Colonial Competitive Algorithm as a Tool for Nash Equilibrium Point Achievement

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Abstract. This paper presents an application of Colonial Competitive Algorithm (CCA) in game theory and multi-objective optimization problems. The recently introduced CCA has proven its excellent capabilities, such as faster convergence and better global optimum achievement. In this paper CCA is used to find Nash Equilibrium points of nonlinear non-cooperative games. The proposed method can also be used as an alternative approach to solve multi-objective optimization problems. The effectiveness of the proposed method, in comparison to Genetic Algorithm, is proven through several static and dynamic example games and also multi-objective problems.

Keywords: Colonial Competitive Algorithm (CCA), game theory, global Nash Equilibrium, evolutionary algorithms.

1 Introduction

Game theory is generally considered to have begun with the publication of "*The Theory of Games and Economic Behaviour"*, by von Neumann & Morgenstern in 1944 [1]. It provided the mathematical analysis of the conflict theory and brought the terminology with which one could analyze it. The development of the "Prisoner's Dilemma" by Tucker and J. Nash's papers on the definition and existence of equilibrium in 1950 laid the foundations of the modern non-cooperative game theory [2, 3]. These papers introduced the idea of the decision makers as players in a game and were designed to solve multi-objective optimization problems using game theory. Nash Equilibrium (NE) defined in [2], is a set of strategies such that no player can improve his payoff, given the strategies of all other players in the game, by changing his strategy. Nash proved that all non-cooperative games have a NE point.

Game theory can today be defined as the analysis of rational behaviour under circumstances of strategic interaction, when a player's best strategy depends on that of the others [4]. The players can be any entities that are able to exhibit strategic behaviour.

Economics is the most interesting field where game theory has been studied and applied. In addition to economics, game theory has been applied to biology, mathematics, political science, management science and even philosophy [5].

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There are different kinds of games resulting in different equilibria points. Some of these equilibria points are Nash, Cournot, Stackelberg, Correlated and Bayesian equilibria. In this paper we will focus on Nash Equilibrium point of nonlinear non-cooperative games.

From a point of view, Games are divided into Dynamic and Static games. *Dynamic Games* mathematically model the interaction among different players, controlling a dynamical system. Instances of such situations happen in armed conflicts and economic competition. These examples are a kind of dynamical systems since the players' actions influence the evolution of the state of a system over time. The difficulty in deciding what should be the behavior of these players relies on the fact that each action a player takes at a given time will influence the reaction of the opponents at later time [6]. A precise mathematical framework to find the NE in linear dynamic games with quadratic cost is introduced in [7]. There have also been made considerable efforts to find the NE Points in nonlinear non-cooperative games. To enhance the poor convergence of simple coevolutionary programming, [8] uses hybrid coevolutionary programming to escape local NE traps and to reach the real NE points. The method used in this paper removes the need for not only such modifications but also the hybrid coevolutionary algorithms themselves. Reference [9] models the behaviour of participants in electricity markets. To study the dynamic behaviour of participants over many trading intervals a coevolutionary approach is developed. Trading agents co-evolve their own populations' strategies using a Genetic Algorithm (GA). [10] proposes a game model based coevolutionary algorithm to solve multi-objective class of problems. It tries to find Evolutionary Stable Strategy (ESS) as a solution to multi-objective problems using game model based coevolutionary algorithm. In [11] efficient approximation algorithms to achieve Nash equilibria in anonymous games, games in which the players' utilities, though different, do not differentiate between other players, are introduced.

In a multi-objective optimization problem including $\boldsymbol{P}$ objectives, there exist $\boldsymbol{P}$ players, each optimizing his own criterion. When a player optimizes his own criterion he knows all other players' actions (strategies) and also knows that these strategies are fixed during his decision making process. This improvement goes on for all players and stops when no player can further improve his criterion. This situation implies that the system has reached to an equilibrium point. This point is called the Nash Equilibrium of the game. If we assume Y_1 as the search space for the first criterion and Y_P the search space for the last criterion. A strategy set $(X_1, X_2, ..., X_P) \in Y_1 \times Y_2 \times ... \times Y_P$ will be called NE point iff:

$$
\begin{aligned}
f_{Y1}(X_1, ..., X_P) &= \inf_{x_1 \in Y_1} f_{Y1}(X_1, ..., X_P) \\
&\vdots \\
f_{Yp}(X_1, ..., X_P) &= \inf_{x_p \in Y_p} f_{Yp}(X_1, ..., X_P)
\end{aligned}
\tag{1}
$$

As an other definition of NE, if the set $\underline{X} = (X_1, X_2, ..., X_p)$ is the strategy set for all players and $J_i(\underline{X})$ is the cost function of player i then $\underline{X}^* = (X^*_1, X^*_2, ..., X^*_p)$ will be a NE point iff: $\forall i,\ \forall x_i$

$$
J_i(X^*_1, ..., X^*_i, ..., X^*_P) \le J_i(X_1, ..., X_i, ..., X_P) \tag{2}
$$

Classical approach to find NE points needs the cost (payoff) function to be concave and differentiable. If this condition is satisfied, solving the following set of equations simultaneously gives the NE point of the game:

$$\frac{\partial J_i(X_1, ..., X_i, ..., X_p)}{\partial X_i} = 0 \quad i = 1, ..., p \tag{3}$$

But there are several games in which the payoff function is not differentiable or is so nonlinear that the calculation of differentiation is really challenging. In these cases the classical approach fails to give the NE point and the use of evolutionary algorithms becomes indispensable. Genetic Algorithms are the most preferred evolutionary methods, applied to such problems. [12] uses GA to find NE points. It has constructed the chromosomes in such a way that the actions of all players are included. Each player updates his own part of chromosomes. This way we can obtain NE point of almost all types of games with nonlinearity in cost (payoff) functions. But the main problem with GA is its poor convergence rate and also poor global optima achievement. In this paper, we apply a newly introduced evolutionary algorithm, called CCA, on non-cooperative games with nonlinear cost functions to eliminate mentioned drawbacks.

The recently introduced CCA [13] has been used to solve many optimization problems in different fields. This global search strategy uses the socio-political competition among empires as a source of inspiration. Like other evolutionary strategies that start with an initial population, CCA begins with initial empires. Any individual of an empire is a country. There are two types of countries; colony and imperialist state that collectively form empires. Imperialistic competition among these empires along with modeled assimilation policy forms the basis of this algorithm. During the imperialistic competition, weak empires collapse and powerful ones take possession of their colonies. Imperialistic competition converge to a state in which there exist only one empire and its colonies are in the same position and have the same cost as the imperialist. This evolutionary optimization strategy has shown great performance in both convergence rate and better global optima achievement. Nevertheless, its effectiveness, limitations and applicability in various domains are currently being extensively investigated. In [14] it has been used to design an optimal controller which not only decentralizes but also optimally controls an industrial Multi Input Multi Output (MIMO) Evaporator system. In [15] CCA is used for reverse analysis of an artificial neural network in order to characterize the properties of materials from sharp indentation test. In order to find the optimal priorities for each user in recommender systems, [16] uses CCA in "Prioritized user-profile" approach to recommender systems, trying to implement more personalized recommendation by assigning different priority importance to each feature of the user-profile in different users.

The most interesting aspect of the proposed method in this paper is its ability to find global Nash Equilibrium more reliably and not stopping at local NE traps. Effectiveness of this method will be shown through several static and dynamic games and some multi-objective problems.

The structure of rest of the paper is as follows:

Section 2 studies the idea of Nash Equilibrium search using evolutionary algorithms. In section 3 a brief description of CCA is presented. Simulation results obtained from some examples are presented in section 4. Finally, the conclusion is done in section 5.

2 Evolutionary Algorithms and Nash Equilibrium

In this section we study the method using with, an evolutionary algorithm can be applied to reach Nash Equilibrium. A P-objective game corresponding to P players each with one action set is considered here. Let each of $\underline{X}_i s$ in the potential solution of the game $\underline{\boldsymbol{X}}=\{\underline{X}_1, \underline{X}_2, \ldots , \underline{X}_p\}$ be the action set of players P_1 to P_P, respectively. Each player can optimize and affect only his own set of actions $\underline{X}_i$. But his action might influence the cost function of other players.

At each iteration of the payoff optimization process, player P_i optimizes his cost function ($J_i(\underline{\boldsymbol{X}})$) knowing that the other players' have chosen and fixed their action sets ($\underline{X}_j$, $j=1,\ldots,P$, $j\neq i$). To start the game, each player randomly chooses his action. Then each player optimizes his action with respect to the actions other players have preferred. When all the players have optimized their actions once, the first iteration is over. Players continue the game using the potential solution of iteration *n, i. e.,* $\underline{\boldsymbol{X}}_n{}^*=\{\underline{X}^*_{1n}, \underline{X}^*_{2n}, \ldots , \underline{X}^*_{pn}\}$ as their initial strategies for next iteration. The game stops when no player is able to change his action and so the cost function stops changing. This stop point is the desired Nash Equilibrium point of the game. Fig. 1 shows diagram of the optimization process used in this paper.

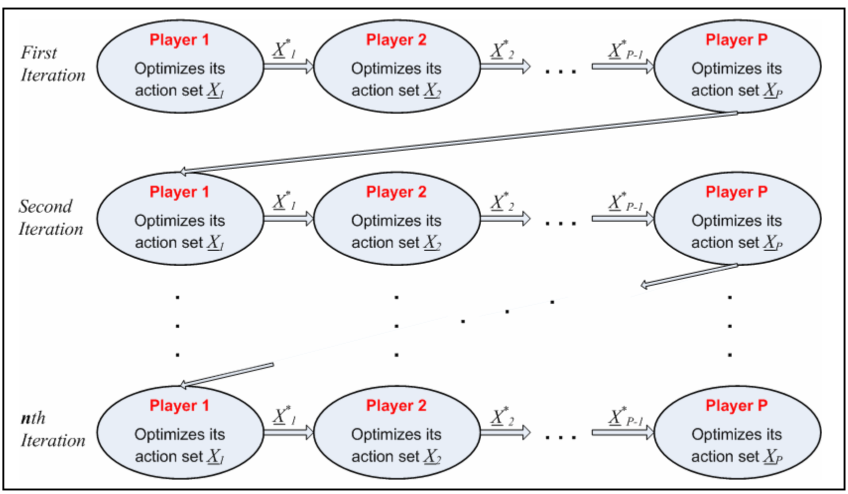

Fig. 1. The strategy scheme used to find Nash Equilibrium

It might seem that this algorithm is similar to that of parallel GA (PGA) introduced in [17]. But it is noteworthy that the principle difference between these two is that PGA uses the same cost function whereas the above mentioned method uses different criteria, resulting in NE point. In this paper we focus on the use of a novel evolutionary algorithm, CCA, to find the NE point of the games.

3 Brief Description of CCA

CCA is a novel global heuristic search method that uses imperialism and imperialistic competition process as a source of inspiration. Fig. 2 shows the pseudo code for this algorithm. This algorithm starts with some initial countries. Some of the best countries are selected to be the *imperialist states* and all the other countries form the *colonies* of these imperialists. The colonies are divided among the mentioned imperialists based on their power.

```
1) Select some random points on the function and initialize the
   empires.
2) Move the colonies toward their relevant imperialist
   (Assimilation).
3) If there is a colony in an empire which has lower cost than
   that of the imperialist, exchange the positions of that
   colony and the imperialist.
4) Compute the total cost of all empires (Related to the power
   of both the imperialist and its colonies).
5) Pick the weakest colony (colonies) from the weakest empires
   and give it (them) to the empire that has the most
   likelihood to possess it (Imperialistic competition).
6) Eliminate the powerless empires.
7) If there is only one empire left, stop, if not go to 2.
```

Fig. 2. Pseudo code of the Colonial Competitive Algorithm

After dividing all colonies among imperialists and creating the initial empires, these colonies start moving toward their relevant imperialist state. This movement is a simple model of assimilation policy that was pursued by some imperialist states [13]. Fig. 2 shows the movement of a colony towards the imperialist. In this movement, θ and *x* are random numbers with uniform distribution as illustrated in (4) and *d* is the distance between colony and the imperialist.

$$\begin{aligned} x &\sim U(0, \beta \times d) \\ \theta &\sim U(-\gamma, \gamma) \end{aligned} \tag{4}$$

where β and γ are arbitrary numbers that modify the area that colonies randomly search around the imperialist. In our implementation β and γ are 2 and 0.5 (rad), respectively.

The total power of an empire depends on both the power of the imperialist country and the power of its colonies. In this algorithm, this fact is modeled by defining the total power of an empire by the power of imperialist state plus a percentage of the mean power of its colonies.

In imperialistic competition, all empires try to take possession of colonies of other empires and control them. This competition gradually brings about a decrease in the power of weak empires and an increase in the power of more powerful ones.

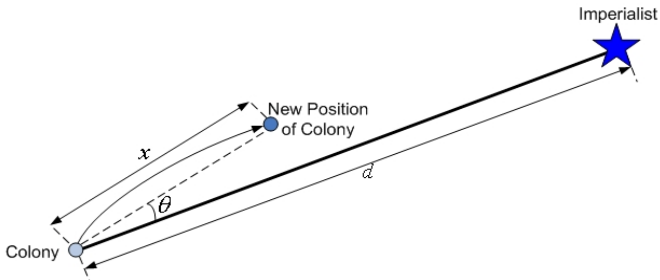

Fig. 3. Motion of colonies toward their relevant imperialist

This competition is modelled by picking some (usually one) of the weakest colonies of the weakest empires and making a competition among all empires to possess these (this) colonies. Fig. 3 shows a big picture of the modelled imperialistic competition. Based on their total power, in this competition, each of empires will have a likelihood of taking possession of the mentioned colonies. The more powerful an empire is, the more likely it will possess these colonies. In other words these colonies will not be certainly possessed by the most powerful empires, but these empires will be more likely to possess them. Any empire that is not able to succeed in imperialist competition and can not increase its power (or at least prevent decreasing its power) will be eliminated.

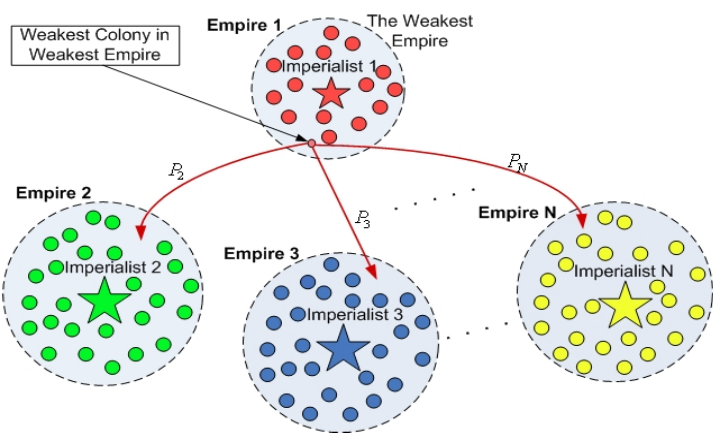

Fig. 4. Imperialistic competition: The more powerful an empire is, the more likely it will possess the weakest colony of the weakest empire

The imperialistic competition will gradually result in an increase in the power of great empires and a decrease in the power of weaker ones. Weak empires will gradually loose their power and ultimately they will collapse.

The movement of colonies toward their relevant imperialists along with competition among empires and also collapse mechanism will hopefully cause all the countries to converge to a state in which there exist just one empire in the world and all the other countries are its colonies. In this ideal new world colonies have the same position and power as the imperialist.

4 Applications and Simulation Results

In this section we focus on some static and dynamic non-cooperative games with nonlinear cost (payoff) functions and also multi-objective problems. Fig. 5 shows a big picture of the CCA applied to games in order to produce global NE points.

4.1 Example of a Simple Nonlinear Static Game

Consider a two player game with the nonlinear cost functions defined as:

$$f_1(x_1, x_2) = (x_1 - 1)^2 + (x_1 - x_2)^2 \tag{5}$$

$$f_2(x_1, x_2) = (x_2 - 3)^2 + (x_1 - x_2)^2 \tag{6}$$

The NE point using analytical method can be obtained as:

$$\begin{cases} \dfrac{\partial f_1(x_1, x_2)}{\partial x_1} = 0 \\ \dfrac{\partial f_2(x_1, x_2)}{\partial x_2} = 0 \end{cases} \Rightarrow \begin{cases} x_2 = 2x_1 - 1 \\ x_2 = \dfrac{x_1 + 3}{2} \end{cases} \Rightarrow \begin{cases} x_1^* = \dfrac{5}{3} = 1.6667 \\ x_2^* = \dfrac{7}{3} = 2.3333 \end{cases} \tag{7}$$

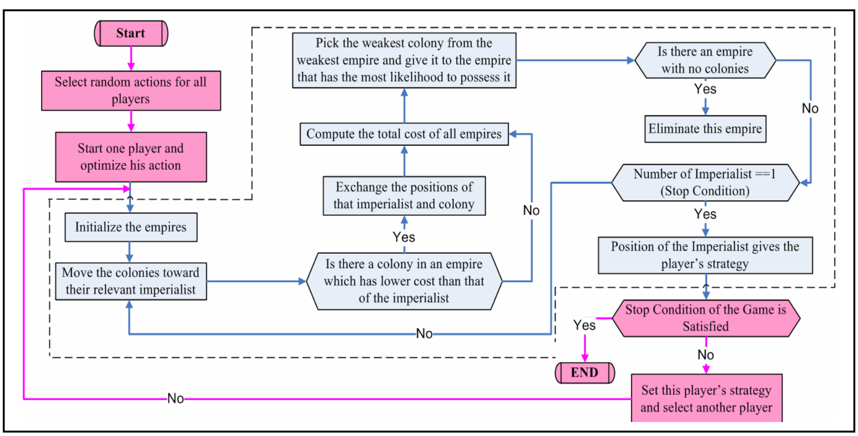

Fig. 5. The flowchart of CCA applied to games for global NE achievement (dashed area is the flowchart of CCA)

So the NE point of this simple game is $(x^*_1, x^*_2) = (1.6667, 2.3333)$ with the corresponding payoff values *(0.88889, 0.88889)*. We will use this value to verify the precision of the answers obtained using both GA and CCA. Fig. 5 and Fig. 6 show the cost function convergence for GA and CCA, respectively.

As it can be seen from Fig. 5 and Fig. 6, even in this simple game CCA has converged in less iterations (5 iterations) compared with GA (8 iterations). Our codes are so that if the payoff values are constant, with the tolerance of 0.0001, for 10 iterations the optimization process stops. In this simple case both the algorithms could achieve the optimal point as computed by analytical method.

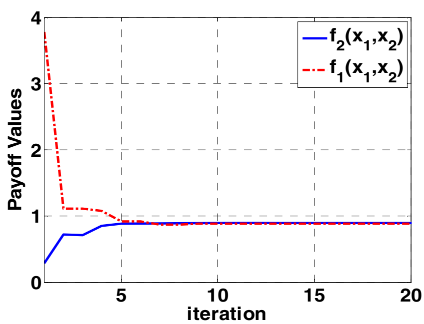

Fig. 6. Payoff convergence for GA

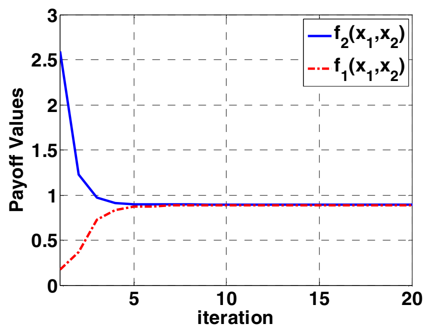

Fig. 7. Payoff convergence for CCA

In this simulation the *Population* size of GA was set to 20 together with the *Mutation* rate of 0.2. On the other hand, *Number of Countries* was set to 20 together with *Number of Imperialists* to 5 for CCA (meaning that 5 out of 20 countries will be chosen as *Imperialists*).

4.2 A Complicated Multi-objective Example

Deb [18] designed some multi-objective problems which their optimization was a challenge for GAs. The following example is one of his. Suppose that there are two criterion functions defined as follows:

$$f_1(x_1) = 4x_1 \tag{8}$$

$$f_2(x_1, x_2) = g(x_2).h(f_1(x_1), g(x_2)) \tag{9}$$

where:

$$g(x_2) = \begin{cases} 4 - 3\exp(-(\frac{x_2 - 0.2}{0.02})^2) & \text{if } 0 \le x_2 \le 0.4 \\ 4 - 2\exp(-(\frac{x_2 - 0.7}{0.2})^2) & \text{if } 0.4 \le x_2 \le 1 \end{cases} \tag{10}$$

and

$$h(f_1, g) = \begin{cases} 1-(\dfrac{f_1}{g})^{\alpha} & \text{if } f_1 \le g \\ 0 & \text{otherwise} \end{cases} \tag{11}$$
$$\alpha = 0.25 + 3.75(g(x_2)-1)$$

We model this optimization problem using game theory and then find its NE point. We suppose that f_1, f_2 are the payoff functions of two players. And also x_1, x_2 are the actions they can take. An investigation of these functions show that for this problem, there is one global Pareto set which is convex and one local Pareto set that is concave. Fig. 8 shows the scatter of (f_1, f_2) for 50000 randomly generated (x_1,x_2) pairs in interval [0,1].

Now we apply both GA and CCA to this multi-objective problem and obtain the results. But this time none of the mentioned algorithms could find the NE using previous settings. So we increased the *Population* size of GA and also the *Number of Countries* and the *Number of imperialists*. But, as seen in Fig. 9, even the increase of *Population* size to 200 and also the increase of its optimization iterations, did not help GA much to find the NE of this problem precisely. On the other hand, CCA could reach the point with *Number of Countries* equal to 35 and *Number of imperialists* to 5. CCA could achieve a NE point on global Pareto front *(0,1)* corresponding to $(x^*_1, x^*_2)=(0, 0.20002)$. Fig. 10 shows the payoff values convergence for CCA.

4.3 An Example of Nonlinear Static Game with Three Players Each with a Single Action to Select

In this example we will focus on a three player static game. There are three market players interconnected with three transmission lines. There are no constraints between transmission lines. The criteria are the profit functions which are going to be maximized. The criteria are defined as:

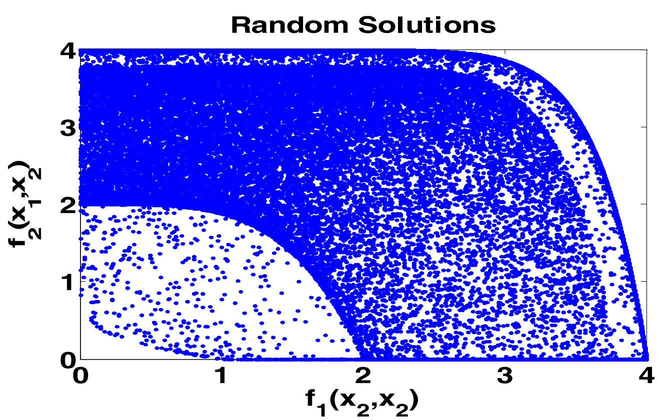

Fig. 8. Randomly generated solutions

$$\pi_i(\underline{x}) = (\theta - \rho(x_1 + x_2 + x_3))x_i - (0.5\phi_i x_i^2 + \gamma_i x_i + \eta_i) \quad i = 1,2,3 \tag{11}$$

This game is in Cournot model format. Connections between the generators are shown in Fig. 9. This system and the market data are quoted from [20]. The transmission lines are assumed to be lossless and have equal reactance [21]. Parameters of the mentioned payoff functions are summarized in table 1. Nash equilibrium is first obtained using analytical method.

$$\frac{\partial \pi_1}{\partial x_1} = 0 \ , \ \frac{\partial \pi_2}{\partial x_2} = 0 \ , \ \frac{\partial \pi_3}{\partial x_3} = 0 \quad \Rightarrow \quad \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 1107.58 \\ 1048.30 \\ 976.73 \end{bmatrix} \tag{12}$$

Now that we've computed NE point of this game we compare it with the results obtained from GA and CCA. Payoff values convergence of GA and CCA are shown in Fig. 12 and Fig. 13, respectively.

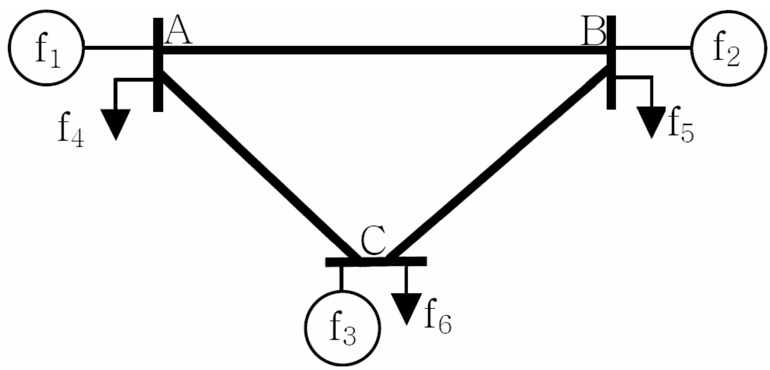

Fig. 9. Sample system for the Cournot model

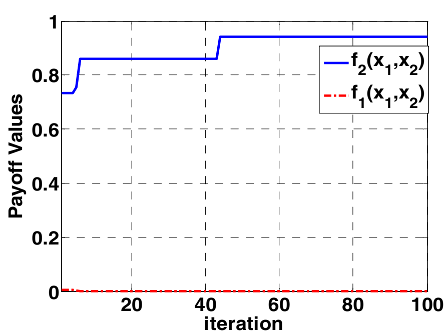

Fig. 10. Payoff convergence for GA

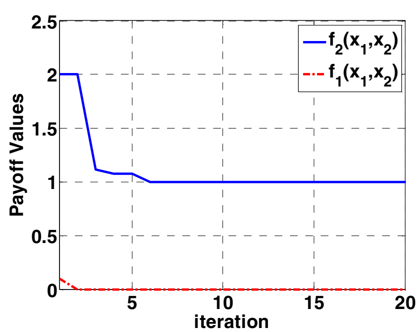

Fig. 11. Payoff convergence for CCA

Table 1. Parameters of the profit functions

	π_1	π_2	π_3
Φ_i	0.015718	0.021052	0.012956
γ_i	1.360575	-2.07807	8.105354
η_i	9490.366	11128.95	6821.482
θ	106.1176	106.1176	106.1176
ρ	0.0206	0.0206	0.0206

Again CCA wins in convergence rate and NE point preciseness, producing the set *(x*1,x*2,x*3)=(1107.5865, 1048.2999, 976.72098)*. This Nash point corresponds to the profit values of *(25421.6369, 23076.6077, 19010.9858)*. GA, in this case, converges to *(x*1,x*2,x*3)=(1104.9563, 1036.1275, 1016.143)* with cost values of *(25256.0385, 22286.6851, 21137.8527)*. As it is seen GA could not find the NE exactly.

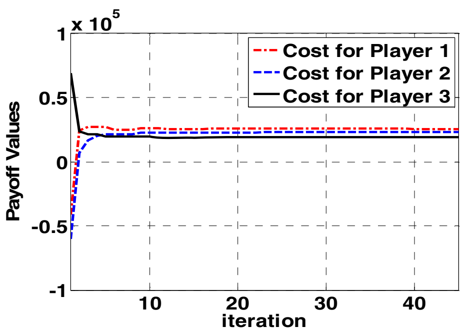

Fig. 12. Payoff convergence for GA

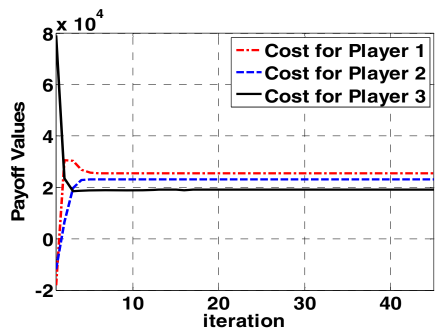

Fig. 13. Payoff convergence for CCA

4.4 An Example of Games with Three Players Each with an Action Set to Select

In this example we extend the players' strategy set so that a three player game each with 2 actions is obtained. Consider the cost functions of the players to be as follows:

$$f_A(x_1,x_2,y_1,y_2,z_1,z_2) = 21 + x\times\sin(5\pi x) + z\times y\times\sin(5\pi yz) \tag{13}$$

$$f_B(x_1,x_2,y_1,y_2,z_1,z_2) = 21 + y\times\sin(5\pi y) + z\times x\times\sin(5\pi xz) \tag{14}$$

$$f_C(x_1,x_2,y_1,y_2,z_1,z_2) = 21 + x\times\sin(5\pi z) + x\times y\times\sin(5\pi yx) \tag{15}$$

where $x = x_1+x_2$, $y = y_1+y_2$ and $z = z_1+z_2$ are the actions of the players P_1 to P_3. Fig. 14 and Fig. 15 depict the payoff function variations for all three players, using GA and CCA. As it can obviously seen from these figures, CCA has converged very faster in just 3 iterations where GA converges after 21 iterations. The payoff values and the NE points obtained by GA and CCA are summarized in table 2.

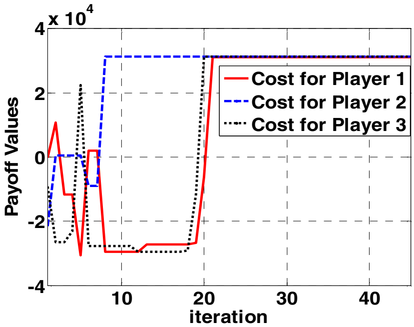

Fig. 14. Payoff convergence for GA

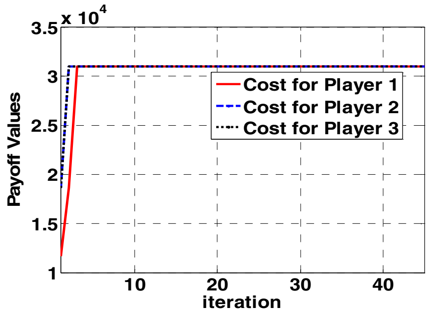

Fig. 15. Payoff convergence for CCA

It can be said that when the players' action sets increase GA losses its convergence speed.

Table 2. NE and payoff values obtained by GA and CCA

	NE obtained by CCA	NE obtained by GA
x^*_1	9.9748	9.9748
x^*_2	9.897	9.897
y^*_1	9.97481	9.9748
y^*_2	9.89703	9.897
z^*_1	9.9748	9.9748
z^*_2	9.897	9.897
Payoff	134.1276	134.4592

4.5 An Example of Static Games with Two Players Each with One Action

In this example a static non-cooperative game is studied. The profit functions of two players are defined as follows:

$$\pi_1 = 21 + x \times \sin(\pi x) + x \times y \times \sin(\pi y) \tag{16}$$

$$\pi_2 = 21 + y \times \sin(\pi y) + y \times x \times \sin(\pi x) \tag{17}$$

Fig. 16 shows these profit functions for different action values that can be chosen by each player. As it can be seen there are several local NE traps and just one global NE in this game.

Fig. 17 and Fig. 18 show the profit functions convergence rates and values for both GA and CCA. It can be seen that CCA converges to NE of *(8.5388, 8.5383)* with corresponding profit values of (x^*, y^*) *=(101.8558, 101.8426)* in just 2 iterations while GA has reached this NE in 9 iterations. The *population* size of GA was set to 30 while the *Number of Countries* and *Number of Imperialists* of CCA were set to 30 and 5, respectively.

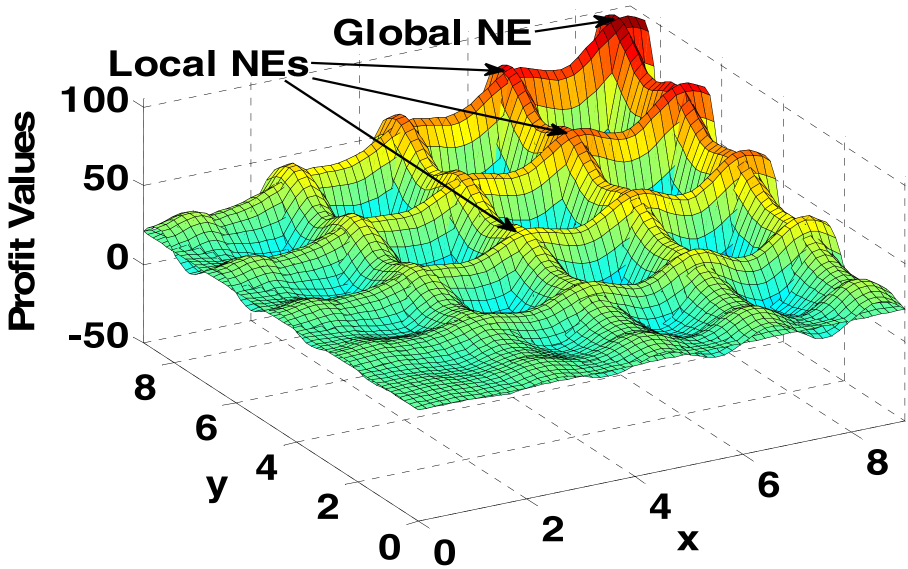

Fig. 16. Profit function values for different actions

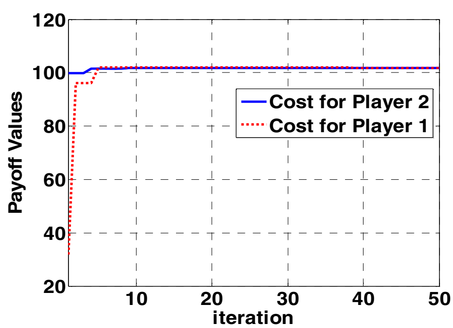

Fig. 17. Profit Values convergence for GA

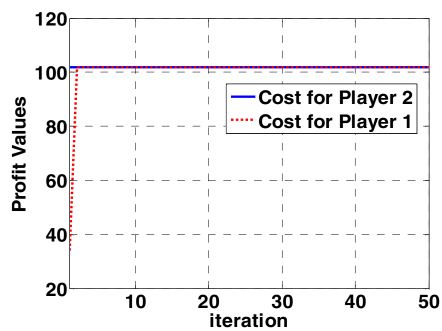

Fig. 18. Profit Values convergence for CCA

4.6 An Example of Games with Three Players Each with an Action Vector of Dimension Three

In this example a game with three players each with three actions is considered and studied. The payoff functions of these three players are as follows:

$$\begin{aligned} f_A = 21 \; + \; x_1 \times \sin(4 \times freq \times x_1) \; + \; 1.1 \times x_2 \times \sin(2 \times freq \times x_2) \; + \\ 1.2 \times x_3 \times \sin(2 \times freq \times x_3) \; + \; z \times y \times \sin(\pi \times y \times z) \end{aligned} \tag{18}$$

$$f_B = 21 + y_1 \times \sin(4\times freq \times y_1) + 1.1\times y_2 \times \sin(2\times freq \times y_2) + 1.2\times y_3 \times \sin(2\times freq \times y_3) + x\times z\times \sin(\pi \times x\times z) \quad (19)$$

$$f_C = 21 + z_1 \times \sin(4\times freq \times z_1) + 1.1\times z2 \times \sin(2\times freq \times z2) + 1.2\times z_3 \times \sin(2\times freq \times z_3) + x\times y\times \sin(\pi \times x\times y) \quad (20)$$

where $x=x_1+x_2+x_3$, $y=y_1+y_2+y_3$ $z=z_1+z_2+z_3$ *and* $freq = 5$. The *freq.* value provides us a tool to make the game more or less complicated. The more *freq.* value is, the more complicated the game will be. Payoffs convergence and precision are depicted in Fig. 19 and Fig. 20. Table 3 shows the achieved NE points together with payoff values obtained for three players. Due to the symmetry in game, looking at payoff functions, a symmetry is expected in payoff values and also in actions. Looking table 3 it can be said that CCA has this symmetry in produced NE point. But GA has not been able to converge to such a symmetry.

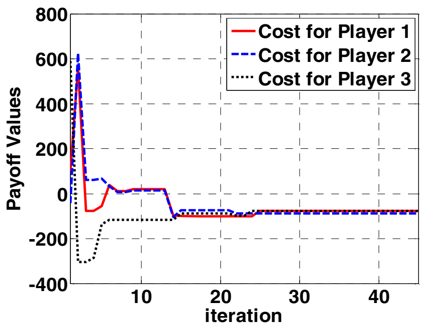

Fig. 19. Payoff Values convergence for GA

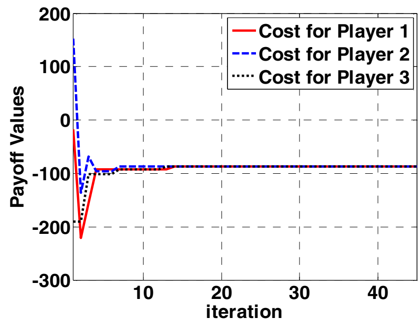

Fig. 20. Payoff Values convergence for CCA

As stated before when the players' action sets increase GA losses its convergence speed. This fact is again seen in Fig. 19 compared with Fig. 20.

Table 3. NE and payoff values obtained by GA and CCA

	NE obtained by CCA	NE obtained by GA
x^*_1	8.7182	8.7182
x^*_2	8.6405	8.6405
x^*_3	8.6405	8.6405
y^*_1	8.7182	8.7187
y^*_2	8.6405	8.6403
y^*_3	8.6405	8.6404
z^*_1	8.7182	8.7182
z^*_2	8.6405	8.6405
z^*_3	8.6405	8.6405
$Payoff_A$	-86.8510	-77.8117
$Payoff_B$	-86.7845	-87.0486
$Payoff_C$	-86.7785	-77.5606

5 Conclusion

In this paper CCA was applied to some multi-objective problems and some non-cooperative games. Better convergence rate and also better optima achievement of CCA, when used in different optimization problems, encouraged us to use it in order to find the NE of non-cooperative games with nonlinear cost functions. Usage of CCA helped to eliminate the drawbacks of GA, such as poor convergence rate and poor optima achievement, while not stopping in local Nash Equilibriums. This method also removed the need for modification of simple coevolutionary algorithms generating hybrid coevolutionary algorithms.

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