Applying a New Grid-Based Elitist-Reserving Strategy to EMO Archive Algorithms

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Abstract. Grid-based measure is an often-used strategy by some MOEAs to maintain the diversity of the solution sets. The well known ε-MOEA, based on the ε-dominance concept, is essentially based on grid-strategy too. Though often gaining an appropriate tradeoff between the aspects of the performance, the ε-MOEA has its inherent vice and behaves unacceptably sometimes. That is, when the PF_{true}'s slope to one dimension changes a lot along the coordinate, the algorithm loses many extreme or representative individuals, that has obvious influence on the diversity of the solution sets. In order to solve this problem, a new δ-dominance concept and the *suppositional optimum point* concept are defined. Then we proposed a new grid-based elitist-reserving strategy and applied it in an EMO archive algorithm (δ-MOEA). The experimental results illustrated δ-MOEA's good performance, which is much better especially for the diversity than NSGA-II and ε-MOEA.

Keywords: Grid, Archive set, ε-dominance, δ-dominance, *suppositional optimum point*, Grid-based Elitist-reserving Strategy, δ-MOEA.

1 Introduction

MOEAs(Multi-Objective Evolutionary Algorithms) have the ability to detect interesting solution candidates for multi-objective optimization problems[1][2], that enables the decision maker to filter efficient solutions and to discover trade-offs between opposing objectives among these solutions.

In practice, the decision maker wishes to evaluate only a limited number of Pareto-optimal solutions. This is due to the limited amount of time for examining the applicability of the solutions to be realized in practice. Hence, how to gain a solution set with good distribution is an important pursuing goal for the MOEA designers. Typically the satisfying solution set should include extreme individuals as well as the ones that are located in important parts of the solution space, where balanced trade-offs can be found.

More than several methods based on grid or hyper-volume measure are used by MOEAs as selection strategy to maintain diversity[3]. The well-known ε-MOEA[5]

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proposed by Deb et is essentially based on the grid measure too. When deciding whether a new generated individual to be reserved in the archive or not, ε-MOEA doesn't employ the general Pareto domination concept[6] but uses the ε- dominance concept instead. Because of the weaker domination relationship than the generally used one, ε-dominance may make the domination relationship between two individuals come into being, though they have no similar relation according to the Pareto domination concept. Furthermore, ε-MOEA just allows only one individual occupying each grid, hence the algorithm converges quickly and the archive set has good diversity. However, the ε-dominance has its inherent vice that when the true PF of the problem has quite discrepant value of slope in different portion of one dimension, some extreme or important representative individuals are lost. Though one can relieve this losing-phenomenon by adjusting the value of ε, the region used to having moderate number of solutions may contain too many individuals, so it can't solve this matter fundamentally.

In order to solve this problem, we defined a new δ-dominance concept, which kept the merit of ε-dominance down but avoided the important individual losing-phenomenon. Then we applied it in the elitist-reserving strategy to update the archive set, which made the new algorithm (δ-MOEA) gain solution sets with better diversity than that by other ones(ε-MOEA and NSGA-II).

The rest of this paper is organized as follows. Section 2 briefly introduces some relating definitions of multi-objective optimization problem. Section 3 explicates the ε-domination and ε-MOEA. Section 4 presents the proposed δ-dominance concept, the new elitist-reserving strategy and δ-MOEA. Section 5 shows experiment results and discussions. Finally, Section 6 concludes with a summary of the paper.

2 Basic Concepts

Definition 1 ***(Multi-objective Optimization Problem(MOP)).*** *A general MOP is described as following:*

$$\text{Min } \vec{f}(X) = (f_1(X), f_2(X), \cdots, f_r(X)) \tag{1}$$

$$g_i(X) \geq 0; (i = 1, 2, ..., k) \tag{2}$$

$$h_i(X) = 0; (i = 1, 2, ..., l) \tag{3}$$

where $\vec{f}(X)$ *is the objective vector,* ***r*** *is the dimension of objectives, (2) and (3) are equality-constraints and inequality-constraints,* $X = (x_1, x_2, ..., x_n)$ *is variable vector,* ***n*** *is the dimension of variables,* $X \in \Omega$, $\Omega \subseteq R^n$ *,where* Ω *is the feasible space, then,* $\vec{f} : \Omega \to \Pi$, $\Pi \subseteq R^r$, Π *is the objective space.*

Definition 2 (*Pareto dominance*). *A solution* $\boldsymbol{x^0}$ *is said to dominate (Pareto optimal) another solution* $\boldsymbol{x^1}$ *(denoted* $\boldsymbol{x^0} \succ \boldsymbol{x^1}$*) if and only if:*
$\forall i \in \{1, \ldots, m\} : f_i(x^0) \leq f_i(x^1) \bigcap (\exists k \in \{1, \ldots, m\} : f_k(x^0) < f_k(x^1))$.

Definition 3 (*Pareto optimal*). *A solution* $\boldsymbol{x^0}$ *is said to be non-dominated (Pareto optimal) if and only if:* $\nexists x^1 \in X : x^1 \succ x^0$.

Definition 4 (*Pareto optimal set*). *The set P_S of all Pareto optimal solutions:* $P_S = \{x^0 \mid \overline{\exists} x^1 \in X : x^1 \succ x^0\}$.

Definition 5 (*Pareto optimal front*). *The set PF of all objective function values corresponding to the solutions in PS:* $P_F = \{f(x) = (f_1(x), \ldots, f_m(x)) \mid x \in P_S\}$.

The optimal result for such multi-objective optimization is no other than the Pareto optimal set PS. However, the size of this set may be infinite, and it is impossible to find this set by using a finite number of solutions. In this case, a representative subset of PS is desired. Generally, the characteristic of MOEAs is to search the decision space by maintaining a finite population of individuals (corresponding to the points in the decision space), which work according to the procedures that resemble the principles of natural selection and evolution. Because we only consider the subset of all the final non-dominated individuals resulted from a MOEA, we call such subset an approximation set and denote it by S, and we call the corresponding objective set a resulting final Pareto optimal front and denote it by PF_{final}. Ideally, we are interested in finding an S of finite size, which contains a selection of individuals from such that the individuals in PF_{final} are diversified as possible. Unfortunately, we usually have no access to PF on beforehand. However, it is common practice to search for a good diversity of the individuals in the objective space because decision makers will ultimately have to pick a single individual as final solution according to its objective vector values. Therefore, it is often best to present a wide variety of tradeoff individuals for the specified goals in constructing MOEAs.

3 Grid-Measure and ε-MOEA

3.1 Generality of Grid-Measure

Generally, the grid-measure divides the objective space into a lot of small grids or hyper-cubes and the size of them usually depends on the value of the objective function. If two individuals are located in the same grid, the difference on each dimension is tolerant and neglectable for the problem.

3.2 The ε-Dominance Concept

Definition 6[4] (*ε-dominance*). *Let f, $g \in \mathbb{R}^{+m}$. Then f is said to ε-dominate g for some ε>0, denoted as $f \succ_\varepsilon g$, if and only if for all $i \in \{1, \ldots, m\}$ (maximizing):*

$$(1+\varepsilon) \cdot f_i \geq g_i \tag{4}$$

When the problem is a *Minimizing*-formulated one, the above inequality formulation (4) should be simply modified.

In the divided objective space, each grid should be endued to ascertain which location of the space that an individual is distributed. And for each candidate solution in the archive set, an identification vector ***B*** (***B***=(B_1, B_2,…B_m)$^{\mathbf{T}}$) is assigned to identify its location and its ε-dominating area. The identification vector is assigned according to the following formulation:

$$B_j(f)=\begin{cases}\left\lfloor \left(f_j - f_j^{\min}\right)/\varepsilon_j \right\rfloor, & f_j \text{ is to be Minimized };\\ \left\lceil \left(f_j - f_j^{\min}\right)/\varepsilon_j \right\rceil, & f_j \text{ is to be Maximized }.\end{cases} \tag{5}$$

Where j is the dimension number, $f_j^{\min}$ is the minimum possible value (default as 0) of the j-th dimension, ε_j is the size of the grid on the j-th dimension. The ε-dominated area of an individual is actually the Pareto dominated area of its identification vector; If two are in the same grid (having the same ***B*** vector), the one has smaller distance to the ***B*** vector is preserved and the other one is deleted. For more detailed description, one can refer to [5].

3.3 ε-MOEA and Its Shortage

The ε-MOEA sets an archive population E(t) and an evolutionary population P(t). In each iteration, an individual by tournament selection from P(t) and another one randomly selected from E(t) are matched. Then they crossover and mutation is processed and finally two new individuals are generated. For each one of them, ε-MOEA uses the general Pareto domination concept to update P(t) while uses the ε-dominance to update E(t). Hence the competitive models are remained in P(t), and the solutions in E(t) are well distributed and the number of them is not too large. Figure 1 shows the results of ZDT1 obtained by ε-MOEA.

As can be seen in the figure, the obtained solution set is nearly well distributed on the PF_{true} (the solid points are scattered in the bolded grids).

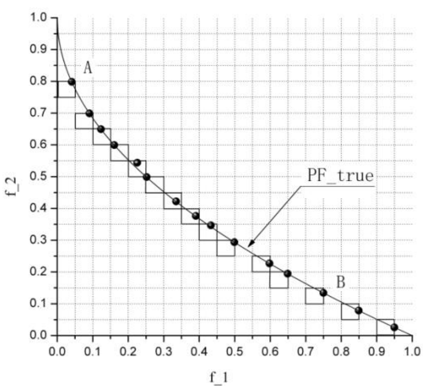


Fig. 1. Results of ZDT1 obtained by ε-MOEA (ε_i=0.05)

However, several grids transited by the true PF contain no solutions, because these grids (for example, the grids above A and the ones besides B in figure 1.) are ε-dominated by the preserved individuals. Obviously, these grid-regions are either extreme or important respective ones, but the algorithm failed to find solutions in these regions. So the diversity of the solution set is not satisfying.

In order to avoid this phenomenon, we proposed a new δ-domination concept and new elitist-reserving strategy. The new strategy based δ-MOEA is also illuminated.

4 New Elitist-Reserving Strategy and δ-MOEA

4.1 The δ-Dominance Concept

As stated above, the ε-dominance uses the identify vector to confirm individual's location, and it allows only one solution preserved in each feasible grid. But, each of the reserved solution has too large dominating ability, which makes some of the extreme and representative solutions lost, and the diversity is dissatisfying. So we improved the dominance concept and allow some individual's (satisfying certain conditions) to be preserved as well as those reserved according to the ε-dominance. It is a more particular concept than the ε-dominance. We call the new dominance concept δ-dominance.

Definition 7 (*δ-dominance*). *Let f, $g \in \mathbb{R}^{+m}$. Then f is said to δ-dominate g for some δ>0, denoted as $f \succ_\delta g$, $\exists \Delta_i$, with $0 \le \Delta_i \le \delta_i$, if and only if for all $i \in \{1,\ldots,m\}$ (minimizing):*

$$(f_i - \Delta_i) \le g_i \tag{6}$$

Where δ_i has similar effect as the ε did in the ε-dominance, it sets the upper extent of possible dominating region; while Δ_i helps to confirm the exact δ-dominating area of the preponderant individual. Specially, when $\Delta_i = 0$ *for each i*, the new concept degenerates to the Pareto domination concept, while when $\Delta_i = \delta_i$ *for each i*, it actually equals the ε-dominance. Figure 2 helps to comprehend this relationship.

4.2 The δ-Dominance Based Elitist-Reserving Strategy

For its inclusion in the archive, an individual is compared with each member in the archive for δ-dominance. Every individual in the archive is assigned an identification vector $(\boldsymbol{B}=(B_1, B_2, \ldots B_m)^{\mathbf{T}}$ too, similar as Formulation (5) stated (with the denominator replaced by δ_i). Figure 2 illustrates that the individual P δ-dominates the entire shadowed region, which is large than that of the Pareto dominance definition but not so large than that of ε-dominance. We only discuss the minimization cases alone for brevity, while similar analysis can be followed for maximization or mixed cases as well.

For the individual P, its identification vector is the coordinates of the point B_p in the objective space and its δ-dominating region can be distinctly partitioned into 2 parts: the partition in the grid and the other out of the grid. If two individuals are in the same grid (having the same identification vector), we check which one is closer to the identification vector (in terms of the Euclidean distance), then delete the farer one (for example, individual 2 is to be deleted and 1 is to be preserved). So the in-grid partition is a rectangle with its left-bottom sector removed. And if the comparing individuals have different B vectors (for example, P and Q1 or and Q2), we check whether the

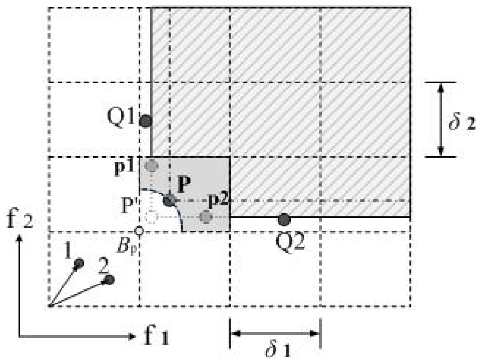


Fig. 2. The δ-dominance concept is illustrated (for minimizing f_1 and f_2)

one with larger B vector is Pareto-dominated by the *suppositional optimum point* of the other's, if so, delete the dominated one, otherwise, both the two are saved in the archive.

The *suppositional optimum point* for P (denoted as P′) is set and updated as:

$$f_i(P'_{t+1}) = \min\{ f_i(P'_t), f_i(P) \} \tag{7}$$

Where *t* denotes the *t*-th updating iteration, *i* is the *i*-th dimension. It should be noticed that each dimension of P′ is actually the *ever-lowest* objective function value (of some individual that has been appeared in the grid). While the *ever-lowest* value for different dimensions can hardly belong to a single individual. Hence, the compositive P′ probably doesn't correspond to any real individual point in the decision space, so we call it ***suppositional*** *optimum point*. The *suppositional optimum point* is related to the real individual and the grid where it stayed, so it should be updated once there is a new individual that entered the archive or replaced an old one in it.

The use of the *suppositional optimum point* insures that the individuals, who should be deleted according to their relationship with the having been deleted former ones, won't be involved in the archive. So the archive set evolves without degradation.

The following procedure explains the elitist-reserving strategy in detail.

Procedure: /* Whether_individual A_enters_the_Archive E(t) */

```
Begin
  If: E(t) is empty
  then A enters E(t), F(A')=F(A);
          // A' is  A's suppositional optimum point, F(A) is A's function value
  Else:
  For each P in E(t)
  {  check whether A and P are in the same grid;
     If (TURE) then whether |BpA|-|BpP|<0 ?
          // compare A and P: which one is closer to the B vector
        Yes: A enters E(t),update A'(P'),discard P,
               end procedure;
```

```
        No: discard A,  end procedure;
     Else: whether A is dominated by P'(δ-dominated by P)
                or |AP|<0.5 |δ|; // or they are too close
       Yes: discard A,  end procedure;
        No: continue;
  }  end For
  A enters E(t), F(A')=F(A);
End.
```

4.3 The Framework of δ-MOEA

The δ-MOEA also sets an evolution population and an archive set that was initialized empty. The crossover operator is SBX and the mutation operator is polynomial mutation. Importantly, we use the new strategy stated above in 4.2 to update the archive set. Figure 3 shows the framework of δ-MOEA.

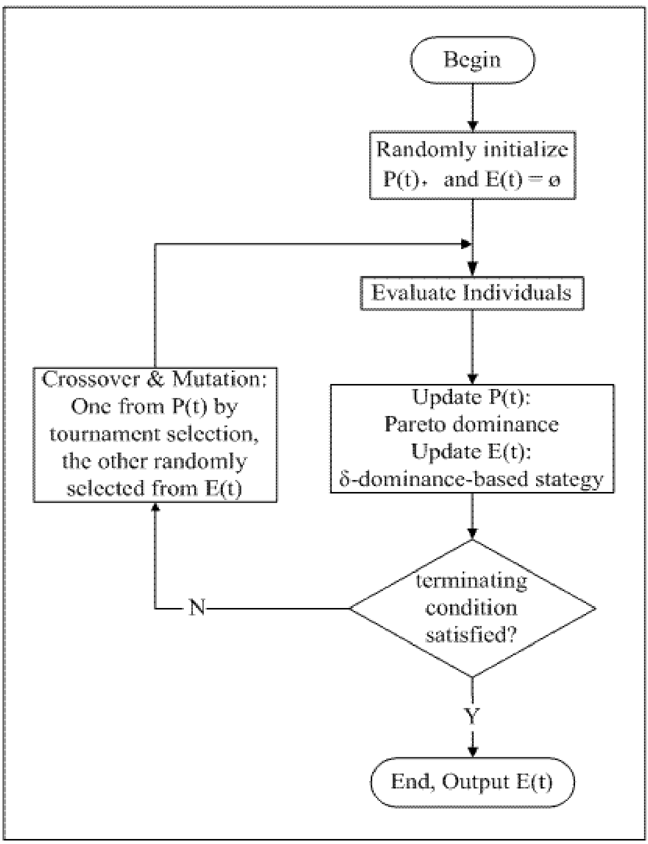


Fig. 3. The flow chart of δ-MOEA

5 Numerical Experiments and Discussion

5.1 Test Functions and Parameter Settings

In order to test the performance of δ-MOEA, we choose some representative and generally used benchmarks. They are: SCH[10], POL[11], FON[12], ZDT2[13][15], DTLZ1[14], DTLZ2[14].The number of decision variables *N*: $N_{SCH}=1$, $N_{POL}=N_{FON}=2$, $N_{ZDT2}=30$, $N_{DTLZ1}=N_{DTLZ2}=7$. SCH, POL, FON, ZDT2 have 2 objectives and DTLZ1, DTLZ2 have 3 ones. About the characteristic of these benchmarks, corresponding literatures can be referenced to.

We use the well know NSGA-II[7] and ε-MOEA as comparing algorithms. The parameter settings for δ-MOEA and them are as follows: function evaluations and population size are stated in Table 1. Other parameters are set as that suggested in [7] and [5]. Withη_c=15 for SBX,η_m=20 for polynomial mutation. The size of final solution set is set to equal the population size.

Table 1. Parameter Setting

Objections	2	3
Population size	100	200
Evaluation	20000 (200 gen)	80000 (400 gen)

5.2 Performance Metrics

The metrics to evaluate the performance of the MOEAs were the Spacing Metric(SP) by Schott[8] and the GD by Veldhuizen[9] respectively. The former was to measure the extent of spread achieved among the obtained solutions, and the latter measured the extent of convergence of known set of Pareto-optimal set. For detail, literature [8] and [9] are suggested to be referred to.

5.3 Results and Discussion

For comparison, the SP and GD of the obtained solutions on all or some of the benchmarks by NSGA-II, ε-MOEA and δ-MOEA are shown in Tables 2 and 3.

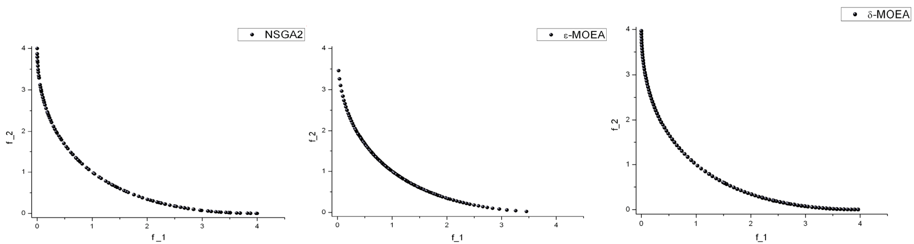
Table 2. Spacing Metric(SP) in 10 runs for NSGA2, ε-MOEA and δ-MOEA

MOEAs	NSGA 2		ε-MOEA		δ-MOEA	
	Sparsity (Avg)	**Std Dev**	**Sparsity (Avg)**	**Std Dev**	**Sparsity (Avg)**	**Std Dev**
SCH	0.03638377	0.00356140	0.04038556	0.00008814	**0.03182887**	0.00012812
POL	0.10597446	0.01097994	0.13732486	0.00950219	**0.03747305**	0.00388055
FON	0.00870666	0.00066549	0.02332467	0.00089394	**0.00434514**	0.00092039
ZDT2	0.00679208	0.00068899	0.00858766	0.00050171	**0.00618374**	0.00052975
DTLZ1	0.04069481	0.00209326	0.01286952	0.00160352	**0.01240354**	0.00061743
DTLZ2	0.09117502	0.02048226	0.03132431	0.00142884	**0.02504852**	0.00106497

Table 3. Convergence Metric(GD) in 10 runs for NSGA2, ε-MOEA and δ-MOEA

MOEAs	NSGA 2		ε-MOEA		δ-MOEA	
	GD (Avg)	**Std Dev**	**GD (Avg)**	**Std Dev**	**GD (Avg)**	**Std Dev**
SCH	0.00041685	0.00002002	**0.00030888**	0.00000282	0.00039228	0.00000651
ZDT2	**0.00014309**	0.00002035	0.00025649	0.00003717	0.00029782	0.00004173
DTLZ1	**0.00004080**	0.00001494	0.00007769	0.00001098	0.00008530	0.00018663
DTLZ2	**0.00028334**	0.00004796	0.00396315	0.00077572	0.00309575	0.00008352

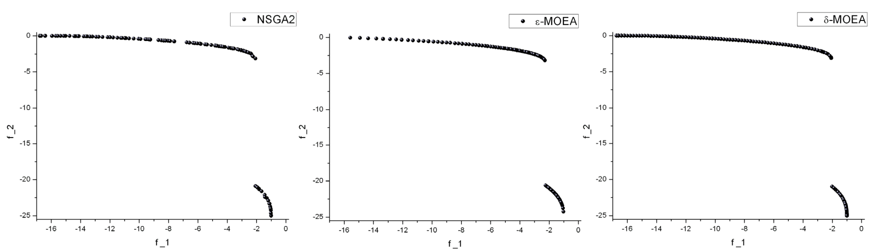
From Table 2, we know that for all problems, δ-MOEA has the best performance in maintaining diversity of the solutions. ε-MOEA is worse than NSGA-II when the objectives are 2 but better than it when the objectives are 3. Table 3 told that NSGA-II gain the best GD of the three on ZDT2, DTLZ1 and DTLZ2, while on SCH, ε-MOEA won the others; Though δ-MOEA is a little weaker than the other two, it is also competitive with them. In the following, we'll give a more intuitionistic illumination by a set of figures (Figure 4-9).



(a) NSGA2 (b) ε-MOEA (c) δ-MOEA

Fig. 4. Distribution of obtained solutions for NSGA2, ε-MOEA and δ-MOEA on SCH

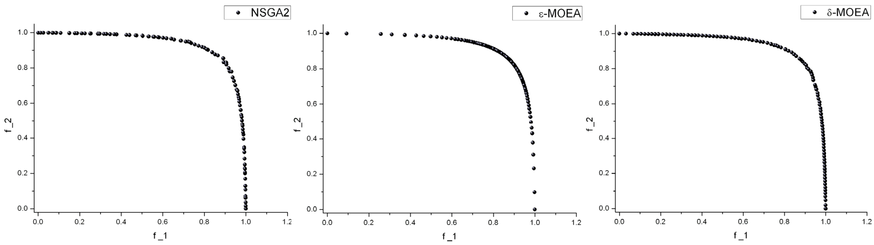
Figure 4 shows that the distribution of obtained solutions by NSGA-II seems discrete and that by ε-MOEA becomes sparser as going to the extremal regions of PF_{true}. Obviously δ-MOEA gained continuous and uniform solutions distributed on the PF_{true}.



(a) NSGA2 (b) ε-MOEA (c) δ-MOEA

Fig. 5. Distribution of obtained solutions for NSGA2, ε-MOEA and δ-MOEA on POL

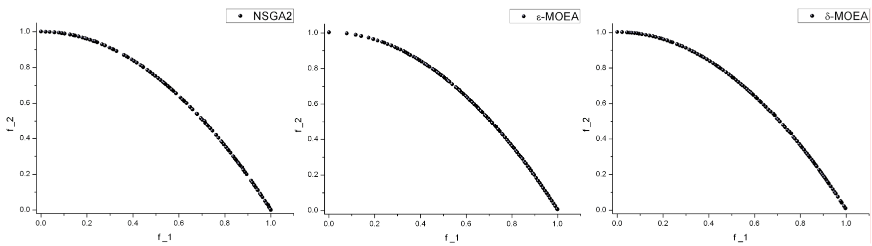
In Figure 5, we can see that on POL, NSGA-II got PF that is not sleek; PF obtained by ε-MOEA is dense in the parted segment but too sparse in the area of PF_{true} closing to f_2. While δ-MOEA gains a well distributed, uniform solution set.



(a) NSGA2 (b) ε-MOEA (c) δ-MOEA

Fig. 6. Distribution of obtained solutions for NSGA2, ε-MOEA and δ-MOEA on FON

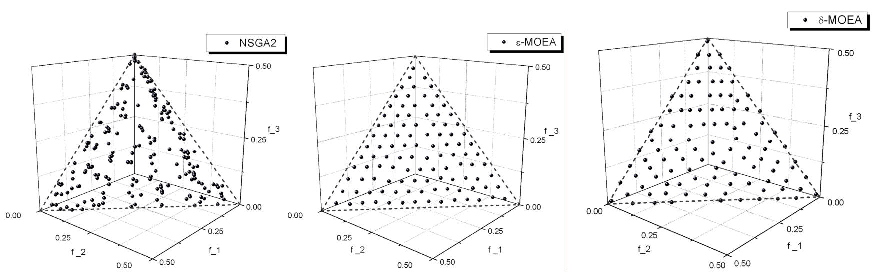
Figure 6 illuminates the shortage of ε-MOEA clearly. It gets too many solutions crowded in the middle part but quite fewer ones in the extremal part. And NSGA-II has dissatisfying results while δ-MOEA is still the best.



(a) NSGA2 (b) ε-MOEA (c) δ-MOEA

Fig. 7. Distribution of obtained solutions for NSGA2, ε-MOEA and δ-MOEA on ZDT2

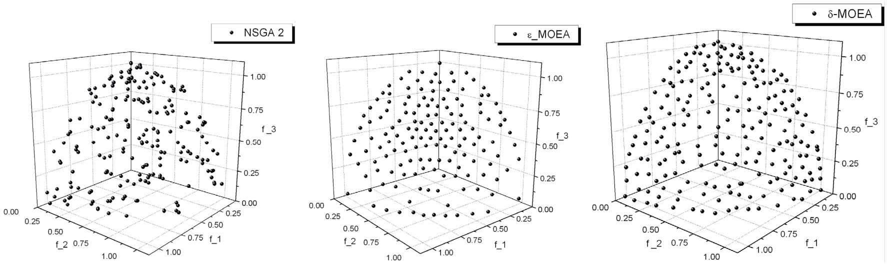
Figure 7 shows the similar situation of results. The PF obtained by NSGA-II is not so good and that by ε-MOEA is obviously sparser when the value of f2 approaching 1. The distribution of solutions obtained by δ-MOEA is acceptable.



(a) NSGA2 (b) ε-MOEA (c) δ-MOEA

Fig. 8. Distribution of obtained solutions for NSGA2, ε-MOEA and δ-MOEA on DTLZ1

For the benchmarks with 3 objectives, DTLZ1 and DTLZ2 are selected. In Figure 8, (a) shows that NSGA-II found solutions asymmetrically distributed; (b) shows the set by ε-MOEA is symmetrical but with no solutions reserved in extremal area;



(a) NSGA2 (b) ε-MOEA (c) δ-MOEA

Fig. 9. Distribution of obtained solutions for NSGA2, ε-MOEA and δ-MOEA on DTLZ2

(c) tells the case that the set by δ-MOEA is satisfying uniformly distributed as well as the extreme solutions were found.

In Figure 9, (a) shows the disorder solution set obtained by NSGA-II; (b) indicates that ε-MOEA still fail to find symmetrical solutions in extremal area; (c) implies that δ-MOEA gained the trade-off between extensiveness and uniformity.

In conclusion, δ-MOEA gained solution sets with better diversity than the other two did. It is because that the new elitist-reserving strategy preserved individuals in the ε-dominated grids especially in those near the extremal region of the PF. Since the individuals that may be eliminated by ε-MOEA are kept down in the grids that are went through by the PF, so the whole solution set is with higher uniformity than that by the ε-MOEA.

6 Conclusions

The proposed δ-dominance concept and new elitist-reserving strategy help δ-MOEA gain a solution set that has good distribution. The δ-MOEA overcomes the difficulties of ε-MOEA in finding and preserving solutions in extremal and some other important regions, it also outperforms the typical NSGA-II as the diversity is concerned. The application of *suppositional optimum point* is also novel and the result of numerical experiments illustrates the good performance of δ-MOEA.

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