{p}(n,t)$ corresponds to the number of occurrences of t in all partitions of n and $\hat{q}(n,m,t)$ corresponds to the number of occurrences of t in all partitions of n where largest term is less than or equal to m. $\hat{p}(n,t)$ can be calculated with the following recurrence relation:

$$\hat{p}(n,t) = \begin{cases} 0 & t > n \\ 1 & t = n \\ q(n-t, n-t) + \hat{p}(n-t,t) & t < n. \end{cases}$$

In the recursion step, we can add t to all possible partitions of $n-t$, that will each have one more t. Since partitions of $n-t$ may contain more t's we add $\hat{p}(n-t,t)$. We now define $\hat{q}(n,m,t)$ as:

$$\hat{q}(n,m,t) = \begin{cases} 0 & t > m \\ q(n-t,m) + \hat{q}(n-t,m,t) & t = m, t < n \\ \hat{q}(n,m-1,t) + \hat{q}(n-m,m,t) & t < m < n, t < n \\ \hat{p}(n,t) & \text{otherwise.} \end{cases}$$

In the first recursion case, we follow a similar construction with $\hat{p}(n,t)$. This time, $q(n-t,m)$ is used instead of $q(n-t,n-t)$ since we want to limit largest aggregate size to m. The second recursion case splits the partitions into two disjoint sets; the first one containing no terms equal to m, and the second one containing at least one term equal to m. The number of occurrences of t is sum of occurrences in these two sets. Note that, $t < m$ in this case, hence the number of occurrences in the second set discounts this mandatory term m.

If there are k *m-aggregate*s, the number of occurrences of t in all such configurations is:

$$\hat{q}(n - k \cdot m, m-1, t).$$

Using $\hat{q}$ and the previous result, we can calculate the total probability of growth for all aggregates of size t. We call this function γ:

$$\gamma(n,m,t,k) = \frac{\hat{q}(n-k\cdot m, m-1, t)}{q(n-m,m)} \cdot \frac{t \cdot P_{leave}}{n} \cdot \frac{k \cdot S_m}{A}; \mathrm{t} < \mathrm{m}.$$

Note that this definition of γ is only valid for $t < m$ since the transitions between *m-aggregate* do not fit the rule explained above. Handling this special case is not very difficult since we know that there are exactly k aggregates of size m. The number of configurations with exactly k aggregates of size m can be computed with $q(n-k\cdot m, m-1)$. In each of these configurations, a robot from k different aggregates can join one of the $k-1$ aggregates to increase the size of largest aggregate. So we extend the definition of γ as follows:

$$\gamma(n,m,k,t) = \begin{cases} \frac{\hat{q}(n-k\cdot m, m-1, t)}{q(n-m,m)} \cdot \frac{t \cdot P_{leave}}{n} \cdot \frac{k \cdot S_m}{A} & ; \mathrm{t} < \mathrm{m} \\ \frac{k \cdot q(n-k\cdot m, m-1)}{q(n-m,m)} \cdot \frac{t \cdot P_{leave}}{n} \cdot \frac{(k-1) \cdot S_m}{A} & ; \mathrm{t} = \mathrm{m} \end{cases}$$

The total probability of growth is the sum of these probabilities for all possible k and t values. For each different number of aggregates of size m, we need to consider all aggregate sizes that can lose a robot that could increase the size of the largest aggregate. These separate cases add up to the total growth probability.

$$P_g(n,m) = \sum_{k=1}^{\lfloor \frac{n}{m} \rfloor} \sum_{t=1}^{m} \gamma(n,m,k,t).$$

5.3 Probabilities to Remain Same

Once the shrinking and the growth probabilities are derived, the probability of remaining in the same equivalence class can be derived as:

$$\boldsymbol{P}_r(n,m) = 1 - \boldsymbol{P}_s(n,m) - \boldsymbol{P}_g(n,m).$$

5.4 Macroscopic Model

The probability distribution of being at different equivalence classes C^m at time t is represented with $\boldsymbol{F}(t)$ which is a probability vector (i.e. have positive real-valued entries summing up to 1) with n entries. Using $\boldsymbol{M}$ we can calculate the value of $\boldsymbol{F}(t+1)$ with:

$$\boldsymbol{F}(t+1) = \boldsymbol{M} \cdot \boldsymbol{F}(t).$$

Here, $\boldsymbol{M}$ is called the system matrix which consist of the probability values derived above as:

$$\boldsymbol{M}_{m,i} = \begin{cases} \boldsymbol{P}_g(n,m) & m = i-1 \\ \boldsymbol{P}_s(n,m) & m = i+1 \\ \boldsymbol{P}_r(n,m) & m = i \\ 0 & \text{otherwise.} \end{cases}$$

The steady state behavior of the system can then be obtained by iterating the system for infinite number of steps:

$$\boldsymbol{F}(\infty) = \boldsymbol{M}^{\infty} \cdot \boldsymbol{F}(0).$$

Notice that $\boldsymbol{M}$ is a *left-stochastic* matrix since its rows are probability vectors. Stochastic matrices can be considered as representations of the transition probabilities of a first-order finite Markov chain. Furthermore, the $\boldsymbol{M}$ matrix is *regular* since the matrix power $\boldsymbol{M^k}$ will contain only strictly positive entries for some k [25]. According to the Perron-Frobenius theorem, such a system has a unique convergence point [26] meaning that it will converge to a steady-state vector representing the distribution of configurations as time goes to infinity. The final steady-state of the system would be the eigenvector of the matrix corresponding to the eigenvalue of unity.

6 Experimental Results

The predictions of the macroscopic model beg to be compared against results obtained from simulated or real robots. We have developed a simple 2D robot simulator for this purpose. The simulator supported simplified physical interactions between the robots, and the robot and the environment based on collision detection and recovery. The simulated robots have a radius of 3 units. They have infrared proximity sensors around them, to detect the existence of other robots and also avoid the walls of the arena. The characteristics of the proximity sensors control the *Robot close* condition which ends the *approach behavior*. The robots

are also equipped with a omnidirectional long-range sensor through which they can sense each other within 30 units. This sensor, gives the center of mass for the robots in the perceptual view of the robot and determine the attraction regions (S_m) of the aggregates. The S_1 is computed as 2826 $unit^2$ (computed as $\pi \cdot 30 \cdot 30$) using the range of the long-range sensor. Through empirical experiments, S_{20} was measured to be approximately 5800 $unit^2$. The S_m values for the aggregate sizes in between was obtained using a linear interpolation between these two values. The random walk behavior duration is determined experimentally to be 20,000 simulation steps where robots can move around 1 units per simulation step. Finally, we would like to note that, the simulator does not restrict the movement of the robots and that more than one robot can be on the move at a given time.

We have conducted two sets of experiments and compared the final aggregate distributions obtained in simulation against those predicted by the macroscopic model. In the first set of experiments, we have studied the effect of the P_{leave} on the performance of the self-organized aggregation. In natural swarm systems, individuals are known to perceive the aggregates that they are in, and to modulate their leaving probability with respect to the size of the aggregate[7]. Specifically, we simulated three different strategies for setting the leave probability: **(1)** Constant: $P_{leave}(i) = G$, **(2)** Inversely proportional to aggregate size: $P_{leave}(i) = G/i$ and **(3)** Inversely proportional to the square of aggregate size: $P_{leave}(i) = G/i^2$. G is chosen to be 0.00002 in all the experiments.

Each strategy was tested with three different swarm and arena sizes: **(1)** 5 robots in a 150×150 arena, **(2)** 10 robots in a 212×212 arena and, **(3)** 20 robots in a 300×300 arena. These setups keep the robot density approximately same while increasing the number of robots. Each simulation run lasted for 50,000,000 simulation steps, which was observed to be sufficient for stabilization.

For each of the nine cases, 50 simulation runs were made, and the largest aggregate formed at the end of the simulations are recorded. Similarly, we used the macroscopic model to predict the final distribution of the aggregates for the same given parameters. The predictions of the model and the histogram of the largest aggregates obtained from simulations are plotted in Figure 5. The results indicate good match between model and the results of the simulation experiments. The model correctly predicts that constant leave probabilities, that is $P_{leave} = G$, should lead to small aggregates, whose mean size is determined by the size of the swarm as shown in the top row of the figure. The plots shown in the middle row indicates a phase transition from the top row to the bottom row. The plots in the bottom row of the figure clearly show that for leave probabilities set to be inversely proportional to the square of the aggregate size, that is $P_{leave}(i) = G/i^2$), the aggregation performance of the swarm is high. For three different arena sizes, it can be seen that the aggregation behavior was able to generate aggregates that contain all of the robots in the swarm for most runs.

These results are in agreement with the results in [7]. In that study, Jeanson et. al reported poor aggregation performance when the agents ignored the number of agents in their close proximity, which corresponds to constant probabilities

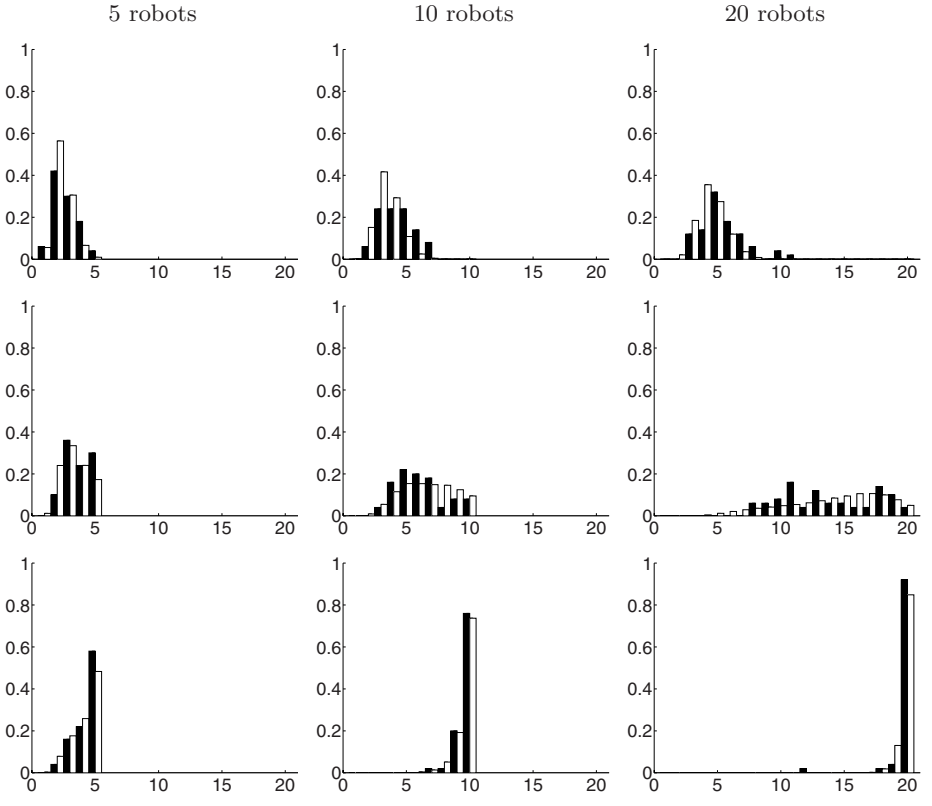

Fig. 5. The comparison of macroscopic model predictions against results obtained from simulations. Each plot shows the histogram of largest aggregates at the end of each run. White boxes show the prediction of the macroscopic model and black boxes show the results of experiments. Top row: $P_{leave}(i) = G$. Middle row: $P_{leave}(i) = \frac{G}{i}$. Bottom row: $P_{leave}(i) = \frac{G}{i^2}$. Note that, the black boxes do not have any error bars, since they represent the normalized histograms obtained from 50 runs.

in our model. They also reported successful aggregation when rest durations are increased more than linearly with respect to aggregate size which is similar to the quadratic case in our experiments. The results with quadratic probabilities are also comparable to results by Lee et. al, in which the controller is defined to use decreasing leave probabilities for aggregates [22].

In a second set of experiments, we investigate the effect of swarm density, which can be defined as the number of robots in the swarm divided by the arena size, on the performance of the aggregation. We studied the aggregation behavior of 5 robots in three different arenas: 150×150, 212×212 and 300×300.

When we used the leave probability setting, $P_{leave}(i) = G/i^2$, which was shown to be the best performer in the first set of experiments. The results of the simulations and the predictions of the model are plotted in Figure 6. It can be

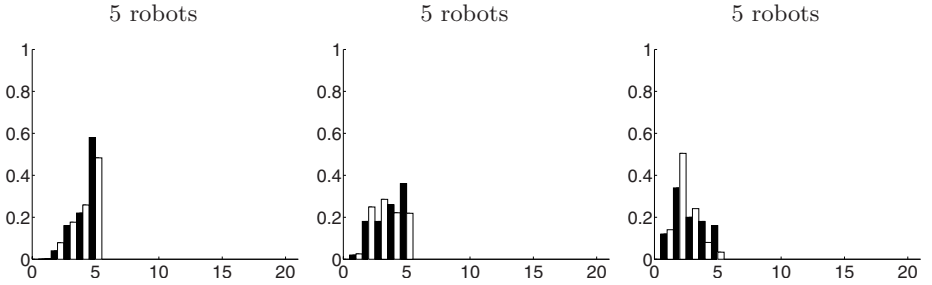

Fig. 6. Comparison of model predictions with experimental results with respect to arena size. Each diagram shows the histogram of largest aggregates at the end of each run. White boxes show the prediction of macroscopic model and black boxes show the results of experiments. $P_{leave}(i) = \frac{G}{i^2}$ where i is the number of robots in the aggregate. Note that, the black boxes do not have any error bars, since they represent the normalized histograms obtained from 50 runs.

seen that there is an approximate match between the model's predictions and the results of the simulations. The results show that the aggregation performance degrades with decreasing robot density, indicating that the constant used in the leave probability is related to the density of the robots in the area for a desired performance.

7 Conclusion

In this study we proposed a macroscopic model for self-organized aggregation behavior in swarm robotic systems and compared its aggregation performance predictions against results obtained from simulations. The macroscopic model, resulting in a simple mathematical form allows detailed analysis, such as the optimization of behavioral parameters for desired aggregation performance if given the number of robots, their perception range and the size of the environment.

However, we would like to explicitly state that the work presented in this paper is not complete yet and the model presented here relies on a number of restricting assumptions. As a possible result of these, there are a number of small discrepancies between the model predictions and the simulation results, such as the middle plot in Figure 6. There are many potential sources for these discrepancies. As stated, the macroscopic model relies on a number of assumptions, which are not fully satisfied in the simulations: **(1)** Single-robot transitions were not explicitly enforced in the simulations, **(2)** spatial information regarding the positions of the aggregates were excluded from the model, and **(3)** attraction regions of the aggregates were based on rough estimations. Other than these, cases like two aggregates joining into a single aggregate through a interconnecting robot were completely neglected. A detailed analysis of these unsatisfied assumptions on the performance of the aggregation remains a future challenge for our studies.

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