

## Conclusion

This conclusion is intended for those who have read the book to the end. We hope that the main text and the notes to separate chapters have furnished a convincing evidence of how rich from the mathematical point of view, both conceptually and technically, is the subject of solitons and integrable partial differential equations. In fact, the inverse scattering method naturally intertwines various branches of mathematics: differential geometry, the theory of Lie groups and Lie algebras and their representations, complex and functional analysis. All of them serve one common purpose, to classify integrable equations and describe their solutions. As a result, the traditional parts of these branches, such as Hamiltonian formalism, affine Lie algebras, or the Riemann problem are seen in a new light.

Moreover, the progress of the inverse scattering method has given rise to new problems and structures in these domains. It will suffice to remind of the general notion of  $r$ -matrix and its interpretation from the Hamiltonian, group-theoretic, and analytical points of view. This is what reflects the modern trend in mathematics when theoretical disciplines that at first sight seem unrelated interact and draw from one another in the joint effort to solve concrete problems having important physical applications.

To a still greater extent this trend persists when the methods developed in our book are generalized to models of quantum mechanics and field theory. Research in this direction has been very active in the last years. And again, the unifying concept proves to be the (quantum)  $R$ -matrix. We hope that this line of study will soon be summarized in yet another monograph like the one we present.