Multi-bidding Strategy in Sponsored Keyword Auction*

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Abstract. The generalized second price auction has recently become a much studied model for sponsored keyword auctions for Internet advertisement. Though it is known not to be incentive compatible, properties of its pure Nash equilibria have been well characterized under the single bidding strategy of each bidder.

In this paper, we study the properties of pure Nash equilibria of the generalized second price auction when each bidder is allowed to submit more than one bid. This multi-bidding strategy is noted to have been adopted by companies for keyword advertisements on search engines. In consideration of the pure Nash equilibria, we completely characterize conditions on the number of selling slots for a pure Nash equilibrium to exist, assuming all the advertisers are allowed to use multi-bidding strategies or only one advertiser will use a multi-bidding strategy.

Our findings reveal interesting properties of limitation and potentials of the market place of online advertisement.

1 Introduction

Sponsored keyword auction is a brand new type of market models adopted by major search engine companies such as Google and Yahoo. It has become a principal source of revenue for those companies. As its name indicates, such kind of auctions mostly sells the advertising positions for web links displayed along with the search results when a user places a related keyword or a few related keywords to find information on those search engines.

Different advertising positions have different *click-through-rates*, the ratio of the number of clicks on the advertising to the number of appearances of the

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advertising web links. Some advertising position draws more attentions from users and generates more clicks than others. For this reason, it is named the *position auction* by Varian [1], which is equivalent to the *generalized second price auction* (GSP for short), a term used by Edelman, Ostrovsky and Schwarz [2]. It is the primary protocol for sponsored link auctions to sell the advertising positions.

Under GSP, each advertiser bid for a price per each click and each winning advertiser is allocated exclusively for a position to place its web-link. The positions are sorted in their click-through-rates (commonly assumed to be the same). For K positions to sell, the K highest bidders win them in the corresponding decreasing order of their bidding prices. A winner pays a price per click, which is equal to the bidding per-click-price of the next highest bidder, i.e., the highest bidding price that is lower than its own bidding price.

If there is only one advertising position to sell, GSP is equal to the popular second price auction/Vickrey auction [3] which is a special case of the more general VCG [3,4,5] mechanisms. Therefore, no bidder can gain any advantage by not bidding its own private value for each click. Any protocol with the property that every agent has a dominant optimal strategy to reveal its own private value is called a *truthful* one. It is also often called an *incentive compatible* protocol.

The GSP allocation method seems simple, intuitive and, arguably, fair. However, if there are more than one position to sell, it will no longer be incentive compatible [2,6]. This observation has inspired further studies of GSP, mostly its pure Nash equilibria in a single bidding strategy.

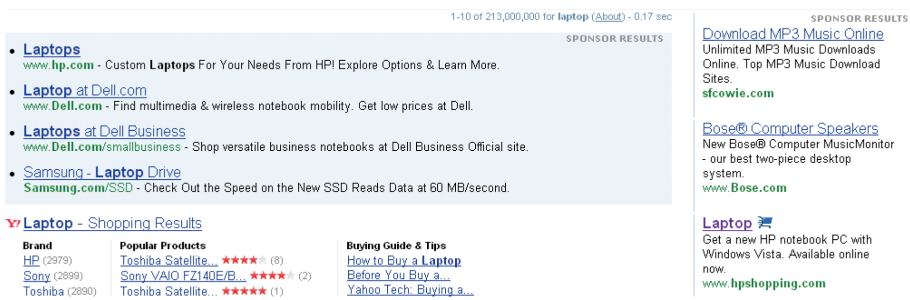

Fig. 1. The sponsor results when keyword “laptop” is typed into the search engine of Yahoo. Both “Dell” and “Hp” adopt the multi-bidding strategy.

In the practice of sponsored keyword auctions, however, bidders may submit multiple bids. For example, when the keyword “laptop” is typed into Yahoo, the sponsor results shown along with the search results page will be similar to the sponsor results shown in Figure 1. In Figure 1, Dell gets two adjacent advertising positions by multiply bids. Although one of the link points to Dell’s homepage and the other link points to Dell’s sub-homepage, it opens up a possibility for Dell to manipulate these bids to decrease the company’s total advertising costs. Similarly, HP also gets two advertising positions in the figure. So this case shows that i) multi-bidding strategy exists in current online advertisement market, ii) usually only the big companies have the competence and the need for the multi-bidding strategy.

We are particularly interested in the GSP auctions with multi-bidding strategy. We aim at studying the pure Nash equilibrium behavior of the bidders in such market conditions.

1.1 Related Work

The considered model for sponsored search auctions were formalized in [2,1]. In [2], Edelman, Ostrovsky and Schwarz name it *generalized second price auction*, while Varian names it *position auction* in [1]. They all discovered that the auction model is not incentive compatible. Then [2,1,7,8] refined the concept of Nash equilibrium of the position auction and study the related properties. Regarding the auction as a static one-shot complete information game, in [2], Edelman, Ostrovsky and Schwarz introduced the concept of *locally envy-free equilibrium*. In [1], Varian proposed *symmetric Nash equilibrium* due to mathematic considerations. In [7], Zhou and Lukose argued for a certain type of *pure strategy Nash equilibrium*, as a result of some *anti-social behavior*, called *vindictive bidding strategy*.

In [8], Bu, Deng and Qi presented the concept of *forward looking Nash equilibrium* as a result of the auction's dynamics and the bidders' strategic manipulations, based on an important property called the forward looking attribute. Furthermore, in [9] they analyzed the convergence of this dynamic system. Coincidentally, convergence is also studied based on the same bidding strategy by Cary, Das, Edelman, Giotis, Heimerl, Karlin, Mathieu and Schwarz in [10] where it is called the *greedy bidding strategy*.

The concept of *false-name bid* (multiple bids by the same agent under different false names) was firstly introduced by [11] in 1999. In [11], Sakurai, Yokoo and Matsubara observed that *generalized Vickrey auction* mechanism, the generalized version of Vickrey auction, is not robust enough against false-name bid behavior in combinatorial auctions. Then they showed that the concavity of a surplus function over bidders is the sufficient condition where the VCG mechanism is false-name-proof in [12].

Note that the multiple biddings we discuss here are related but not exactly the same as the false name biddings. The agents are submitting their bids under their own true identities where false name biddings are the bids under assumed different identities by the same agent.

1.2 Our Contributions

We study the next technical issue when multiple biddings are allowed. Note that this is different from false name bids in that multiple bidding bidders reveal their true identities but false name bidders do not.

We completely characterize conditions on the number of selling slots for a pure Nash equilibrium to exist, if the advertisers are allowed to use multiple-bidding strategies. We find that there always exists a pure Nash equilibrium when the maximum allowed number of submitted bids is not less than the number of slots. Otherwise, a pure Nash equilibrium need not exist. Even there is only one advertiser using the multi-bidding strategy, the property of non-existence of pure Nash equilibria still be discovered when the number of multiple bids is greater than 2.

As commented above, when the number of positions to bid for is one, the GSP auction is the same as Vickrey auction which is known to be incentive compatible. When the number of positions is at least two, the GSP auction is no longer incentive compatible even if everyone is allowed for at most one bid. Furthermore, we prove that no other auction protocols selling two or more slots can have the properties of incentive compatibility, social efficiency and individual rationality if multiple biddings are allowed.

Furthermore, we study the most general case when a single bidder may have several advertisements to bid with different private values for each of them. We develop a complete characterization of the existence conditions of equilibrium in this model.

Therefore, in general, we have the existence of Nash equilibrium and the impossibility result in the multiple-bidding market.

The paper is organized as follows. In Section 2 we present the standard GSP model and the extended version with multi-bidding strategy. Section 3 completely characterize conditions on the number of selling slots for a pure Nash equilibrium to exist, if the advertisers are allowed to use multi-bidding strategies. Section 4 discusses the existence of pure Nash equilibria when only one bidder is allowed to use multi-bidding strategy. We present the impossible result in section 5. In Section 6, we discuss the issue of biddings of agents each with multiple advertisement needs of different values.

2 Model and Notation

We follow the GSP auction model presented in [2,1]. For some keyword, there are $\mathcal{N} = \{1, 2, \ldots, N\}$ advertisers who bid $\mathcal{K} = \{1, 2, \ldots, K\}$ advertisement slots ($K < N$). If the indexes of slots satisfy $k_1 < k_2$, then slot k_1's expected *click-through-rate* (CTR for short) c_{k_1} is larger than c_{k_2}. Namely, $c_1 > c_2 > \cdots > c_K > 0$. Moreover, each bidder $i \in \mathcal{N}$ has a privately known information, $v^{(i)}$, which represents the maximum price he is willing to pay for per-click of his advertisement.

According to each bidder i's submitted bid $b^{(i)} \geq 0$, the auctioneer decides how to distribute the advertisement slots among the bidders and how much they should pay for per-click. In particular, the auctioneer firstly sorts the bidders in decreasing order according to their submitted bids. Then the slot with smaller index will be allocated to the bidder with higher bidding value. The last $N - K$ bidders would lose and get nothing. Finally, each winner would be charged for per-click the next bid to his in the descending bid queue. The losers would pay nothing. In the case of ties, we assume that the auctioneer would break ties according to a prior notice he declares. For example, ties could be broken randomly. Another method of breaking ties is to allocate the higher slot to the bidder with a prior time stamp.

Let b_k denote k^{th} highest bid in the descending bid queue and v_k the true value of the k^{th} bidder in the descending queue. So if bidder i got slot k, i's payment would be $b_{k+1} \cdot c_k$. Otherwise, his payment would be zero. Hence, for any bidder $i \in \mathcal{N}$, if i were on slot $k \in \mathcal{K}$, his utility (payoff) could be represented as

$$u_k^i = (v^{(i)} - b_{k+1}) \cdot c_k \quad .$$

2.1 Multi-bidding Model

We consider the extended GSP model associated with the multiple bidding strategy. In other words, each bidder is allowed to submit several bids instead of only one bid. We refer to the extended GSP model as M-GSP if each bidder is allowed to submit at most M bids.

In M-GSP, every bidder i submits at most M non-negative bidding prices to the auctioneer, despite having a unique $v^{(i)}$. We denote bidder i's j^{th} bidding price by $b^{(i,j)}$. Without loss of generality, if the number of submitted bids of i is less than M, the extra dummy bids will be added to make sure that bidder i submits exactly M bids. So for any bidder i, his bidding vector could be written as $\mathbf{b}^{(i)} = \{b^{(i,1)}, \ldots, b^{(i,M)}\}$.

Similarly, if bidder i's j^{th} bidding price is the k^{th} highest among all the bids of bidders, i would be on slot k and the utility of bidding $b^{(i,j)}$ could be represented as

$$u_k^{i,j} = (v^{(i)} - b_{k+1}) \cdot c_k \quad .$$

As a result, the total utility of bidder i submitting $\mathbf{b}^{(i)}$ is

$$u^{(i)} = \sum_{j=1}^{M} u^{(i,j)} \quad .$$

Additionally, the following lemma states that each bidder would never overbid his true value in the sponsored keyword auctions. Let $\mathbf{b}^{(-i)} = (\mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(i-1)}, \mathbf{b}^{(i+1)}, \ldots, \mathbf{b}^{(N)})$. $u^{(i)}(\mathbf{b})$ represents the total utility of bidder i given all the bidders' bidding vector $\mathbf{b}$.

Lemma 2.1. *In M-GSP, $\forall i \in \mathcal{N}$, for any fixed $\mathbf{b}^{(-i)}$, if*

$$\bar{\mathbf{b}}^{(i)} \in \arg \max_{\{\mathbf{b}^{(i)} | b^{(i,j)} \leq v^{(i)}, \forall j \in \{1,\ldots,M\}\}} u^{(i)}(\mathbf{b}^{(-i)}, \mathbf{b}^{(i)}) \quad ,$$

then

$$\bar{\mathbf{b}}^{(i)} \in \arg \max_{\mathbf{b}^{(i)}} u^{(i)}(\mathbf{b}^{(-i)}, \mathbf{b}^{(i)}) \quad .$$

As a result, our model adopts the similar assumption in [13].

Assumption 2.2. (Non-overbidding strategy) In M-GSP, $\forall i \in \mathcal{N}$, $\forall j \in \{1, \ldots, M\}$, $b^{(i,j)} \leq v^{(i)}$.

At last, we give the formal definition of pure Nash equilibrium in M-GSP.

Definition 2.3. ***(Pure Nash equilibrium)*** *In M-GSP, the pure Nash Equilibrium is a set of biddings $\widehat{\mathbf{b}} = (\widehat{\mathbf{b}}^{(1)}, \widehat{\mathbf{b}}^{(2)}, \ldots, \widehat{\mathbf{b}}^{(N)})$, in which $\forall i$, $\widehat{\mathbf{b}}^{(i)} \in \arg\max_{\mathbf{b}^{(i)}} u^{(i)}(\widehat{\mathbf{b}}^{(-i)}, \mathbf{b}^{(i)})$.*

In other words, no bidder can benefit from changing any of his or her bids unilaterally. It should be note that since the bidder could decrease one of his bids from some value to 0 or increase one of his bids from 0 to some value, no bidder could benefit even from adding or removing any bids unilaterally in any pure Nash equilibrium.

3 The Existence of Nash Equilibrium

In this section, we focus on the existence of pure Nash equilibrium in GSP auction with multiple bidding strategy.

3.1 Preliminaries

Firstly, The following lemma gives some necessary conditions for the existence of Nash equilibrium, which is helpful to verify the (non)existence of Nash equilibrium later.

Lemma 3.1. ***(Necessary conditions)*** *If there exists a pure Nash equilibrium $\widehat{\mathbf{b}}$ in M-GSP, then the following propositions must be true.*

1. *If $v^{(i)} \neq v^{(j)}$ for any $i, j \in \mathcal{N}$, then bidder i gets at least one slot except slot K (the last slot) in $\widehat{\mathbf{b}} \Rightarrow$ bidder i gets exactly M slots in $\widehat{\mathbf{b}}$;*
2. *$\forall i \in \mathcal{N}$, bidder i gets slot $k, k+1, \ldots, k+l$ ($l < m$) in $\widehat{\mathbf{b}} \Rightarrow b_{k+1} = b_{k+2} = \cdots = b_{k+l+1} + \varepsilon$ for arbitrarily small $\varepsilon > 0$;*
3. ***(Winner monotone)*** *[14] $\forall i, j \in \mathcal{N}$, $v^{(i)} < v^{(j)}$ and bidder i gets at least one slot in $\widehat{\mathbf{b}} \Rightarrow$ bidder j must also gets at least one slot in $\widehat{\mathbf{b}}$;*
4. *If the owner of slot K is bidder i, and $\bar{v} = \max\{v^{(j)} | j \neq i$ and j gets less than M slots in $\widehat{\mathbf{b}}\}$, then $b_K \geq \bar{v}$.*

3.2 Simple Setting

We first consider a simple setting where $K = 3$, $M = 2$. I.e, there are totally 3 slots and each bidder can submit 2 bids. Let the three slots be slot 1, 2, and 3 with CTR $c_1 > c_2 > c_3$. Assume N bidders compete for these three slots and $v^{(1)} > v^{(2)} > \cdots > v^{(N)}$. According to Condition 1 and 3 of Lemma 3.1, in the equilibrium, the winners must be bidder 1 and bidder 2. Either bidder 1 gets slot 1, 2 and bidder 2 gets slot 3 or bidder 2 gets slot 1, 2 and bidder 1 gets slot 3. By Condition 2 and 4 of Lemma 3.1, in equilibrium, bidder 3 bids $b_4 \leq v^{(3)}$, and $b_2 = b_3 \geq v^{(3)}$. Now assume $b_2 = b_3 = x$.

Case I: Bidder 1 gets slot 1, 2 and bidder 2 gets slot 3.

This allocation is an equilibrium if and only if for some fixed $c_1 > c_2 > c_3$, $v^{(1)} > v^{(2)} > v^{(3)}$, all the following inequalities are satisfied.

$$\begin{aligned} (v^{(1)} - x)c_2 &\geq (v^{(1)} - b_4)c_3 \\ (v^{(1)} - x)(c_1 + c_2) &\geq (v^{(1)} - b_4)(c_2 + c_3) \\ (v^{(2)} - b_4)c_3 &\geq (v^{(2)} - x)(c_2 + c_3) \\ b_4 &\leq v^{(3)} \\ v^{(2)} \geq x &\geq v^{(3)} \end{aligned} \tag{3.1}$$

Case II: Bidder 2 gets slot 1, 2 and bidder 1 gets slot 3.

Similarly, it is an equilibrium if and only if all the following inequalities are satisfied.

$$\begin{aligned} (v^{(2)} - x)c_2 &\geq (v^{(2)} - b_4)c_3 \\ (v^{(2)} - x)(c_1 + c_2) &\geq (v^{(2)} - b_4)(c_2 + c_3) \\ (v^{(1)} - b_4)c_3 &\geq (v^{(1)} - x)(c_2 + c_3) \\ (v^{(1)} - b_4)c_3 &\geq (v^{(1)} - v^{(2)})(c_1 + c_2) \\ b_4 &\leq v^{(3)} \\ v^{(2)} \geq x &\geq v^{(3)} \end{aligned} \tag{3.2}$$

Thus, an equilibrium exists in this setting if and only if one of the above inequality set is satisfied.

Let $A = \{x | x$ is a feasible solution to inequality set (3.1)$\}$, $B = \{x | x$ is a feasible solution to inequality set (3.2)$\}$. We observe that $B \subset A$. Thus the case $K = 3$, $M = 2$ has an equilibrium if and only if the inequality set (3.1) has a solution. By solving the inequality set (3.1), we obtain the following proposition.

Proposition 3.2. *For $K = 3$, 2-GSP has equilibria if and only if one of the following inequality satisfies,*

1. $\frac{v^{(1)} - v^{(2)}}{v^{(1)} - v^{(3)}} \geq \frac{c_3^2}{c_2^2}, c_1 c_3 \geq c_2^2$;
2. $\frac{c_1 - c_3}{c_1 + c_2} v^{(1)} + \frac{c_2 + c_3}{c_1 + c_2} v^{(3)} \geq \frac{c_2}{c_2 + c_3} v^{(2)} + \frac{c_3}{c_2 + c_3} v^{(3)}, c_1 c_3 \leq c_2^2$,

where $c_1 > c_2 > c_3$ are CTRs of the three slots and $v^{(1)} > v^{(2)} > v^{(3)}$ are the three highest private values among all the bidders.

When some bidders share a same value, it could be verified similarly that the above proposition is still true.

3.3 Existence of Pure Nash Equilibria

When the number of submitted bids of each bidder is unlimited, the following theorem shows that there always exist a pure Nash equilibrium.

Theorem 3.3. *M-GSP always has pure Nash equilibria when $M \geq K$.*

Theorem 3.4. *(Revenue in M-GSP) In M-GSP, the auctioneer's revenue is $R = v^{(2)} \sum_{j=1}^{K} c_j$ in any equilibrium when $M \geq K$.*

Obviously, the revenue under this situation is trivially equal to the revenue under VCG mechanism.

3.4 Non-existence of Nash Equilibria

Now, we explore the case where the maximum number of allowed submitted bids of each bidder is less than the number of advertising positions. We first fix $M = 2$ and study the relationship between the number of slots and the (non)existence of Nash equilibrium. The important lemmas obtained are as follows.

Lemma 3.5. ***(The (non)Existence of Nash Equilibrium when** $M = 2$**)*** *2-GSP always has Nash equilibria for any $K \leq 2$; 2-GSP doesn't always have a Nash equilibrium for any $K \geq 3$.*

After studying the case of $M = 2$, we try to generalize our observation to any M and we get some interesting results as follows.

Algorithm 1. Counter Example Generator (K, M)

if $(K \leq M)$ **then**
    exit
end if
if $(K/M == 2)$ **then**
    Let $a = 0$
else
    Let $a = \lfloor K/M \rfloor - 1$
end if
Let the click through rates of K slots be $\mathbf{c} = (c_1, c_2, \cdots, c_K)$
for $i = 1 : aM + M - 2$ **do**
    Let $c_i = 200 + aM + M - 2 - i$
end for
Let $c_{aM+M-1} = 20$, $c_{aM+M} = 11$ and $c_{aM+M+1} = 10$
for $i = aM + M + 2 : K$ **do**
    Let $c_i = 10^{-i}$
end for
Let the true values of $a + 3$ bidders be $\mathbf{v} = (v^{(1)}, v^{(2)}, \cdots, v^{(a+3)})$
for $i = 1 : a$ **do**
    Let $v^{(i)} = 6 + a - i$
end for
Let $v^{(a+1)} = 5, v^{(a+2)} = 4, v^{(a+3)} = 1$
Output $\mathbf{c}$, $\mathbf{v}$

Algorithm 1 is a counter example generator. For any input $K > M$, the algorithm will output K slots with click through rates $c_1 > c_2 > \cdots > c_K$ and $\lfloor K/M \rfloor + 2$ bidders with true values $v^{(1)} > v^{(2)} > \cdots > v^{(\lfloor K/M \rfloor + 2)}$. And in fact, there doesn't exist a Nash equilibrium for multiple-bidding position auctions with input $\mathbf{c}, \mathbf{v}$ generated by the above algorithm. So we called the algorithm *Counter Example Generator*. The idea of the algorithm comes from the proof of lemma 3.5. In order to prove there may not exist a Nash equilibrium for the case: $K = 4$ in lemma 3.5, we add one more slot with very small click through rate based on the counter example for $K = 3$. And later, to prove the nonexistence of Nash equilibrium for $K > 4$, we add some more slots with very large click through rates. Similarly, here we add $K - 3$ more slots with very large or very small click through rates based on $c_i = 20, c_{i+1} = 11, c_{i+2} = 10$ for some i and add $\lfloor K/M \rfloor - 1$ more bidders with high true values to $v^{(\lfloor K/M \rfloor)} = 5, v^{(\lfloor K/M \rfloor + 1)} = 4, v^{(\lfloor K/M \rfloor + 2)} = 1$. It's easy to verify the correctness of the algorithm. And from the above algorithm, we obtain the following lemma.

Lemma 3.6. *M-GSP doesn't always have a Nash equilibrium when $M < K$.*

From the above two lemmas, we obtain the theorem of (non)existence of Nash equilibrium as follows.

Theorem 3.7. ***(The (non)Existence of Nash Equilibrium)*** *M-GSP always has Nash equilibria when $M \geq K$; It doesn't always have a Nash equilibrium when $M < K$.*

4 Only One Bidder Multi-bidding

In the above sections, we study the M-GSP model in which every bidder can submit at most M bids in the auction. We prove that when the number of slots $K > M$, there doesn't always exist a Nash equilibrium in that model. Now consider the case that only one bidder can submit at most M bids and each of the other bidders can only submit one bid, i.e., only one bidder multi-bidding. Does there always exist a Nash equilibrium in this situation? In this section, we try to find the solution to this question. We use $M^{(i)}$-GSP to represent the extended GSP model in which only bidder i is allowed submitting at most M bids, and each of the other bidders can only submit one bid. In the following part, without loss of generality, we always assume $v^{(1)} \geq v^{(2)} \geq \cdots \geq v^{(N)}$.

Lemma 4.1. ***(The (non)Existence of Nash Equilibrium when $M = 2$)***

1. *$\forall\ i > K$, $2^{(i)}$-GSP always has Nash equilibria;*
2. *$\forall\ i \leq K$, $2^{(i)}$-GSP always has Nash equilibria for any $K \leq 2$; $2^{(i)}$-GSP doesn't always have a Nash equilibrium for any $K \geq 3$.*

Next, we relax the constraint $M = 2$ and study the existence of Nash equilibrium in $M^{(i)}$-GSP model for any given M.

Theorem 4.2. ***(The (non)Existence of Nash Equilibrium in $M^{(i)}$-GSP)***

1. *$\forall\ i > K$, $M^{(i)}$-GSP always has Nash equilibria;*
2. *$\forall\ i \leq K$, $M^{(i)}$-GSP always has Nash equilibria for any $K \leq 2$; $M^{(i)}$-GSP doesn't always have a Nash equilibrium for any $K \geq 3$.*

5 An Impossibility Result

As we mentioned, the sponsored keyword auction is not incentive compatible. However, if we replace the allocation method and pricing method in the position auction by other methods, does there exist a mechanism satisfying not only truthful but also multiple-bidding proof? The following theorem answers the question.

Theorem 5.1. *There exists no mechanism (with allocation method and pricing method) which is truthful, multiple-bidding proof, social efficiency and individual rationality.*

6 Multiple Biddings of Players with Multiple Private Values

Up to this point, we have regarded that each bidder's value per-click is the same for all the submitted bids. It may more adequately called the single value multi-bidding model. In general, there could be a multi-value multi-bidding model. In the multi-value multi-bidding model, a bidder submits more than one bid and his value per-click is not unique. For example, a computer company sells both business laptops and home laptops. The company wants to buy two slots for the keyword 'laptop', one for his business laptops and the other for his home laptops. The value per click for the advertise of the business laptops may be different from that of the home laptops. The company cares about his total profit and in the auction he can always cooperate with himself. And indeed it is often the case as in the examples of the multiple advertisement displays of Dell and HP in Figure 1.

Proposition 6.1. *(Individual Efficiency Property) At any Nash equilibrium, an agent's winning biddings are ordered in their privates values for the corresponding advertisements. That is, the higher is the private value, the higher is the bid.*

In the single-value multi-bidding GSP auction, there may not always exist a Nash equilibrium. Unfortunately, there may not exist a Nash equilibrium in the multi-value multi-bidding GSP auction neither. This can be shown by the following counter example.

	Business Laptop	Home Laptop
Merchant A	10	8
Merchant B	9	6

For the same keyword both Merchant A and B bid for two slots, one for business laptop and the other for home laptop. Their values per-click are shown in the above table. However, there are totally three slots with $c_1 = 12, c_2 = 11, c_3 = 10$. In this setting, there doesn't exist a Nash equilibrium.

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