Changing Legal Systems: Abrogation and Annulment
Part I: Revision of Defeasible Theories

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Abstract. In this paper we investigate how to model legal abrogation and annulment in Defeasible Logic. We examine some options that embed in this setting, and similar rule-based systems, ideas from belief and base revision. In both cases, our conclusion is negative, which suggests to adopt a different logical model.

1 Introduction

Mainly inspired by [1], most formal models of norm change usually focus on the dynamics of obligations and permissions. However, as rightly noted on the occasion of a recent workshop on this topic[1], "these systems did not explicitly refer to possible changes in the underlying norms [. . .]". In fact, "new norms may be created and old norms may need to be retracted. In this dynamic setting, it is essential to distinguish norms from obligations and permissions as studied by deontic logic, to understand the formal properties specific for the dynamics of norms, and to describe how such objects can be manipulated [. . .]". Unfortunately, "a formal model that captures the relevant features of norm change is still lacking".

The aim of our work is to make some steps in this direction by investigating the notion of legal modification. Legal modifications are the ways through which the law implements norm dynamics [10]. Modifications can be either explicit or implicit. In the first case, the law introduces norms whose peculiar objective is to change the system by specifying what and how other existing norms should be modified. In the second case, the legal system is revised by introducing new norms which are not specifically meant to modify previous norms, but which change in fact the system because they are incompatible with such existing norms. The most interesting case is when we deal with explicit modifications, which permit to classify a large number of modification types.

In general, we have different types of modifying norms, as their effects (the resulting modifications) may concern, for example, the text of legal provisions, their scope, or their time of force, efficacy, or applicability [10,8,9]. Derogation is an example of scope change: a norm n supporting a conclusion P and holding at the national level may be derogated by a norm n' supporting a different conclusion P' within a regional context.

Hence, derogation corresponds to introducing one or more exceptions to *n*. Temporal changes impact on the target norm in regard to its date of force (the time when the norm is "usable"), date of effectiveness (when the norm in fact produces its legal effects) or date of application (when conditions of norm applicability hold). An example of change impacting on time of force is when a norm *n* is originally in force in 2007 but a modification postpones *n* to 2008. Substitution replaces some textual components of a provision with other components. For example, some of its applicability conditions are replaced by other conditions.

We are interested here in studying the concepts of *abrogation* and *annulment*.

Annulment is usually seen as a kind of repeal, as it makes a norm invalid and removes it from the legal system. Its peculiar effect applies *ex tunc*: annuled norms are prevented to produce all their legal effects, independently of when they are obtained. The nature of *abrogation* is most controversial. In some cases, it is important to see whether the abrogation is the result of judicial review, legislation, or referenda. But again, despite domestic peculiarities, abrogations, too, are seen as a type of norm removal, even though they are different from annulments; the main point is usually that abrogations operate *ex nunc* and so do not cancel the effects that were obtained before the modification. If so, it seems that abrogations cannot operate retroactively. However, this is not always true. Even where retroactive abrogations are prohibited (such as in the Italian system), the problem is open in some contexts. Suppose an ordinary court is called upon to decide a case in which a norm *n* applies, but the court argues that *n* infringes some fundamental rights and so it suspends the trial proceedings referring to the constitutional court to decide on the illegitimacy and abrogation of *n*. Constitutional court's decision and abrogation of *n* is necessarily posterior to the case. Hence, what is the difference between these modifications?

Suppose that a norm n_1 in force in 2006 states that, if your annual income is less than 5,000 euros, you are a needy person and norm n_2 says that a needy person has the right to live for free in a council house. If *n* is retroactively *annuled* in 2007, this counts as *n*'s removal since 2006, and all its effects are blocked. Imagine now that two norms n_3 and n_4 are added in 2007 stating that needy people's income is less than 3,000 euros and that needy people are eligible for medical aid. Even if *n* is retroactively *abrogated* in 2007, jurists may argue that its indirect effect (obtained via n_2: right to house) should not be estinguished in 2007, whereas the propagation of the qualification "needy person" (with an income of less than 5,000 euros) cannot propagate from 2006 to 2007, since this would make n_4 applicable. Note that, in other cases, indirect effects should propagate whereas the direct effect should be blocked, or all past effects should propagate, or, again, norm removal should apply after in 2007 and only blocking some effects retroactively holds. In fact, jurists [10] say that abrogations can at most block some, but not *all*, past effects (otherwise, we would have annulments).

To sum up, and independently of terminological issues, what we have to bear in mind is that here the law implements different reasoning patterns: in one case norms are removed with all their effects, whereas in other cases norms are removed but some or all their effects propagate if obtained before the modification.

How to model these scenarios? Clearly, a temporal representation may help, but the point is whether we can abstract from this aspect and move to a general analysis

(e.g., based on theory revision) where time is not considered. We address this issue using Defeasible Logic (DL) [12,2], but analogous considerations can be extended to other nonmonotonic (sceptical) rule-based systems. Although other options are available, rule-based systems seem a natural way to represent legal systems: legal norms are usually viewed as rules specifying some applicability conditions and a legal effect.

In this paper we discuss whether it is possible to adjust belief and theory revision in DL to capture abrogation and annulment. The layout is as follows. Section 2 provides an overview of DL. Section 3 considers an immediate method to adjust revision of belief sets in DL in order to capture annulment. Section 4 examines a possible alternative in which all operations, including contraction, are captured by only adding a suitable set of new rules. Even though this second option is better for modelling abrogation and annulment, some basic problems remain unsolved. Section 5 takes advantage of some ideas from the previous section and discusses how base revision in DL can be applied to capture norm removals. However, also this approach is not fully satisfactory, which suggests to adopt a different conceptual model, whose general features are illustrated in Section 6. This is the new model we have used for our initial investigation on modelling norm changes in DL [8,9].

2   Overview of Defeasible Logic

DL is based on a logic programming-like language and it is a simple, efficient but flexible non-monotonic formalism capable of dealing with many different intuitions of non-monotonic reasoning. An argumentation semantics exists [7]. DL has a linear complexity [11] and also has several efficient implementations [3]. In addition, some preliminary works on legal modifications in DL have been recently proposed [8,9].

A *defeasible theory* D is a structure $(F, R, \succ)$ where F is a finite set of facts, R a finite set of rules, and $\succ$ an acyclic superiority relation on R. *Facts* are represented as literals and are indisputable statements. A *rule* expresses a relationship between a set of premises and a conclusion. We have in DL three types of rules conveying the strength of the relationships: strict rules, defeasible rules and defeaters. A *strict* rule has the form $A_1, \ldots, A_n \rightarrow B$ and states the strongest kind of relationship since its conclusion always holds when the premises are indisputable. *Defeasible* rules have the form $A_1, \ldots, A_n \Rightarrow B$ and cover the case when the conclusion normally holds when the premises tentatively hold; *defeaters* have the form $A_1, \ldots, A_n \rightsquigarrow B$ and consider a situation where the premises do not warrant the conclusions: in defeaters the premises simply prevent another rule to support the opposite.

Accordingly, a conclusion can be labelled either as definite or defeasible. A definite conclusion is an indisputable conclusion, while a defeasible conclusion can be retracted if additional premises become available. DL is based on a constructive proof theory for conclusions. Hence, we can say that a derivation for a conclusion exists and that it is not possible to give a derivation for a conclusion. Based on these two ideas conclusions will be tagged according to their strength and type of derivation:

- $+\Delta B$, meaning that we have a definite proof for B (a definite proof is a proof where we use only facts and strict rules);
- $-\Delta B$, meaning that it is not possible to build a definite proof for B;

- $+\partial B$, meaning that we have a defeasible proof for B;
- $-\partial B$, meaning that it is not possible to give a defeasible proof for B.

Provability is based on the concept of a *derivation* (or proof) in $D = (F,R,\succ)$. A derivation is a finite sequence $P = (P(1),\ldots,P(n))$ of tagged literals satisfying four conditions (which correspond to inference rules for each of the four kinds of conclusion). $P(1..i)$ denotes the initial part of the sequence P of length i.

Proof conditions for strict derivations are here omitted. Strict proofs are just derivations based on detachment for strict rules. Given a strict rule $A_1,\ldots,A_n \to B$, where we have definite proofs for all A_i's, we can deduce B ($+\Delta B$).

DL is a sceptical non-monotonic formalism: with a possible conflict between two conclusions (i.e., one is the negation of the other), DL refrains to take a decision and we deem both as not provable unless we have some more pieces of information that can be used to solve the conflict. One way to solve conflicts is to use a superiority relation over rules. The superiority relation gives us a preference over rules with conflicting conclusions. In case we have a conflict between two rules we prefer the conclusion of the strongest of the two rules. The superiority relation is applied in defeasible proofs.

Some notational conventions before presenting proof conditions for defeasible derivations. Each rule is identified by a unique label. $A(r)$ denotes the set of antecedents of a rule r, while $C(r)$ denotes its consequent. If R is a set of rules, R_s is the set of all strict rules in R, R_{sd} the set in R of strict and defeasible rules, R_d the set of defeasible rules, and R_{dft} the set of defeaters. $R[B]$ denotes the set of rules in R with consequent B. If B is a literal, $\sim B$ denotes the complementary literal (if B is a positive literal C then $\sim B$ is $\neg C$; and if B is $\neg C$, then $\sim B$ is C).

Defeasible proofs proceed in three phases: we first look for an argument supporting the conclusion we want to prove (an applicable rule for the conclusion). Second, we look for arguments for the opposite of what we want to prove. Third, we rebut the counterarguments. This can be done by showing that the counterargument is not founded (i.e., some of the premises do not hold), or by defeating the counterargument, i.e., the counterargument is weaker than an argument for the conclusion we want to prove. Formally,

$+\partial$: If $P(i+1) = +\partial B$ then either
(1) $+\Delta B \in P(1..i)$ or
(2.1) $\exists r \in R_{sd}[B] \forall A \in A(r) : +\partial A \in P(1..i)$ and
(2.2) $-\Delta \sim B \in P(1..i)$ and
(2.3) $\forall s \in R[\sim B]$ either
   (2.3.1) $\exists A \in A(s) : -\partial A \in P(1..i)$ or
   (2.3.2) $\exists t \in R_{sd}[B]$ such that
      $\forall A \in A(t) : +\partial A \in P(1..i)$ and $t \succ s$.

$-\partial$: If $P(i+1) = -\partial B$ then
(1) $-\Delta B \in P(1..i)$ and
(2.1) $\forall r \in R_{sd}[B] \exists A \in A(r) : -\partial A \in P(1..i)$ or
(2.2) $+\Delta \sim B \in P(1..i)$ or
(2.3) $\exists s \in R[\sim B]$ such that
   (2.3.1) $\forall A \in A(s) : +\partial A \in P(1..i)$ and
   (2.3.2) $\forall t \in R_{sd}[B]$ either
      $\exists A \in A(t) : -\partial A \in P(1..i)$ or $t \not\succ s$.

3   Revising Extensions of Normative Systems

In the remainder of this paper we address the problem of how to embed in DL some ideas from belief and base revision in order to capture annulment and abrogation. We attack two different problems raised by these modifications: (i) how to block either some or all norm effects; (ii) how to model norm removals in legal systems. As we

argued, even though such modifications have a temporal flavour, we move to a general analysis where time is not considered.

We assume that a defasible theory can represent the basic logical structure of a legal system [8,9]. It is a general tenet in the literature that one reason why legal reasoning is defeasible depends on the fact that, in may cases, norm conclusions can be obtained only if we do not have stronger norms attacking them [14]. DL theories consist of a set of rules (which may be defeasible), a set of facts, and a set of priorities over rules (which establish their relative strength). In this perspective, rules naturally correspond to legal norms, while priorities represent the criteria used to solve legal conflicts. Hence, a general picture like this provides a standard for capturing the basics of legal systems [13]. With this said, let us begin with our discussion on annulment and abrogation.

Approaches based on AGM usually assume that a belief set B is a theory, i.e., a set of formulas closed under a logical consequence relation, thus $B = \text{Cn}(B)$. Let us consider the equivalent of this notion in DL.

Let HB_T be the Herbrand Base for a Defeasible Theory T. In [2], the extension of a Defeasible Theory T is defined as the 4-tuple

$$E(T) = (\Delta^+(T), \Delta^-(T), \partial^+(T), \partial^-(T)),$$

where $\#^\pm(T) = \{p | p \in HB_T, T \vdash \pm \# p\}, \# \in \{\Delta, \partial\}$.

Definition 1. *Let $T = (F, R, \succ)$ be a Defeasible Theory. We define another Defeasible Theory $T' = (\emptyset, R', \emptyset)$ such that R' is the smallest set satisfying the following conditions*

- *if $p \in \Delta^+(T)$, then $\rightarrow p \in R'$;*
- *if $p \in \partial^+(T)$, then $\Rightarrow p \in R'$;*
- *if $p \notin \Delta^+(T) \cup \Delta^-(T)$, then $p \rightarrow p \in R'_s$;*

- *if $p \in \Delta^-(T)$, then $R'_s[p] = \emptyset$;*
- *if $p \in \partial^-(T)$, then $R'_d[p] = \emptyset$;*
- *if $p \notin \partial^+(T) \cup \partial^-(T)$, then $p \Rightarrow p \in R'$.*

We will say that T' is the theory generated by the extension *of T.*

Proposition 1. *Let T be a defeasible theory. For every $p \in HB_T$, $T \vdash \# \pm p$ iff $T' \vdash \# \pm p$.*

The above result gives us an immediate way to define contraction for revision based on belief sets. We define $T_c^\ominus = T'$ such that $E(T) = (\Delta^+(T), \Delta^-(T), \partial^+(T), \partial^-(T))$ and T' is the theory generated by the extension

$$(\Delta^+(T) - \{c\}, \Delta^-(T), \partial^+(T) - \{c\}, \partial^-(T)).$$

It is easy to verify that the above way to define contraction satisfies all AGM postulates. The meaning of the result in Proposition 1 is that for every theory (and so every set of conclusions), we can generate a new equivalent theory without looking at the structure of the original theory: In fact, classically two theories are equivalent if they have the same extension (the same set of conclusions).

How can the procedure described in Definition 1 be used to cover abrogation and annulment?

Let examine *annulment*. When we annul a norm in a legal system, this means that all (direct and indirect) legal effects deriving from it must be cancelled as well. For

example, if we have a normative system T containing only the rules $A \Rightarrow B$ and $B \Rightarrow C$, then the annulment of the former rule (assuming the fact A) should block both B and C. Intuition suggests that contraction is the right operation to capture annulment. Hence, the question is how to use contraction in this case. What one could do here is simply to remove the consequent of the rule. However, the (positive defeasible) extension of T (i.e., $\partial^+(T)$) is $\{A,B,C\}$,[2] and contracting B leaves C in the extension. Hence, this immediate use of contraction is not representative of legal annulment. As we said, we have to consider all consequences of the formula to be contracted. In the above example, C can only be derived if B does. Accordingly, annulment of any rule $A_1,\ldots,A_n \Rightarrow B$ could be defined as follows. Let $T = (F,R,\succ)$ be a Defeasible Theory. Then

$$
T^{\ominus}_{A_1,\ldots,A_n \Rightarrow B} =
\begin{cases}
T & \text{if } A_1,\ldots,A_n \Rightarrow B \notin R \text{ or } \{A_1,\ldots A_n\} \not\subseteq \partial^+ \\
(F',R',\succ') & \text{otherwise}
\end{cases}
$$

$$\text{such that}$$
$$(1)$$
$$(F',R',\succ') \text{ is the theory generated by } E(T) - E(T')$$
$$\text{and } T' = (F = \{B\},R,\succ).$$

The contraction operation reflecting annulment is defined by "removing" the consequent of the rule. In addition, the theory T' generates all consequences of B with respect to T. Then $T^{\ominus}_{A \Rightarrow B}$ is the theory generated by the extension $E(T) - E(T')$. However, let us consider another example.

Example 1. Assume to work with the following theory:

$$T = (F = \{A\}, R = \{A \Rightarrow B, B \Rightarrow C, A \Rightarrow C\}, \emptyset).$$

Thus,

$$T' = (F = \{B\}, R = \{A \Rightarrow B, B \Rightarrow C, A \Rightarrow C\}, \emptyset).$$

Hence, $(\partial^+(T) = \{A,B,C\}) - (\partial^+(T') = \{B,C\}) = \{A\}$, and this leads (by applying Definition 1) to obtain that $T^{\ominus}_{A \Rightarrow B}$ corresponds to

$$T'' = (\emptyset, R = \{\Rightarrow A\}, \emptyset).$$

This procedure is not satisfactory unless more sophisticated measures are added. Example 1 shows that the procedure does not properly work, as C has *multiple causes* (B and A): with T'' we exclude $A \Rightarrow B$ by dropping B (and its consequences), but this leads to drop, too, C and so to exclude $A \Rightarrow C$, which is too much.

In addition, the above procedure requires to change the set of facts, which seems to us meaningless. Why cannot we change the set of facts? The facts of a theory are only those pieces of evidence in a case used to *apply* rules (norms) and not to *change* them: hence they should not be considered when one modifies norms. Accordingly, if norms are represented as rules, then reasoning only on the consequences of a theory is not representative of norm change. For example, the norm *HighIncome $\Rightarrow$ TopMarginalRate*

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[2] From now on, whenever clear from the context, we will use the term 'extension of a theory' as either the positive defeasible extension of it or the full extension of the theory (see Definition 1).

says that if the income of a person is in excess of the threshold for high income, then the top marginal rate must be applied. If it is a fact that Nino exceeded the threshold (i.e., *HighIncome* $\in F$) then he has to pay the top marginal rate. Thus the extension is {*HighIncome*, *TopMarginalRate*}; contracting with *HighIncome* results in the theory just consisting in $\Rightarrow$ *TopMarginalRate*, namely in a rule stating that, no matter what your income is, you will have to pay taxes at the top marginal rate. Thus, revising the evidence on which a case is based results in a change in the legislation, which seems a non-sense when applied to real legal systems.

The idea behind Definition 1 and (1) is that we have to generate a new normative system from the revised extension of corresponding source normative system. However, there are at least three reasons why Definition 1 and (1) do not seem satisfactory:

1. they may change the set of facts, and so do not differentiate between norms and instances of cases;
2. they revise theories regardless of the logical structure of the source theories;
3. they do not correctly account for *ex tunc* modifications, such as annulment.

Changing facts or generating new theories whose structure does not reflect the theories from which they have been obtained trivialise the concept of legal change. Indeed, it is crucial in the law to establish *what rules* generate *which effects*. Therefore, the contraction function defined in this section does not offer a suitable method for modelling annulment (and, in general, norm changes), even if it satisfies all AGM postulates.

4   Revising Normative Systems by Adding Exceptions

The difficulties under points 1 and 2 above can be alleviated by adopting in DL the approach proposed in [4] to deal with belief revision of rule-based non-monotonic formalisms, where change operators are not applied to the set of facts and are all implemented by adding new rules and changing priorities. This permits to incrementally modify the legal system, taking into account the logical structure of the source theory. Let us briefly recall the basic features of this approach.

Let us examine *expansion*. Following [6], expansion adds a formula A to $\partial^+(T)$ *only if* $\neg A \notin \partial^+(T)$. Hence, the case where $\neg A \in \partial^+(T)$ is irrelevant. However AGM decided to also add A in this case. In [4] T is kept unchanged, following [6] rather than [1]. Let $c = P_1, \ldots P_n$ be the formulas to be added. Expansion can be defined as follows:

$$
T_c^+ = \begin{cases} T & \text{if } \sim P_i \in \partial^+(T) \text{ or } \sim P_i = P_j \text{ for some } i, j \in \{1, \ldots, n\} \\ (F, R', \succ') & \text{otherwise} \end{cases}
$$

where
$$
R' = R \cup \{\Rightarrow P_1, \ldots, \Rightarrow P_n\}
$$
$$
\succ' = (\succ \cup \{\Rightarrow P_i \succ r \mid i \in \{1, \ldots, n\}, r \in R[\sim P_i]\}) -
$$
$$
\{r \succ \Rightarrow P_i \mid i \in \{1, \ldots, n\}, r \in R[\sim P_i]\}.
$$

(2)

Thus, rules are added that prove each of the literals P_i, and it is ensured that these are strictly stronger than any possibly contradicting rules.

Let us examine *contraction*, which seems the right candidate to capture at least some aspects of abrogation and annulment[3]:

$$T_c^- = \begin{cases} T & \text{if } P_1,\ldots,P_n \notin E(T) \\ (F,R',\succ') & \text{otherwise} \end{cases}$$

where   (3)

$$R' = R \cup \{P_1,\ldots,P_{i-1},P_{i+1},\ldots,P_n \rightsquigarrow \sim P_i \mid i \in \{1,\ldots,n\}\}$$
$$\succ' = \succ - \{s \succ r \mid r \in R' - R\}.$$

Intuitively, (3) aims to prevent the proof of all the P_is. To achieve this it is ensured that at least one of the P_is will not be proven. The new rules in R' ensure that if all but one P_i have been proven, a defeater with head $\sim P_j$ will fire. Having made the defeaters not weaker than any other rules, the defeater cannot be "counterattacked" by another rule, and P_j will not be proven, as an inspection of the condition $+\partial$ in Section 2 shows.

This approach slightly deviates from the AGM postulates, in particular from those for contraction. The second AGM postulate states that we contract a formula only by deleting some formulas, but not by adding new ones. This postulate cannot be adopted here because it contradicts the sceptical nonmonotonic nature of DL. To see this, suppose that we know A, and we have rules $\Rightarrow B$ and $A \Rightarrow \neg B$. Then A is sceptically provable and B is not. But if we decide to contract A, B becomes sceptically provable. Note that this behaviour is not confined to DL but holds in any sceptical nonmonotonic formalism [4]. Another peculiarity of this approach is the clear distinction between facts and rules and that facts are indisputable and cannot be changed. Thus, the negation of facts correspond to contradictions, and contracted facts are still included in the extension of the theory.

The advantages of [4]'s proposal are clear, as legal systems are changed by only adding new rules. In this sense, even though it works on theory extensions (suitable new rules ensure that some literals are included in extensions, or are excluded from them), this approach seems closer to base revision (see Section 5). But, independently of this question, one problem is still open: how to adjust this approach to account for legal modifications? A legal system T is modified by selecting, as a target, one or more norms of T, whereas [4]'s proposal parametrises operations to sets of literals. Let us bear in mind these points and proceed with our discussion.

5   Revising Normative Bases

The main problem with revision based on belief sets is that this approach does not mimic how the law implements norm changes, since "new" rules are generated to reflect the changes. Legal effects of rules can be used to guide how norms should be changed, but they should not determine *what* and *how* rules are changed. Therefore the alternative to revision based on belief sets is base revision. As is well-known, base revision does not operate on the extension of a theory, but rather applies to the theory "generators" (i.e., the non-logical axioms of the theory). This idea can be naturally coupled with

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[3] For space reasons, [4]'s treatment of revision is omitted.

partitioning the elements of a theory into "facts" and "rules", where the former cannot be revised (unless update is used), while the latter may be subject to revision.

Usually, belief revision operations are defined as contraction followed by expansion (according to Levi's Identity). Therefore, revision often results in some rules to be removed from the base of a theory. Base revision allows us to adopt different strategies, namely, to modify rules. In the law there are different types of norm changes: some directly correspond to the removal of rules (e.g., abrogation and annulment), while others amount to introducing new rules (e.g., derogation), and finally some are the result of partial modifications of provisions. In this perspective, assuming a rule-based representation of norms, revision on bases using modification techniques seems closer to the legal practice in so far as it allows for the conceptual distinction of these types of changes. In addition, as argued e.g. in [5], base revision results in theories that are closer to the structure of the theory to be revised.

Let us consider an example to introduce the idea of modification of bases. Suppose we want to revise a theory with a rule $r_1 : A \Rightarrow B$ and contract B when C is the case (let us say that C implies $\neg B$). The revision of the rule is $r'_1 : A, \neg C \Rightarrow B$. This means that we have modified the original rule taking into account the exception provided by C. DL has an elegant mechanism to deal with exceptions. An exception is simply implemented by a rule capturing the connection between the exceptional antecedent and the conclusion to be blocked. Thus, in the example above, instead of changing r_1 into r'_1, we may simply add a new rule such as $r_2 : C \Rightarrow \neg B$ or $r_2 : C \rightsquigarrow \neg B$, and state that $r_2 \succ r_1$. As we have seen in Section 4, this idea has been originally proposed for DL in [4], but there was still the open problem of how to set change operations in such a way to parametrise them with respect to the proper target of legal modifications, namely, legal rules.

Let us see how to adjust [4]'s definitions for norm changes, and in particular for annulment and abrogation. Let T be a theory and $A_1, \ldots, A_n \Rightarrow B$ be the rule to be removed. For *annulment*:

$$T^{annul1}_{A_1,\ldots,A_n \Rightarrow B} = T^-_B \tag{4}$$

Hence, the annulment of a rule is the contraction of the head of the rule. This solution directly applies (3) to the head of the rule to be annulled. However, (4) is too strong since it forces the removal of B from the extension (unless B is a fact). If we have two different (and independent) rules applicable at the same time and with the same head, and we just annul one of them, the other should still be able to produce its effect. But (4) affects the second rule as well. Thus, we have to give an alternative annulment operation based on a variant of the contraction operation.

$$T^{annul2}_{r:A_1,\ldots,A_n \Rightarrow B} = \begin{cases} T & \text{if } B \notin \partial^+(T) \\ (F, R', \succ') & \text{otherwise} \end{cases}$$

$$\text{where} \tag{5}$$

$$R' = R \cup \{r' : \rightsquigarrow \sim B\}$$
$$\succ' = \succ \cup \{(r', r)\} \cup \{(s, r') | s \in R[B] - \{r\}\}$$

Consider the following examples.

Example 2. Let us consider the following theory:

$$T = (F = \{A\}, R = \{r_1 : A \Rightarrow B, \ r_2 : B \Rightarrow C\}, \emptyset).$$

Clearly, $\partial^+(T) = \{B, C\}$. Hence,

$$T^{\prime annul2}_{r_1 : A \Rightarrow B} = (F, R \cup \{r'_1 : \leadsto \neg B\}, \emptyset).$$

In the resulting theory we prove $-\partial B$, which makes r_2 inapplicable, thus preventing the positive conclusion of C.

Example 3. Let us consider again the theory in Example 1:

$$T = (F = \{A\}, R = \{r_1 : A \Rightarrow B, \ r_2 : B \Rightarrow C, \ r_3 : A \Rightarrow C\}, \emptyset).$$

The annulment of r_1 still amounts to adding $r'_1 : \leadsto \neg B$ to R, which prevents the conclusion of all literals depending *only* on B. Accordingly, C will be in the extension, as it is obtained through r_3. In addition, if $r_4 : \ \Rightarrow B$ were in R, r_4 would be stronger than r^-, thus obtaining B.

In general this approach is closer to the legal practice, as it precisely focuses on modifications of norms and not on the modification of the normative positions (effects) of norms. First, it does not depend on facts. Second, it offers a seamless solution to *ex tunc* modifications. However, things can be viewed from a different perspective. Even though this approach can simulate *ex tunc* modification like annulment (since it allow us to block norm effects), the actual operation fails to remove norms. (Hence, this approach is appealing for modifications corresponding essentially to exceptions, such as derogations: see Section 1.) When a norm is annulled, it is "removed" from the legal system, whereas here we just remove the effects of the norm and its consequences.

Accordingly, we can simply remove the rule to be annulled from the set of rules:

$$T^{annul3}_r = (F, R - \{r\}, \succ) \tag{6}$$

But, then, we have another problem: How to deal with *ex nunc* modifications, such as abrogations? In this case, the modification of a rule should not necessarily prevent the derivation of its conclusions. Let us consider Example 2 and assume that the abrogation of r_1 does not prevent the derivation of B and C. This means that, if B and C were derivable before the modification, then they should remain in the extension of the revised theory. Here, we have two options. First, we can argue, as done above with annulment, that when a norm is abrogated, it is "removed" from the legal system. But, if r_1 is removed following a similar procedure to that stated in (6), the extension of the revised theory will *not* contain B as well as C, whereas abrogations can also admit cases where both conclusions should be maintained. Thus, a second option does not remove the rule, but adds a suitable set of new rules which allow to derive what should not be blocked. However, what can we do in this case if both B and C should not be dropped? It seems hard to adjust (5) in order to maintain both B and C. At most, what we can do

is preventing the derivation of B and maintaining C. Only in this case, if $T = (F, R, \succ)$ is a defeasible theory, then the *abrogation* of a norm $r : A_1, \ldots, A_n \Rightarrow B$ runs as follows:

$$T^{abr}_{r:A_1,\ldots,A_n \Rightarrow B} = \begin{cases} T & \text{if } r \notin R \\ (F, R', \succ') & \text{otherwise} \end{cases}$$

where

$$
\begin{aligned}
R' = R \cup \{&r^- : \rightsquigarrow \neg B, \ r' : \ \Rightarrow B'\} \\
\cup \{&t' : (A(t) - \{B\}) \cup \{B'\} \rightarrow C(t) | t \in R_s \text{ and } B \in A(t)\} \\
\cup \{&t' : (A(t) - \{B\}) \cup \{B'\} \Rightarrow C(t) | t \in R_d \text{ and } B \in A(t)\} \\
\cup \{&t' : (A(t) - \{B\}) \cup \{B'\} \rightsquigarrow C(t) | t \in R_{dft} \text{ and } B \in A(t)\} \\
\succ' = \succ \cup \{&(w, r^-) | w \in R[B] - \{r\}\} \cup \{(t', s) | (t, s) \in \succ\} \cup \{(s, t') | (s, t) \in \succ\}
\end{aligned}
$$

(7)

where B' is a new literal not appearing in T.

Proposition 2. *Given a theory T and a rule $r : A_1, \ldots, A_n \Rightarrow B$, for every $C \in HB_T - \{B\}$, $T \vdash C$ iff $T^{abr}_r \vdash C$.*

Example 4. Consider the following theory:

$$T = (F = \{A, D\}, R = \{r : A \Rightarrow B, \ t : B \Rightarrow C, \ s : D \Rightarrow \neg C, \ w : E \Rightarrow B\}, (t, s) \in \succ)$$

Hence, according to (7), $T^{abr}_{r:A_1,\ldots,A_n \Rightarrow B}$ is as follows:

$$
\begin{aligned}
T^{abr}_{r:A_1,\ldots,A_n \Rightarrow B} = (F = \{A, D\} \\
R = \{r : A \Rightarrow B, \ t : B \Rightarrow C, \ s : D \Rightarrow \neg C, \ w : E \Rightarrow B \\
r^- : \rightsquigarrow \neg B, \ r' : \Rightarrow B', \ t' : B' \Rightarrow C\} \\
\succ = \{(t, s), (t', s), (w, r^-)\})
\end{aligned}
$$

The fact A makes r applicable, but the introduction of r^- blocks the derivation of B using r. However, C is derived via r' and t' (which is stronger than s). Note that (7) is such that the defeater r^- attacks only r (we are abrogating rule r only): hence, if E were in F, B would be obtained from w.

In sum, we have the following possibilities:

- We omit to model annulments and abrogations as corresponding to rule removals. Hence, we represent them working only on rule conclusions and so adopt (5) and (7). However, (7) is partially satisfactory, as it blocks the derivation of the head of the abrogated rule; but an abrogation may remove only norms and not the already obtained effects of the norms to be abrogated.
- We address the issue that annulments and abrogations correspond to rule removals. Thus, (6) works for annulments, but it seems quite hard to find an adequate counterpart for abrogation.
- We do not care whether annulments and abrogations correspond to rule removals and are free to adopt, together with (7), either (5) or (6). But, as we said, (7) is problematic.

Of course, we do not exclude that the above problems can be settled. For example, some limits of (7) can avoided by combining the introduction of exceptions and the removal of the abrogated rule. This can be done by applying the idea in (6) and subsequently reinstate the conclusions that should not be blocked. This can be done by simply using expansion $^+$ as defined in (2). More precisely, suppose $c = C_1, \ldots C_n$ are the consequences of the rule to be abrogated which we want to maintain.

Definition 2. *Let* $T = (F, R, \succ)$ *be a Defeasible Theory such that* $r : A_1, \ldots, A_n \Rightarrow B \in R$. *Then*
$$T^{abr'}_{r:A_1,\ldots,A_n \Rightarrow B} = (T')^+_c$$
such that $T' = (F, R - \{r\}, \succ)$ *and* $c = C_1, \ldots C_n \in E(T'')$, *where*

- $T'' = (F = \{B\}, R, \succ)$;
- *for every* C_k, $1 \leq k \leq n$, $C_k \notin E(T')$.

But, even in that case, another difficulty arises when we have to deal with *retroactive* modifications: as we already mentioned, retroactivity is a typical feature of legal modifications. This problem is discussed in the following section.

6   A Temporal Model for Legal Systems and Norm Change

6.1   Revision and Retroactivity

A norm modification is an operation such that a normative system (consisting of norms and the consequences of cases) is transformed into a different normative system. Accordingly, dynamics of a normative system are described by a sequence of operations.

Suppose we have a system, let us call it T_0, in which we introduce a new rule r and subsequently we remove another rule, let us say s. The system obtained from the first operation is T_1, while the final system is T_2. Thus $T_2 = ((T_0)^+_r)^-_s$. So far so good. But let us suppose that that the removal of s is retroactive. How can we model this case? The idea is that every time we have a retroactive modification we should reconstruct the normative system at the time when the retroactive modification is effective. For example, if the modification is effective since yesterday, we have to recover the system as it was yesterday by undoing the operations leading to the normative system we have today, then we have to apply the retroactive modification and finally redo the other modifications. So, if in the example above s is a retroactive modification effective from T_0, the sequence of modifications still adds r and removes s, but the sequence of theories is $T'_1 = (T_0)^-_s$ and $T_2 = ((T_0)^-_s)^+_r$. Is this procedure in agreement with the intuition behind retroactive legal modifications? Our answer is negative. The point is that it is possible to define transformations moving from one normative system T_i to T_{i+1} where the transformation is effective at T_i itself, thus the system to be changed is not the target of the modification but the source of it. Let us consider the following example. The normative system T_0 is just the fact A. T_1 is obtained from T_0 by retroactively adding two rules $A \Rightarrow B$ and $B \Rightarrow C$ and these rules are effective in T_0. Then the next transformation, leading to T_2 is the removal of $A \Rightarrow B$ from T_0. But then we have two different versions of T_0. Analogous considerations apply when we work on rule consequences and model modifications adding defeaters.

   The reason why we have multiple versions of a normative system is that norms have different temporal dimensions: the time of validity of a norm (when the norm enters in the normative system) and the time of effectiveness (when the norm can produce legal effects). Thus, if one wants to model norm modifications, then normative systems must be modelled by more complicated structures. In particular, a normative system is not just the set of norms valid in it, but it should also consider the normative systems where the norms are effective. Accordingly, a normative system is a structure $N_i = (T_i, \langle T_0, T_1, \dots \rangle)$, where T_i is the theory modelling the set of norms/rules and facts valid in the normative system N_i, and $\langle T_0, T_1, \dots \rangle$ is the sequence of theories encoding the effective norms for all "versions" of the normative system.

   A revision of a legal system is an operation that transforms a normative system into another normative systems by 'changing' the rules in it. In particular, the operation should specify what rules are to be changed and when they are changed, and when the changes are effective. Thus a norm change can be seen as a transaction from a normative system $N_i = (T_i, \langle T_0^i, T_1^i, \dots \rangle)$ to a normative system $N_{i+1} = (T_{i+1}, \langle T_0^{i+1}, T_1^{i+1}, \dots \rangle)$, where there exists some j such that $T_j^{i+1} = change(T_j^i)$ for some *change* operation. For example, the abrogation of a rule r may be modeled as $T_{i+1}^{i+1} = (T_i^i)_r^{abr}$, and the retroactive annulment of r, as $T_j^{i+1} = (T_j^i)_r^{annul}$ for $j < i$. In addition, in general, once a norm has been introduced in a normative system the norm continues to be in the normative system unless it is explicitly removed. This means that the norm must be included in all theories succeeding the theory in which it has been first introduced. Accordingly, it could be very cumbersome to keep track of the changes and where the changes have to been applied. In real normative systems norms are introduced at a particular time, they are effective at a particular time, and so are changes –changes are norms themselves. Thus, to obviate the issue of keeping track of the changes, and at the same time to offer a conceptual model of norm changes, we have proposed in [8,9] an extension of DL with time, where we consider the two temporal dimensions of relevance for norm change (effectiveness and validity). This is done by labelling rules with two time values, one for the validity time of the norms, and the other for their effectiveness time; furthermore the labels indicate whether these 'changes' persist or not. The idea that changes are norms themselves is captured by the notion of meta-rule, i.e., a rule whose elements can be rules themselves and not only literals. The next section offers the conceptual background of the proposal presented in [8,9].

6.2   Inner and Outer Time of Legal Systems

The above discussion suggests that the dynamics of a legal system *LS* are more correctly captured by a time-series $LS(t_1), LS(t_2), \dots, LS(t_j)$ of its versions. Each version of *LS* is called a *norm repository* [8,9]. The passage from one repository to another is effected by legal modifications or simply by persistence [9]. But dynamics of norm change and retroactivity need to introduce another time-line within each version of *LS* (see Figure 1). Clearly, retroactivity
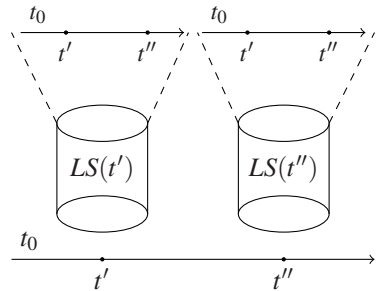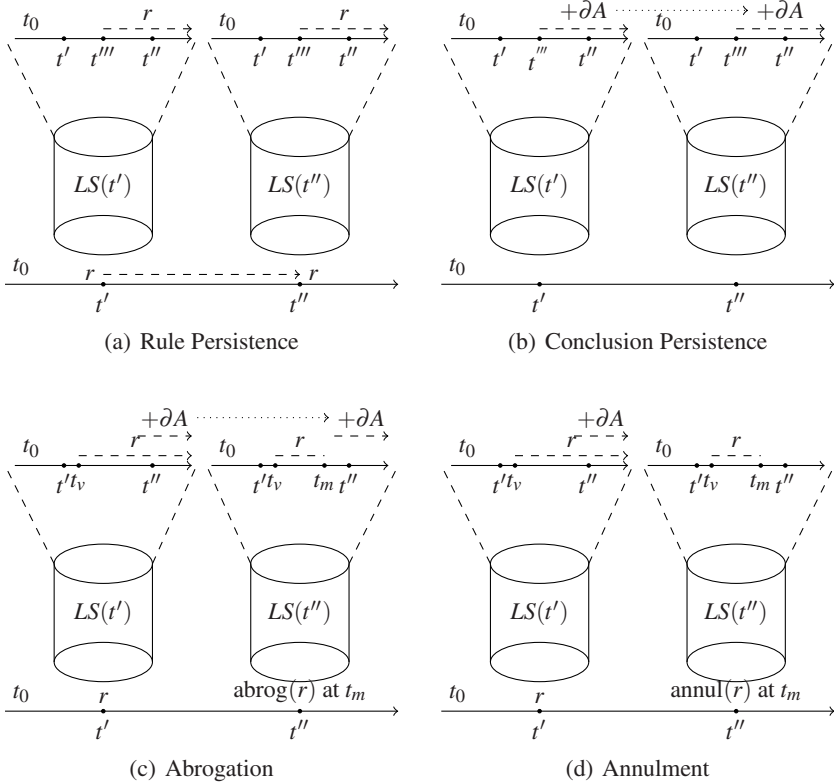

Fig. 1. Legal System at t' and t''

(a) Rule Persistence

(b) Conclusion Persistence

(c) Abrogation

(d) Annulment

does not imply that we can really change the past, but it rather requires that we have to reason on the legal system from the viewpoint of its current version as it were revised in the past: when we change some $LS(i)$ retroactively, this does not mean that we modify some $LS(k)$, $k < i$, but that we move back from the perspective of $LS(i)$. Hence, we can "travel" to the past along this inner time-line, i.e. from the viewpoint of the current version of LS where we modify norms.

Elements contained in, or derived from, theories can propagate across these time-lines. Hence, propagation concerns the derived conclusions of rules (when some consequent P holds), the rules themselves, and also derivations (i.e., queries: $+\partial P$). This introduces several options regarding how modifications affect a legal system over time:

– conclusions may persist within a certain repository or across different repositories;
– derivations may persist within a certain repository or across different repositories;
– rules may persist within a certain repository or across different repositories.

For example, Figure 2(a) shows how rule persistence works. A persistent rule r enacted at time t' and in force at t''' carries over from the legal system $LS(t')$ to the legal system $LS(t'')$, where it is still in force at t'''. Figure 2(b) illustrates conclusion persistence: a conclusion A persists from $LS(t')$ to $LS(t'')$ even if the rules used to derive it are no longer effective in $LS(t'')$. Figure 2(c) presents a case of abrogation: in $LS(t')$ rule

r, in force from t_v onwards, produces a persistent effect A. The effect carries over by persistence to $LS(t'')$ even if the rule r is abrogated at t_m and is no longer in force to produce the effect. Finally, Figure 2(d) illustrates a case of annulment: in $LS(t')$ rule r, in force since t_v, is applied and produces a persistent effect A. Since the rule is annulled in $LS(t'')$ at t_m, the effect of A must be undone as well. While the intuition in Figures 2(c) and 2(d) seems clear, its precise implementation in DL is not simple and only a partial solution was offered in [9]. The development of a complete DL temporal model for abrogation and annulment is a matter of future work.

7   Summary

In this paper we investigated how to model in DL legal abrogation and annulment. Terminology may vary from one legal system to another, but, despite this, it is possible to identify in general two different reasoning patterns: in one case norms are removed with all their effects, whereas in other cases norms are removed but all or some of their effects propagate if obtained before the modification. We examined some ways to capture these intuitions in DL using techniques from revision based on belief sets and from base revision. We concluded that abrogation and annulment can only be partially represented in these settings. In addition, we argued that it is hard, if not impossible, to simulate retroactivity, which clearly refers to the temporal dimension. Hence, we illustrated a different conceptual starting point from which the problem can be addressed.

References

1. Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change: Partial meet contraction and revision functions. Journal of Symbolic Logic 50, 510–530 (1985)
2. Antoniou, G., Billington, D., Governatori, G., Maher, M.J.: Representation results for defeasible logic. ACM Transactions on Computational Logic 2(2), 255–287 (2001)
3. Bassiliades, N., Antoniou, G., Vlahavas, I.: DR-DEVICE: A defeasible logic system for the Semantic Web. In: Bry, F., Henze, N., Małuszyński, J. (eds.) PPSWR 2003. LNCS, vol. 2901. Springer, Heidelberg (2004)
4. Billington, D., Antoniou, G., Governatori, G., Maher, M.J.: Revising nonmonotonic belief sets: The case of defeasible logic. In: Burgard, W., Christaller, T., Cremers, A.B. (eds.) KI 1999. LNCS (LNAI), vol. 1701. Springer, Heidelberg (1999)
5. Di Giusto, P., Governatori, G.: A New Approach to Base Revision. In: Barahona, P., Alferes, J.J. (eds.) EPIA 1999. LNCS (LNAI), vol. 1695. Springer, Heidelberg (1999)
6. Gärdenfors, P.: Knowledge in Flux: Modeling the Dynamics of Epistemic States. MIT, Cambridge (1988)
7. Governatori, G., Maher, M.J., Billington, D., Antoniou, G.: Argumentation semantics for defeasible logics. Journal of Logic and Computation 14(5), 675–702 (2004)
8. Governatori, G., Palmirani, M., Riveret, R., Rotolo, A., Sartor, G.: Norm modifications in defeasible logic. In: Proc. JURIX 2005. IOS Press, Amsterdam (2005)
9. Governatori, G., Palmirani, M., Riveret, R., Rotolo, A., Sartor, G.: Variants of temporal defeasible logic for modelling norm modifications. In: Proc. ICAIL 2007. ACM Press, New York (2007)

10. Guastini, R.: Teoria e dogmatica delle fonti, Giuffré, Milan (1998)
11. Maher, M.J.: Propositional defeasible logic has linear complexity. Theory and Practice of Logic Programming (6), 691–711 (2001)
12. Nute, D.: Defeasible reasoning. In: Proceedings of 20th HICSS. IEEE press, Los Alamitos (1987)
13. Prakken, H.: Logical Tools for Modelling Legal Argument. Kluwer, Dordrecht (1997)
14. Sartor, G.: Legal Reasoning. Springer, Dordrecht (2005)