Causal Dataflow Analysis for Concurrent Programs

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Abstract. We define a novel formulation of dataflow analysis for concurrent programs, where the flow of facts is along the *causal* dependencies of events. We capture the control flow of concurrent programs using a Petri net (called the *control net*), develop algorithms based on partially-ordered unfoldings, and report experimental results for solving causal dataflow analysis problems. For the subclass of distributive problems, we prove that complexity of checking data flow is linear in the number of facts and in the *unfolding* of the control net.

1 Introduction

Advances in multicore technology and the wide use of languages that inherently support threads, such as Java, foretell a future where concurrency will be the norm. Despite their growing importance, little progress has been made in static analysis of concurrent programs. For instance, there is no standard notion of a control-flow graph for concurrent programs, while the analogous notion in sequential programs has existed for a long time [10]. Consequently, dataflow analysis problems (arguably the simplest of analysis problems) have not been clearly understood for programs with concurrency.

While it is certainly easy to formulate dataflow analysis for concurrent programs using the *global product state space* of the individual threads, the usefulness of doing so is questionable as algorithms working on the global state space will not scale. Consequently, the literature in flow analysis for threaded programs concentrates on finding tractable problem definitions for dataflow analysis. A common approach has been to consider programs where the causal relation between events is *static* and apparent from the structure of the code (such as fork-join formalisms), making feasible an analysis that works by finding fixpoints on the *union* of the individual sequential control flow graphs. These approaches are often highly restrictive (for example, they require programs to have no loops [23] or at least to have no loops with concurrent fork-join constructs [13,14]), and cannot model even simple shared-memory program models. In fact, a coherent

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formulation of control-flow that can capture programs with dynamic concurrency (including those with shared memory) and a *general definition* of dataflow analysis problems for these programs has not been formulated in the literature (see the end of this section for details on related work).

The goals of this paper are (a) to develop a formal *control-flow* model for programs using Petri nets, (b) to propose a novel definition of dataflow analyses based on *causal flows* in a program, (c) to develop algorithms for solving causal flow analyses when the domain of flow facts is a finite set $\mathbb{D}$ by exploring the partially-ordered runs of the program as opposed to its interleaved executions, and (d) to provide provably efficient algorithms for the class of *distributive* CCD problems, and support the claim with demonstrative experiments. The framework we set forth in this paper is the first one we know that defines a formal general definition of dataflow analysis for concurrent programs.

We first develop a Petri net model that captures the control flow in a concurrent program, and give a translation from programs to Petri nets that explicitly abstracts data and captures the control flow in the program. These nets, called *control nets*, support dynamic concurrency, and can model concurrent constructs such as lock-synchronizations and shared variable accesses. In fact, we have recently used the same model of control nets to model and check *atomicity* of code blocks in concurrent programs [7]. We believe that the control net model is an excellent candidate for capturing control flow in concurrent programs, and can emerge as the robust analog of *control-flow graphs* for sequential programs.

The causal concurrent dataflow (CCD) framework is in the flavor of a *meet-over-all-paths* formulation for sequential programs. We assume a set of dataflow facts $\mathbb{D}$ and each statement of the program is associated with a *flow transformer* that changes a subset of facts, killing some old facts and generating new facts. However, we demand that the flow transformers respect the concurrency in the program: we require that if two *independent* (concurrent) statements transform two subsets of facts, D and D', then the sets D and D' must be *disjoint*. For instance, if there are two local variable accesses in two different threads, these statements are independent, and cannot change the same dataflow fact, which is a very natural restriction. For example, if we are tracking *uninitialized variables*, two assignments in two threads to local variables do affect the facts pertaining to these variables, but do not modify the same fact. We present formulations of most of the common dataflow analysis problems in our setting.

The structural restriction of requiring transformers to respect causality ensures that dataflow facts can be inferred using partially ordered traces of the control net. We define the dataflow analysis problem as *a meet over partially ordered traces* that reach a node, rather than the traditional meet-over-paths definition. The meet-over-traces definition is crucial as it preserves the concurrency in the program, allowing us to exploit it to solve flow analysis using *partial-order* based methods, which do not explore all interleavings of the program.

Our next step is to give a solution for the general causal dataflow analysis problem when the set of of facts $\mathbb{D}$ is finite by reducing the problem to a reachability problem of a Petri net, akin to the classic approach of reducing meet-over-paths to graph reachability for sequential recursive programs [21]. Finally, the reachability/coverability problem is solved using the optimized partial-order *unfolding* [16,6] based tool called PEP [9].

For the important subclass of *distributive* dataflow analysis problems, we develop a more efficient algorithm for checking flows. If N is the control net of a program and the size of its finite unfolding is n, we show that any distributive CCD problem over a domain $\mathbb{D}$ of facts results in an augmented net of size $n|\mathbb{D}|$ (and hence in an algorithm working within similar bounds of time and space). This is a very satisfactory result, since it proves that the causal definition does not destroy the concurrency in the net (as that would result in a blow-up in n), and that we are exploiting distributivity effectively (as we have a linear dependence on $|\mathbb{D}|$). The analogous result for sequential recursive programs also creates an augmented graph of size $n|\mathbb{D}|$, where n is the size of the control-flow graph.

Related Work. Although the majority of flow analysis research has focused on sequential software [1,19,17,20], flow analysis for concurrent software has also been studied to some extent. Existing methods for flow-sensitive analyses have at least one of the following restrictions: (a) the programs handled have simple static concurrency and can be handled precisely using the union of control flow graphs of individual programs, or (b) the analysis is sound but not complete, and solves the dataflow problem using heuristic approximations.

A body of work on flow-sensitive analyses exists in which the model for the program is essentially a collection of CFGs of individual threads (tasks, or components) together with additional edges among the CFGs that model inter-thread synchronization and communication [15,18,22]. These analyses are usually restricted to a class of behaviors (such as detecting deadlocks) and their models do not require considering the set of interleavings of the program. More general analyses based on the above type of model include [12] which presents a *unidirectional* bit-vector dataflow analysis framework based on abstract interpretation (where the domain $\mathbb{D}$ is a *singleton*). This framework comes closest to ours in that it explicitly defines a meet-over-paths definition of dataflow analysis, can express a variety of dataflow analysis problems, and gives sound and complete algorithms for solving them. However, it cannot handle dynamic synchronization mechanisms (such as locks), and the restriction to having only one dataflow fact is crucially (and cleverly) used, making multidimensional analysis impossible. For example, this framework cannot handle the problem of solving *uninitialized variables*. See also [23] for dataflow analysis that uses flow along causal edges but disallows loops in programs and requires them to have static concurrency. The works in [13,14] use the extension of the static single assignment form [3] for concurrent programs with emphasis on optimizing concurrent programs as opposed to analyzing them.

In [4], concurrent models are used to represent interleavings of programs, but the initial model is coarse and refined to obtain precision, and efficiency is gained by sacrificing precision. Petri nets are used as control models for Ada programs in [5], although the modeling is completely different form ours. In [2], the authors combine reachability analysis with symbolic execution to prune the infeasible paths in order to achieve more effective results.

This paper presents only the gist of the definitions and proofs. For more detailed definitions of Petri nets, unfoldings, the framework for backward flow analyses and the non-distributive framework, for further examples and detailed proofs, we refer the reader to the technical report [8].

2 Preliminaries

A Simple Multithreaded Language. We base our formal development on the language SML (Simple Multithreaded Language). Figure 1 presents the syntax of SML. The number of threads in an SML program is fixed and preset. There are two kinds of variables: local and global, respectively identified by the sets *LVar* and *GVar*. All variables that appear at the definition list of the program are global and shared among all threads. Any other variable that is used in a thread is assumed to be local to the thread.

We assume that all variables are integers and are initialized to zero. We use small letters (capital letters) to denote local (global, resp.) variables. *Lock* is a global set of locks that the threads can use for synchronization purposes through `acquire` and `release` primitives. The semantics of a program is the obvious one and we do not define it formally.

P ::=	defn thlist	(program)
thlist ::=	null | stmt || thlist	(thread list)
defn ::=	`int` Y | `lock` l | defn ; defn	(variable declaration)
stmt ::=	stmt ; stmt | $x := e$ | `skip`	
	| `while` (b) { stmt } | `acquire`(l) | `release`(l)	
	| `if` (b) { stmt } else { stmt }	(statement)
e ::=	$i \mid x \mid Y \mid e+e \mid e*e \mid e/e$	(expression)
b ::=	`true` | `false` | $e\ op\ e \mid b \vee b \mid \neg b$	(boolean expression)
	$op \in \{<, \leq, >, \geq, =, !=\}$	
	$x \in$ LVar, $Y \in$ GVar, $i \in$ Integer, $l \in$ Lock	

Fig. 1. SML syntax

Petri Nets and Traces

A Petri net is a triple $N = (P, T, F)$, where P is a set of places, T (disjoint from P) is a set of transitions, and $F \subseteq (P \times T) \cup (T \times P)$ is the flow relation.

For a transition t of a (Petri) net, let ${}^\bullet t = \{p \in P | (p,t) \in F\}$ denote its set of pre-conditions and $t^\bullet = \{p \in P | (t,p) \in F\}$ its set of post-conditions. A marking of the net is a subset M of positions of P.[1] A marked net is a structure (N, M_0), where N is a net and M_0 is an initial marking. A transition t is *enabled* at a marking M if ${}^\bullet t \subseteq M$. The transition relation is defined on the set of markings: $M \xrightarrow{t} M'$ if transition t is enabled at M and $M' = (M \setminus {}^\bullet t) \cup t^\bullet$. Let $\xrightarrow{*}$ denote the reflexive and transitive closure of $\longrightarrow$. A marking M' *covers* a marking M if $M \subseteq M'$. A *firing sequence* is a finite sequence of transitions $t_1 t_2 \ldots$ provided we have a sequence of markings $M_0 M_1 \ldots$ and for each i, $M_i \xrightarrow{t_{i+1}} M_{i+1}$. We denote the set of firing sequences of (N, M_0) as $FS(N, M_0)$. Given a marked net (N, M_0), $N = (P, T, F)$, the *independence* relation of the net I_N is defined as $(t, t') \in I$ if the neighborhoods of t and t' are disjoint, i.e. $({}^\bullet t \cup t^\bullet) \cap ({}^\bullet t' \cup t'^\bullet) = \emptyset$. The *dependence* relation D_N is defined as the complement of I_N.

Definition 1. *A* trace *of a marked net* (N, M_0) *is a labeled poset* $Tr = (\mathcal{E}, \preceq, \lambda)$ *where* $\mathcal{E}$ *is a finite or a countable set of events,* $\preceq$ *is a partial order on* $\mathcal{E}$*, called the* causal order*, and* $\lambda : \mathcal{E} \longrightarrow T$ *is a labeling function such that the following hold:*

- $\forall e, e' \in \mathcal{E}, e \lessdot e' \Rightarrow \lambda(e) D_N \lambda(e')$.[2] *Events that are immediately causally related must correspond to dependent transitions.*
- $\forall e, e' \in \mathcal{E}, \lambda(e) D_N \lambda(e') \Rightarrow (e \preceq e' \ \vee e' \preceq e)$. *Any two events with dependent labels must be causally related.*
- *If* σ *is a linearization of* Tr *then* $\sigma \in FS(N, M_0)$.

For any event e in a trace $(\mathcal{E}, \preceq, \lambda)$, define $\downarrow e = \{e' \in \mathcal{E} \mid e' \preceq e\}$ and let $\Downarrow e = \downarrow e \setminus \{e\}$.

3 The Control Net of a Program

We model the flow of control in SML programs using Petri nets. We call this model the *control net* of the program. The control net formally captures the concurrency between threads using the concurrency constructs of a Petri net, captures synchronizations between threads (e.g.. locks, accesses to global variables) using appropriate mechanisms in the Petri net, and formalizes the fact that data is abstracted in a sound manner.

We describe the main ideas of this construction but skip the details (see [8] for details). Transitions in the control net correspond to program statements, and places are used to control the flow, and to model the interdependencies and synchronization primitives. Figure 2 illustrates a program and its control net.

[1] Petri nets can be more general, but in this paper we restrict to 1-safe Petri nets where each place gets at most one token.

[2] $\lessdot$ is the immediate causal relation defined as: $e \lessdot e'$ iff $e \prec e'$ and there is no event e'' such that $e \prec e'' \prec e'$.

There is a place l associated to each lock l which initially has a token in it. To acquire a lock, this token has to be available which then is taken and put back when the lock is released.

For each global variable Y, there are n places $\mathtt{Y}_1, \ldots, \mathtt{Y}_n$, one per thread. Every time the thread T_i reads the variable Y (Y appears in an expression), it takes the token from the place $\mathtt{Y}_i$ and puts it back immediately. If T_i wants to write Y (Y is on the left side of an assignment), it has to take one token from each place $\mathtt{Y}_j$, $1 \leq j \leq n$ and put them all back. This ensures correct causality: two read operations of the same variable by different threads will be independent (as their neighborhoods will be disjoint), but a read and a write, or two writes to a variable are declared dependent.

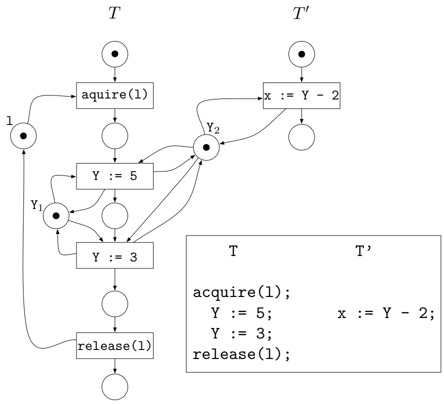


Fig. 2. Sample Net Model

4 Causal Concurrent Dataflow Framework

We now formulate our framework for dataflow analysis of concurrent programs based on causality, called the Causal Concurrent Dataflow (CCD) framework.

A property space is a *subset lattice* $(\mathcal{P}(\mathbb{D}), \sqsubseteq, \sqcup, \bot)$ where $\mathbb{D}$ is a finite set of *dataflow facts*, $\bot \subseteq \mathbb{D}$, and where $\sqcup$ and $\sqsubseteq$ can respectively be $\cup$ and $\subseteq$, or $\cap$ and $\supseteq$. Intuitively, $\mathbb{D}$ is the set of dataflow facts of interest, $\bot$ is the initial set of facts, and $\sqcup$ is the *meet* operation that will determine how we combine dataflow facts along different paths reaching the same control point in a program. "May" analysis is formulated using $\sqcup = \cup$, while "must" analysis uses the $\sqcup = \cap$ formulation. The property space of an IFDS (interprocedural finite distributive subset) problem [21] for a sequential program (i.e. the subset lattice) is exactly the same lattice as above.

For every transition t of the control net, we associate two subsets of $\mathbb{D}$, D_t and D_t^*. Intuitively, D_t^* is the set of dataflow facts relevant at t, while $D_t \subseteq D_t^*$ is the subset of relevant facts that t may modify when it executes. The *transformation function* associated with t, f_t, maps every subset of D_t to a subset of D_t, reflecting how the dataflow facts change when t is executed.

Definition 2. *A causal concurrent dataflow (CCD) problem is a tuple* $(N, \mathcal{S}, \mathcal{F}, \mathcal{D}, \mathcal{D}^*)$ *where:*

– $N = (P, T, F)$ *is the control net model of a concurrent program,*
– $\mathcal{S} = (\mathcal{P}(\mathbb{D}), \sqsubset, \sqcup, \bot)$ *is a property space,*
– $\mathcal{D} = \{D_t\}_{t \in T}$ *and* $\mathcal{D}^* = \{D_t^*\}_{t \in T}$*, where each* $D_t \subseteq D_t^* \subseteq \mathbb{D}$.

– *$\mathcal{F}$ is a set of functions* $\{f_t\}_{t \in T} : 2^{D_t} \rightarrow 2^{D_t}$ *such that:*
(*) $\forall t, t' : (t, t') \in I_N \Rightarrow (D_t \cap D^*_{t'} = D^*_t \cap D_{t'} = \emptyset)$.[3]

We call a CCD problem *distributive* if all transformation functions in $\mathcal{F}$ are distributive, that is $\forall f_t \in \mathcal{F},\ \forall X, Y \subseteq D_t : f_t(X \sqcup Y) = f_t(X) \sqcup f_t(Y)$.

Remark 1. Condition (*) above is to be specially noted. It demands that for any two concurrent events e and e', e cannot change a dataflow fact that is relevant to e'. Note that if e and e' are events in a trace such that $D_{\lambda(e)} \cap D^*_{\lambda(e')}$ is non-empty, then they will be causally related.

4.1 Meet over All Traces Solution

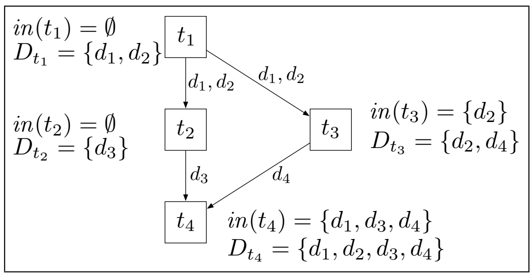


Fig. 3. Flow of facts over a trace

In a sequential run of a program, every event t has at most one predecessor t'. Therefore, the set of dataflow facts that hold before the execution of t (let us call this $in(t)$) is exactly the set of dataflow facts that hold after the execution of t' ($out(t')$). This is not the case for a trace (a partially ordered run). Consider the example in Figure 3. Assume t_1 generates facts d_1 and d_2, t_2 generates d_3 and t_3 kills d_2 and generates d_4. The corresponding D_t sets appear in the Figure. Trying to evaluate the "*in*" set of t_4, we see three important scenarios: (1) t_4 inherits *independent* facts d_3 and d_4 respectively from its immediate predecessors t_2 and t_3, (2) t_4 inherits fact d_1 from t_1 which is not its immediate predecessor, and (3) t_4 does not inherit d_2 from t_1 because t_3, which is a (causally) later event and the last event to modify d_2, kills d_2.

This example demonstrates that in a trace the immediate causal predecessors do not specify the "*in*" set of an event. The indicating event is actually the (causally) last event that can change a dataflow fact (eg. t_3 for fact d_2 in computing $in(t_4)$). We formalize this concept by defining the operator $maxc^d_{\preceq}(Tr)$, for a trace $Tr = (E, \preceq, \lambda)$ as $maxc^d_{\preceq}(Tr) = max_{\preceq}(\{e \mid e \in E \wedge d \in D_{\lambda(e)}\})$. Note that this function is undefined on the empty set, but well-defined on non-empty sets because all events that affect a dataflow fact d are causally related due to (*) in Definition 2.

Remark 1 suggests that for each event e it suffices to only look at the facts that are in the "*out*" set of events in $\Downarrow e$ (events that are causally before e), since events that are concurrent with e will not change any fact that's relevant to e.

[3] And hence $D_t \cap D_{t'} = \emptyset$.

Definition 3. *For any trace $Tr = (E, \preceq, \lambda)$ of the control net and for each event $e \in E$, we define the following dataflow sets:*

$$\begin{cases} in^{Tr}(e) = \bigcup_{d \in D^*_{\lambda(e)}} (out^{Tr}(maxc^d_{\preceq}(\Downarrow e)) \cap \{d\}) \\ out^{Tr}(e) = f_{\lambda(e)}(in^{Tr}(e) \cap D_{\lambda(e)}) \end{cases}$$

where $in^{Tr}(e)$ (respectively $out^{Tr}(e)$) indicates the set of dataflow facts that hold before (respectively after) the execution of event e of trace Tr.

In the above definition, $maxc^{d_i}_{\preceq}(\Downarrow e))$ may be undefined (if $\Downarrow e = \emptyset$), in which case we assume $in^{Tr}(e)$ evaluates to the empty set.

We can now define the **meet over all traces** solution for a program Pr, assuming the $\mathcal{T}(N)$ denotes the set of all traces induced by the control net N.

Definition 4. *The set of dataflow facts that hold before the execution of a transition t of a control net N is $MOT(t) = \bigcup_{Tr \in \mathcal{T}(N), e \in Tr, \lambda(e)=t} in^{Tr}(e)$.*

The above formulation is the concurrent analog of the meet-over-all-paths formulation for sequential programs. Instead of the above definition, we could formulate the problem as a meet-over-all-paths problem, where we take the meet over facts accumulated along the *sequential runs* (interleavings) of the concurrent program. However, due to the restriction (*) in Definition 2, we can show that the dataflow facts accumulated at an event of a trace is precisely the same as that accumulated using any of its linearizations. Consequently, for dataflow problems that respect causality by satisfying the condition (*), the meet-over-all-paths and the meet-over-traces formulations coincide. The latter formulation however yields faster algorithms based on partial-order methods based on unfoldings to solve the dataflow analysis problem.

4.2 Formulation of Specific Problems in the CCD Framework

A wide variety of dataflow analysis problems can be formulated using the CCD framework, including reaching definitions, uninitialized variables, live variables, available expressions, copy constant propagation, very busy expressions, etc. Some of these are *backward flow analysis* problems that can be formulated using an adaptation of CCD for backward flows. Due to lack of space, we detail only a couple of representative forward flow problems here; formulation of several others, including formulation of backward flows can be found in [8].

Reaching Definitions. The reaching definitions analysis determines: "*For each control point, which relevant assignments may have been made and not overwritten when program execution reaches that point along some path*". The relevant assignments are the assignments to variables that are referred to in that control point. Given the control net $N = (P, T, F)$ for a program Pr, define $Defs = \{(v,t) \mid t \in T, v \in (\text{GVar} \cup \text{LVar}), \text{ and } v \text{ is assigned in } t\}$. The property space is $(Defs, \subseteq, \cup, \emptyset)$, where presence of (v,t) in $D^{in}(t')$ means that the definition of v at t may reach t'.

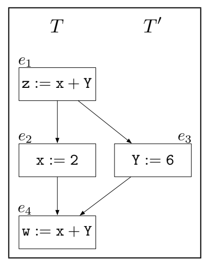
Let $D_t = \{(v,t') \mid v \text{ is assigned in } t\}$; $D_t^* = \{(v,t') \mid v \text{ is assigned or accessed by } t\}$.

For each transition t and each set $S \subseteq D_t$:

$$f_t(S)(= \begin{cases} S & \text{if } t \text{ is not an assignment} \\ S - \ \{(v,t')|t' \in T\} \ \cup \ \{(v,t)\} & \text{if } t \text{ is of the form v := e} \end{cases}$$

The construction of the control net ensures that two accesses of a variable v where one of them is a write, are dependent (neighborhoods intersect). This guarantees that the condition (*) of Definition 2 holds, i.e. our formulation of reaching-definitions ensures that information is inherited only from causal predecessors. Note that the above formulation is also distributive.

Available Expressions. The available expressions analysis determines: "*For a program point containing $x := Exp(x_1, \ldots, x_k)$ whether Exp has already been computed and not later modified on all paths to this program point*". In the standard (sequential) formulation of available expressions analysis, dataflow facts are defined as pairs (t, Exp), where Exp is computed at t. This formulation does not work for the concurrent setting. To see why consider the trace on the right where `x` is a local variable in T and `Y` is a global variable. Events e_2 and e_3 are independent (concurrent), but they both can change (kill) the dataflow fact associated with $\texttt{x} + \texttt{Y}$, which is not in accordance with the condition (*) of Definition 2. The natural remedy is to divide this fact into two facts, one for `x` and another for `Y`. Let us call these two facts $\texttt{x} + \texttt{Y} : \texttt{x}$ and $\texttt{x} + \texttt{Y} : \texttt{Y}$. The fact $\texttt{x} + \texttt{Y} : \texttt{x}$ (respectively $\texttt{x} + \texttt{Y} : \texttt{Y}$) starts to hold when the expression $\texttt{x} + \texttt{Y}$ is computed, and stops to hold when a definition to `x` (respectively `Y`) is seen. The problem is that $\texttt{x} + \texttt{Y}$ holds when $\texttt{x} + \texttt{Y} : \texttt{x}$ holds **and** $\texttt{x} + \texttt{Y} : \texttt{Y}$ holds, which makes the framework *non-distributive*. Although we can solve non-distributive problems in the CCD framework (see Appendix), distributive problems yield faster algorithms (see Section 5).



The analysis can however be formulated as a distributive CCD problem by looking at the dual problem; that is, for *unavailability* of expressions. The dataflow fact $\texttt{x} + \texttt{Y}$ indicates the expression being unavailable, and accordingly the presence of $\texttt{x} + \texttt{Y} : \texttt{x}$ **or** $\texttt{x} + \texttt{Y} : \texttt{Y}$ can make it hold. We are now in a distributive framework. Assume EXP presents the set of all expressions appearing in the program code, and define $\mathbb{D} = \{exp : x_i \mid exp \in EXP \ \wedge \ x_i \text{ appears in } exp\}$. The property space is the subset lattice $(\mathbb{D}, \subseteq, \cup, \mathbb{D})$, where presence of exp in $D^{in}(t')$ means that exp is unavailable at t. We have $D_t = D_t^* = \{exp : x \mid x \text{ is assigned in } t \text{ or } exp \text{ appears in } t\}$. For each transition t and each set $S \subseteq \mathbb{D}$:

$$f_t(S) = \begin{cases} S & t \text{ is not an assignment} \\ S \cup \ \{exp' : x \mid \forall exp' \in EXP, x \in V(exp')\} & \\ \quad - \{exp : y \mid y \in V(exp)\} & t \text{ is } x := exp \end{cases}$$

where $V(exp)$ denotes the set of variables that appear in *exp*.

5 Solving the Distributive CCD Problem

In this section, we show how to solve a dataflow problem in the CCD framework. The algorithm we present is based on augmenting a control net to a larger net based on the dataflow analysis problem, and reducing the problem of checking whether a dataflow fact holds at a control point to a reachability problem on the augmented net. The augmented net is carefully constructed so as to not destroy the concurrency present in the system (crucially exploiting the condition (*) in Definition 2). Reachability on the augmented net is performed using net unfoldings, which is a partial-order based approach that checks traces generated by the net as opposed to checking linear runs.

Due to space restrictions, we present only the solution for the distributive CCD problems where the meet operator is union, and we prove upper bounds that compare the unfolding of the augmented net with respect to the size of the unfolding of the original control net.

In order to track the dataflow facts, we enrich the control net so that each transition performs the transformation of facts as well. We introduce new places which represent the dataflow facts. The key is then to model the transformation functions, for which we use *representation relations* from [21].

Definition 5. *The representation relation of a distributive function* $f : 2^D \rightarrow 2^D$ *(*$D \subseteq \mathbb{D}$*) is* $R_f \subseteq (D \cup \{\bot\}) \times (D \cup \{\bot\})$*, a binary relation, defined as follows:*

$$R_f = \{(\bot, \bot)\} \ \cup \ \{(\bot, d) \mid d \in f(\emptyset)\} \ \cup \ \{(d, d') \mid d' \in f(\{d\}) \wedge \ d' \notin f(\emptyset)\}$$

The relation R_f captures f faithfully in that we can show that $f(X) = \{d' \in D \mid (d, d') \in R_f, \ \textit{where } d = \bot \textit{ or } d \in X\}$, for any $X \subseteq D$.

Given a CCD framework $(N, \mathcal{S}, \mathcal{F}, \mathcal{D}, \mathcal{D}^*)$ with control net $N = (P, T, F)$, we define the net representation for a function f_t as below:

Definition 6. *The net representation of* f_t *is a Petri net* $N_{f_t} = (P_{f_t}, T_{f_t}, F_{f_t})$ *defined as follows:*

- *The set of places is* $P_{f_t} = {}^\bullet t \cup t^\bullet \cup \{\bot_m \mid m \in [1, n]\} \cup \bigcup_{d_i \in D_t} \{p_i, \overline{p}_i\}$ *where a token in* p_i *means the dataflow fact* d_i *holds, while a token in* $\overline{p}_i$ *means that* d_i *does not hold, and* n *is the number of dataflow facts.*
- *The set of transitions* T_f *contains exactly one transition per pair* $(d_i, d_j) \in R_{f_t}$*, and is defined as:*

$$T_{f_t} = \left\{ s^t_{(\bot,\bot)} \right\} \cup \left\{ s^t_{(\bot,j)} \mid (\bot, d_j) \in R_{f_t} \right\} \cup \left\{ s^t_{(i,j)} \mid (d_i, d_j) \in R_{f_t} \right\}$$

Note that if $D_t = \emptyset$ *then* $T_{f_t} = \left\{ s^t_{(\bot,\bot)} \right\}$.

– *The flow relation is defined as follows:*

$$F_{f_t} = \bigcup_{s \in T_{f_t}} \left(\bigcup_{p \in {}^\bullet t} \{(p,s)\} \cup \bigcup_{p \in t^\bullet} \{(s,p)\} \right) \cup \bigcup_{d_k \in D_t} \left\{ (\overline{p}_k, s^t_{(\bot,\bot)}), (s^t_{(\bot,\bot)}, \overline{p}_k) \right\}$$

$$\cup \bigcup_{(\bot,d_j) \in R_{f_t}} \left(\left\{ (\bot_m, s^t_{(\bot,j)}) \mid t \in T_m \right\} \cup \left\{ (s^t_{(\bot,j)}, p_j) \right\} \right.$$

$$\left. \cup \bigcup_{d_k \in D_t} \left\{ (\overline{p}_k, s^t_{(\bot,j)}) \right\} \cup \bigcup_{k \neq j} \left\{ (s^t_{(i,j)}, \overline{p}_k) \right\} \right)$$

$$\cup \bigcup_{\substack{(d_i,d_j) \in R_{f_t} \\ i \neq j}} \left(\left\{ (p_i, s^t_{(i,j)}), (s^t_{(i,j)}, p_j), (\overline{p}_j, s^t_{(i,j)}), (s^t_{(i,j)}, \overline{p}_i) \right\} \right)$$

$$\cup \bigcup_{(d_i,d_i) \in R_{f_t}} \left(\left\{ (p_i, s^t_{(i,i)}), (s^t_{(i,i)}, p_i) \right\} \right)$$

The idea is that each transition $s^t_{(i,j)}$ is a copy of transition t that, besides simulating t, models one pair (d_i, d_j) of the relation R_{f_t}, by taking a token out of place p_i (meanwhile, also checking that nothing else holds by taking tokens out of each $\overline{p}_k$, $k \neq i$) and putting it in p_j (also returning all tokens $\overline{p}_k$, $k \neq j$). Thus if d_i holds (solely) before execution of t, d_j will hold afterwards. The transitions $s^t_{\bot,j}$ generate new dataflow facts, but consume the token $\bot_m$ associated with the thread. We will engineer the net to initially contain only one $\bot_m$ marking (for some thread m), and hence make sure that only one fact is generated from $\bot$.

For every t, transitions $s^t_{(i,j)}$ are in *conflict* since they have ${}^\bullet t$ as common predecessors. This means that only one of them can execute at a time, generating a single fact. If we assume that initially nothing holds (i.e., initial tokens are in every $\overline{p}_i$'s and no initial tokens in any of the p_i's), then since each transition consumes one token and generates a new token, the following invariant always holds for the system: "At any reachable marking of the augmented net, exactly one position p_i corresponding to some dataflow fact d_i holds". We use this observation later to argue the complexity of our analysis.

Definition 7. *The augmented marked net $N^{\mathcal{S},\mathcal{F}}$ of a CCD problem $(N, \mathcal{S}, \mathcal{F})$ is defined as $\bigcup_{f \in \mathcal{F}} N_f$ where the union of two nets $N_1 = (P_1, T_1, F_1)$ and $N_2 = (P_2, T_2, F_2)$ is defined as $N_1 \cup N_2 = (P_1 \cup P_2, T_1 \cup T_2, F_1 \cup F_2)$. It is assumed that N_f's have disjoint set of transitions, and only the common places are identified in the union. Furthermore we add a new position p^*, make each $\bar{p}_i$ initial, and also introduce n initial transitions t^*_m, one for each thread, that removes p^* and puts a token in $\bot_m$ and a token in the initial positions of each thread.*

The above construction only works when $\bot = \emptyset$. When $\bot = D_0$, for some $D_0 \subseteq \mathbb{D}$, we will introduce a new initial set of events (all in conflict) that introduce nondeterministically a token in some $p_i \in D_0$ and remove $\bar{p}_i$.

The problem of computing the *MOT* solution can be reduced to a *coverability* problem on the augmented net. To be more precise, fact d_i *may* hold before the execution of transition t of the control net if and only if $\{p_i, p_t\}$ is coverable from the initial marking of the control net where p_t is the local control place associated to transition t in its corresponding thread.

Theorem 1. *A dataflow fact d_i holds before the execution of a transition t in the control net N of a program if and only if $d_i \in D_t^*$ and the marking $\{p_i, p_t\}$ is coverable from the initial marking in the augmented net $N^{\mathcal{S},\mathcal{F}}$ constructed according to Definition 7.*

Checking coverability: While there are many tools that can check reachability/coverability properties of Petri nets, tools that use *unfolding* techniques [16,6] of nets are particularly effective, as they explore the state space using partially ordered unfoldings and give automatic reduction in state-space (akin to partial-order reduction for model checking of concurrent systems). We assume the reader is familiar with net unfoldings and refer to [6] for details.

Complexity of distributive CCD: Algorithms for Petri nets which use finite unfoldings essentially produces a finite unfolding of the net, from which coverability of one position can be checked in linear time. For every transition $t' \in T_{f_t}$ and every fact $d_i \in D_t^*$, we can create a new transition whose preconditions are those of t' plus p_i, and outputs a token in a new position (t, d_i). By Theorem 1, coverability of this single position is equivalent to fact d_i holding at t. Furthermore, we can argue that the unfolding of this net introduces at most $n|\mathbb{D}|$ new events compared to the unfolding of the augmented net.

Let us now analyze the size of the unfolding of the augmented net in terms of the size of the unfolding of the original control net; let us assume the latter has n events. We can show that (a) every marking reachable by a local configuration of the control net has a corresponding event in its finite unfolding that realizes this marking, and (b) that for every marking reached by a local configuration of the control net, there are at most $|\mathbb{D}|$ corresponding local configurations in the augmented net (at most one for each dataflow fact), and this covers all local configurations of the augmented net. Since the number of events in the unfolding is bounded by the number of markings reachable by local configurations, it follows that the size of the unfolding of the augmented net is at most $|\mathbb{D}|$ times that of the control net. This argues the efficacy of our approach in preserving the concurrency inherent in the control net and in exploiting distributivity to its fullest extent.

Theorem 2. *Let $(N, \mathcal{S}, \mathcal{F})$ be a distributive CCD problem, with $\mathcal{S} = (\mathcal{P}(\mathbb{D}), \subseteq, \cup, \bot)$. Let n be the size of the unfolding of N. Then the size of the unfolding of the augmented net $N^{\mathcal{S},\mathcal{F}}$ (and even the complexity of checking whether a fact holds at a control point) is at most $O(n|\mathbb{D}|)$.*

6 Experiments

We have applied the techniques from Section 5 to perform several dataflow analyses for concurrent programs. Unfortunately, there is no standard benchmark for concurrent dataflow programs. We have however experimented our algorithms with sample programs for the primary dataflow analysis problems, and studied performance when the number of threads is increased.

The motive of the experiments is to exhibit in practice the advantages of concurrent dataflow that exploit the causal framework set forth in this paper. While the practical efficacy of our approach on large programs is still not validated, we believe that setting up a general framework with well-defined problems permitting reasonable algorithms is a first step towards full-scale flow analysis. Algorithms that work on large code may have to implement approximations and heuristics, and we believe that the our framework will serve as a standard for correctness.

In many of our examples, there is an exponential increase in the set of reachable states as one increases the number of threads, but the partial order methods inherent to these techniques substantially alleviate the problem. We use the PEP tool [9] to check the coverability property on the augmented net to answer the relevant coverability queries.

For each example, we have included the sizes of the unfolding for the program's control net and of the augmented net (see Table 1). The *construction* time refers to the time to build the unfolding, and the *checking* time refers to the time for a single fact checking. Note the huge differences between the two times in some cases, and also note that the unfolding is only built once and is then used to answer several coverability queries. All experiments were performed on a Linux machine with a 1.7GHz processor and 1GB of memory. The numbers are all in seconds (with a precision of 0.01 seconds).

Uninitialized Variables. This set of examples contains a collection of n threads with n global variables $\mathtt{X}^0, \ldots, \mathtt{X}^n$. One uninitialized variable $\mathtt{X}^0$ in one thread can consequently make all $\mathtt{X}^i$s uninitialized. Concurrency results in many possible interleavings in this example, a few of which can make a certain variable $\mathtt{X}^j$ uninitialized.

Reaching Definitions. This example set demonstrates how our method can successfully handle synchronization mechanisms. There are two types of threads: (1) those which perform two consequent writes to a global variable Y, and (2) those which perform a read of Y. There are two variations of this example: (1) where none of the accesses is protected by a lock, which we call RD, and (2) where the read, and the two writes combined are protected by the same lock, which we call RDL (the code on the right). The main difference between the two versions is that Y := 1 will reach the read in the lock-free version, but cannot reach it in the presence of the locks. In a setting with one copy of T' and n copies of T, there are $2n$ definitions where only n of them can reach the line x := Y + 1 of T'.

```
      T                T'

acquire(l);      acquire(l)
  Y := 1;          x := Y + 1;
  Y := 2;        release(l)
release(l);
```

Table 1. Programs and Performances

Example	$\lvert\mathbb{D}\rvert$	#Threads	Unfolding Control Net	Unfolding Augmented Net	Checking Time (sec)	Construction Time (sec)
UV(10)	11	11	906	4090	< 0.01	<0.01
UV(20)	21	21	3311	16950	< 0.01	0.70
UV(60)	61	61	40859	156390	0.01	60.11
RD(3)	4	6	410	1904	< 0.01	0.03
RD(4)	5	8	1545	9289	0.01	1.5
RD(5)	6	10	5596	41186	0.01	133.16
RDL(3)	6	4	334	1228	< 0.01	0.01
RDL(4)	8	5	839	3791	< 0.01	29
RDL(5)	10	6	2024	10834	< 0.01	5.35
RDL(6)	12	7	4745	29333	0.01	121.00
AE(50)	2	50	250	650	< 0.01	< 0.01
AE(150)	2	150	750	1950	< 0.01	0.34
AE(350)	2	350	1750	4550	< 0.01	4.10

Available Expressions. The example set AE shows how the unfolding method can fully benefit from concurrency. The threads here do not have any dependencies. Each thread defines the same expression $\mathtt{X} + \mathtt{Y}$ twice, and therefore, the expression is always available for the second instruction of each thread. Table 1 shows that in the case of zero dependencies, the size of the unfolding grows linearly with the number of threads (understandably so since new threads do not introduce new dataflow facts).

7 Conclusions

The main contribution of this paper lies in the definition of a framework that captures dataflow analysis problems for concurrent program using partial orders that preserves the concurrency in the system. The preserved concurrency has been exploited in the partial-order based analysis, but could instead have been exploited in other ways, for example using partial-order reduction strategies as those used in SPIN.

As for future directions, the first would be to study *local* or *compositional* methods to solve the CCD problems and deploy them on large real world programs. This would have to handle (approximately) complex data such as pointers and objects. Our algorithms do not work for programs with *recursion*, and it is well known that dataflow analysis for concurrent programs with recursion quickly leads to undecidability. Structural restrictions like nested locking (see [11]) would be worth studying to obtain decidable fragments. Studying a framework based on computing *minimal fixpoints* for concurrent programs would be also interesting. Extending our approach to decide flow problems with *infinite domains of finite height* is challenging as well (they can be handled in the sequential setting [20]).

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